Decay mechanisms in bound state interaction kernels

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Der Wissenschaftsfonds.

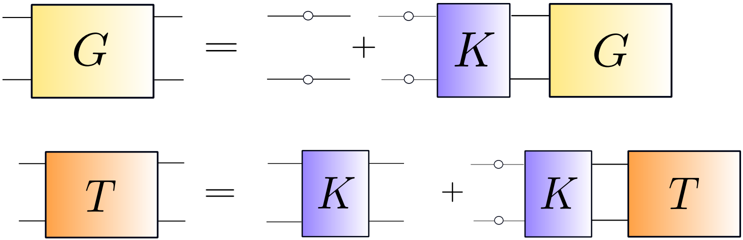
Outline

Bethe Salpeter equations

Properties of bound states are encoded by a set of n-point Green's functions.

- Mesons as a bound states of $q\bar{q} \longrightarrow$ four point functions.
- Baryons as a bound states of $qqq \longrightarrow$ six point functions.

We will concentrate in mesons.



If the n-particles form a bound state a pole appears in the Green function for $P^2 = -M^2$

$$G \to \frac{\Psi\bar{\Psi}}{P^2 + M^2}$$

with Ψ the wave function.

$$\Psi = KG_0\Psi$$

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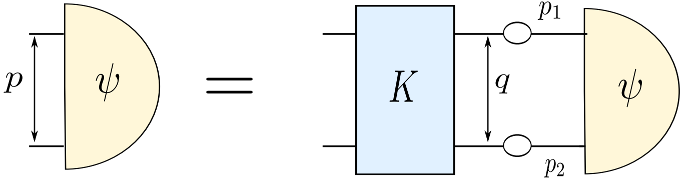
$$\Psi = K G_0 \Psi$$

- G_0 is the product of dressed quark propagator.
- Bethe Salpeter equations (BSEs): homogeneous equation for Ψ.
- The interaction kernel contains an infinite sum of diagrams.

Bethe Salpeter equations

Diagrammatic BSE

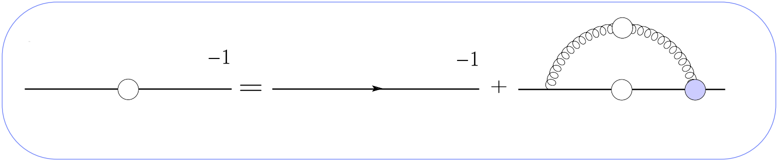


Ingredients:

- Dressed quark propagator,
- Interaction Kernel.

Dyson-Schwinger equations (DSEs)

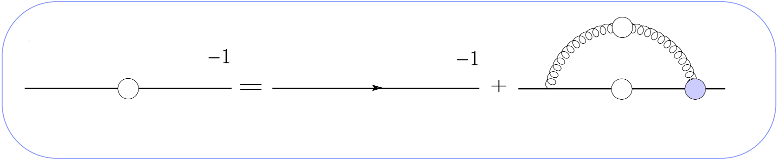
Quark propagator DSE



- The loop diagram contains the sum of all possible processes where the quarks emits and reabsorbs a gluon.
- The gluon is fully dressed.
- Dressed quark-gluon vertex sum all possibles vertex corrections.

Dyson-Schwinger equations (DSEs)

Quark propagator DSE



- Gluon propagator $D^{\mu\nu}$
- Quark-gluon vertex Γ^{μ}
- In most phenomenological applications, the quark DSE has been truncated

Rainbow ladder truncation

- Preserve chiral symmetry,
- Quark-gluon vertex $\Gamma^{\mu} \sim \gamma^{\mu}$
- Full gluon propagator $D^{\mu\nu} \sim \alpha(k^2)\frac{T^{\mu\nu}}{k^2}$, $\quad T^{\mu\nu} = \left(\delta^{\mu\nu} - \frac{k^{\mu}k^{\nu}}{k^2}\right)$
- Collect the dressings in an effective coupling $\alpha(k^2)$.

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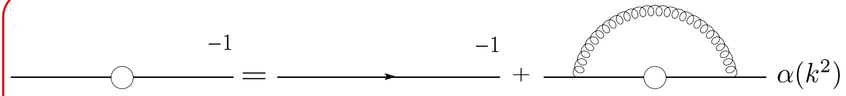
DSE quark propagator with truncation:



One frequently used effective interactions is the Maris-Tandy model[1]

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right)^2 \exp^{-\eta^2 \frac{k^2}{\Lambda^2}} + \alpha_{UV}$$

- Reproduces the one-loop QCD behaviour of the quark propagator at large momenta,
- Enough strength for dynamical chiral symmetry breaking to take place.
- Energy scale $\Lambda = 0{,}72$
- $\eta = 1{,}8$

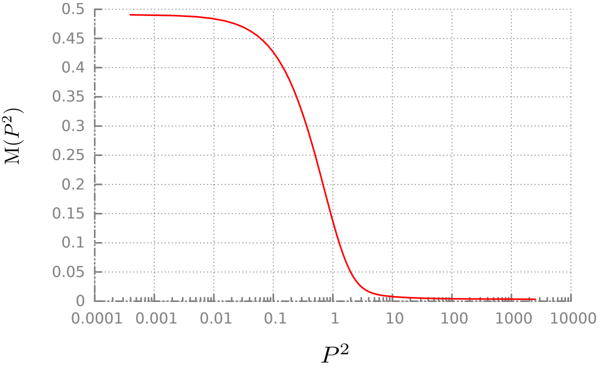
Λ and η fitted to reproduce the decay constant from pion BSE.

[1] P. Maris and C. Tandy, Phys. Rev. C60 (1999) 055214

Dressed propagator is given by

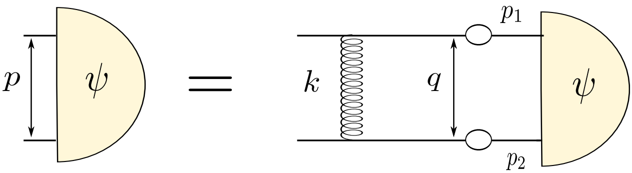
$$S(p) = \frac{1}{A(p^2)} \frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)} = -i\not{p}\sigma_v(p^2) + \sigma_s(p^2);$$

Renormalization conditions: $A(\mu^2) = 1, \quad M(\mu^2) = m_q, \quad \mu = 19GeV$



- The quark mass function encodes dynamical chiral symmetry breaking and displays the transition from constituent quark mass to current quark mass.

We need to specify the interaction kernel. We use the BSE with rainbow ladder truncation



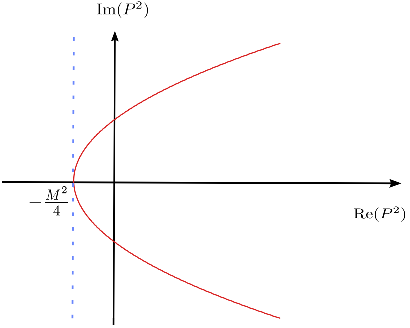
- The resulting BSE kernel is a gluon exchange.
- Can be solved numerically.

$$\Psi(p,P) = \int \frac{d^4q}{(2\pi)^4} \gamma^\mu S(p_1) \Psi(q,P) S(p_2) \gamma^\nu D_{\mu\nu}(k)$$

For technical reasons we work in euclidean space time.

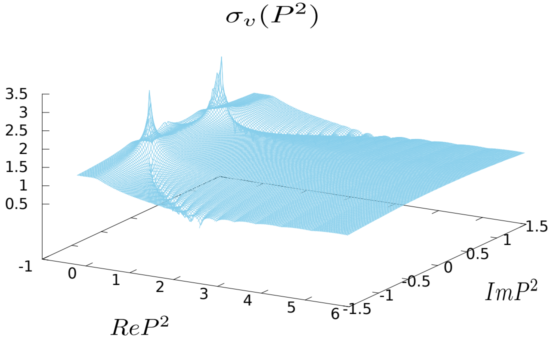
- In euclidean space to get $P^2 = -M^2$ for bound states we need to use complex momentum,
- Therefore we need the propagator for complex momentum.

The propagators are sampled within complex parabolas



The propagator carries a singularity structure

- Complex conjugate poles



- Rainbow ladder truncation + Maris-Tandy model works very well for ground states. [2] [3]

[2] G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, C. S. Fischer. Baryons as relativistic three quark bound states. (arXiv:1606.09602 [hep-ph])

[3] T. Hilger, M. Gmez-Rocha, A. Krassnigg, W. Lucha. Aspects of open-flavour mesons in a comprehensive DSBSE study. arXiv:1702.06262 (2017)

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Most hadrons are resonances and they decay

- $\rho \to \pi\pi$
- $\Delta \to N\pi$

A complete description of hadrons should incorporate these properties.

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The main goal of the PhD thesis project will be to develop BSE kernels that contain explicit decay mechanisms.

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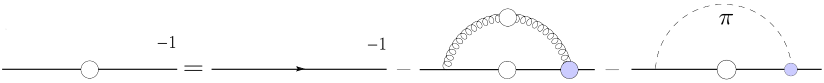
Beyond the Rainbow: Pion Exchange

We will introduce explicit pionic degrees of freedom in the system, in addition to quarks and gluons. Besides the gluon part of the quark DSE, an emission and absorption of the pion appears[4,5]



[4] H. Sanchis-Alepuz, C. S. Fischer, S. Kubrak, Phys. Lett. B733,151 (2014)

[5] C. S. Fischer, R. Williams, Phys. Rev. D78, 074006 (2008)

The corresponding Bethe-Salpeter equation that we need to solve is,

$$\Psi(p;P) = \int \frac{d^4k}{(2\pi)^4} \left[K^{gluon}(p,k;P) + K^{pion}(p,k;P)\right] \left[S(k_1)\Psi(k;P)S(k_2)\right]$$

with the the kernel

$$\begin{aligned} K^{pion}_{tu,sr}(q,p;P) &= \frac{1}{4}[\Gamma^j_\pi]_{ru}\left(\frac{p+q-P}{2};p-q\right)[Z_2\tau^j\gamma^5]_{ts}D_\pi(p-q) \\ &+ \frac{1}{4}[\Gamma^j_\pi]_{ru}\left(\frac{p+q-P}{2};q-p\right)[Z_2\tau^j\gamma^5]_{ts}D_\pi(p-q) \\ &+ \frac{1}{4}[\Gamma^j_\pi]_{ts}\left(\frac{p+q-P}{2};p-q\right)[Z_2\tau^j\gamma^5]_{ru}D_\pi(p-q) \\ &+ \frac{1}{4}[\Gamma^j_\pi]_{ts}\left(\frac{p+q-P}{2};q-p\right)[Z_2\tau^j\gamma^5]_{ru}D_\pi(p-q) \end{aligned}$$

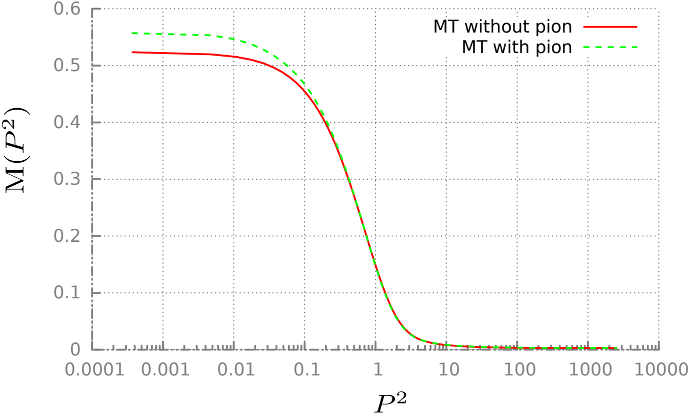
We approximate the pion Bethe-Salpeter amplitude in the quark DSE and the kernel of the BSE by the leading amplitude in the chiral limit,

$$\Gamma_\pi^j(q;P) = \tau^j \gamma_5 \frac{B(p^2)}{f_\pi}$$

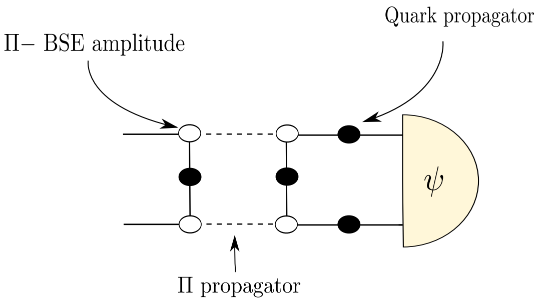
For the pion corrected kernel we fit $\Lambda = 0{,}84$ GeV to reproduce f_π.

	MT	MT + pion	MT + pion (refitted)	PDG
m_π	140	145	138	138

Quark mass function as a function of the squared momentum.



Next step, ρ meson



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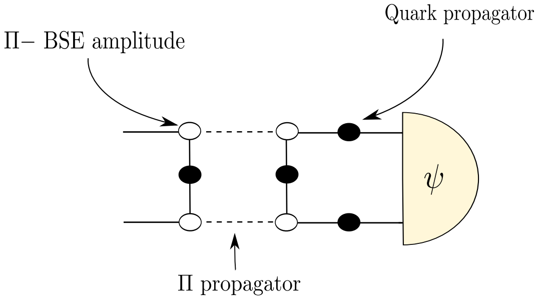

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- Dealing with them will be very difficult.

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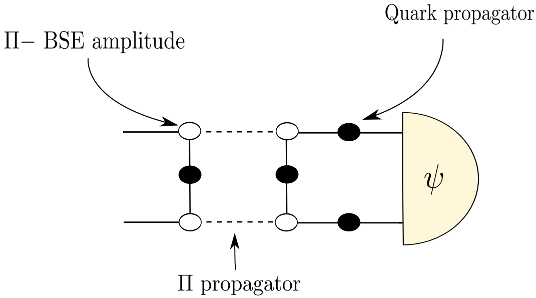


- Potentially a lot of singularities,
- Dealing with them will be very difficult.
- Check whether analytic parametrizations of quark propagators, BSE amplitudes, etc. capture in afaithful enough manner the necessary physics to describe bound state properties.

Poles parametrization

One way to represent the quark propagator is based on a complex conjugate pole model.

$$S(p) = \frac{1}{A(p^2)} \frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)} = -i\not{p}\sigma_v(p^2) + \sigma_s(p^2)$$

$$\sigma_v = \sum_i^n \left[\frac{\alpha_i}{p^2 + m_i} + \frac{\alpha_i^*}{p^2 + m_i^*} \right]$$

$$\sigma_s = \sum_i^n \left[\frac{\beta_i}{p^2 + m_i} + \frac{\beta_i^*}{p^2 + m_i^*} \right]$$

where the parameters m_i, α_i, β_i can be obtained by fitting the corresponding solution along the real axis of p^2

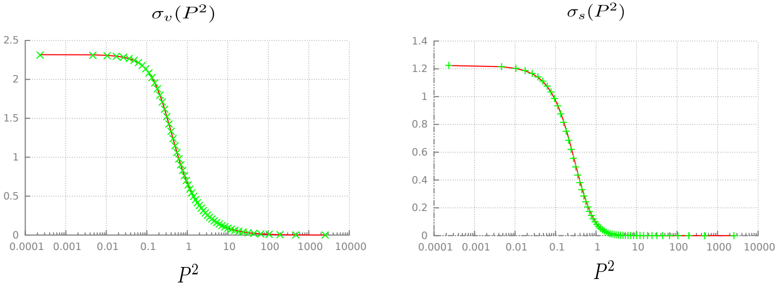
$$\sigma_v = \sum_i^n \left[\frac{\alpha_i}{p^2 + m_i} + \frac{\alpha_i^*}{p^2 + m_i^*} \right]$$

$$\sigma_s = \sum_i^n \left[\frac{\beta_i}{p^2 + m_i} + \frac{\beta_i^*}{p^2 + m_i^*} \right]$$

The complex conjugate pole parameters are ($i = 3$)

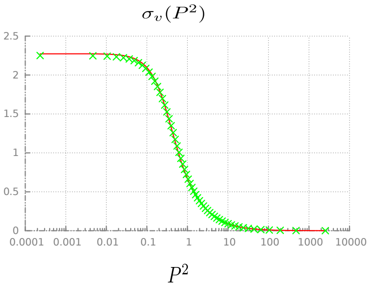
	Re	Im
m_1	-0.22	$\pm$ 0.39
m_2	-0.39	$\pm$ 0.53
m_3	-1.34	$\pm$ 1.61

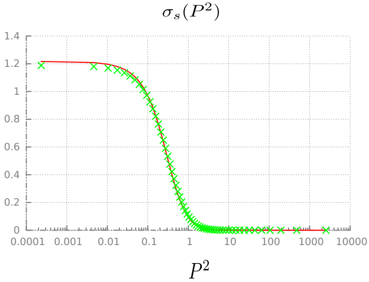
Fit along the parabola.



Green points corresponds to the fit

Fit along the real axis.





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- The knowledge of the position of the singularities in the complex plane will allow one to develop effective algorithms adequate for numerical calculations.
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Feynman parametrization:

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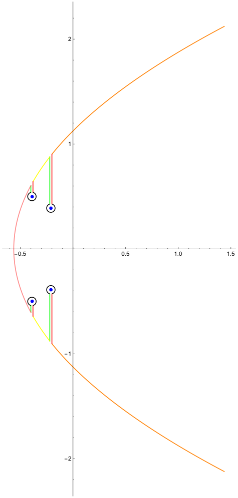
- Solving the BSE with the pole ansatz together with the Feynman parametrization we find agreement for the pion mass.

- Contour deformation [6] [7] [8]



[6] G. Eichmann, C. Fischer, E. Weil, and R. Williams. On the large-Q^2 behavior of the pion transition form factor, arXiv:1704.05774 [hep-ph].

[7] G. Eichmann. PhD thesis.

[8] E. Weil, G. Eichmann, C. S. Fischer, and R. Williams. Electromagnetic decays of the neutral pion, arXiv:1704.06046 [hep-ph].

Outlook

Near future

- Compare ground-states masses with true propagators and fits,
- Study all possible non-analyticities of the decay kernel,

Long term

- Compare Form Factors
- Use this kernel in a baryon.

Thank you!

Backup

Diagrammatic representation of the two body kernel including pion exchange contribution