Real-space renormalization group and entanglement

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Non-Perturbative Methods in Quantum Field Theory
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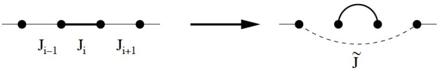
An easy start: Ma-Dasgupta-Hu-Fisher RG

- Let us consider a disordered Heisenberg XXZ chain

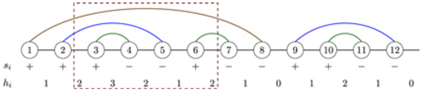
$$H = \sum_i J_i (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta_i S_i^z S_{i+1}^z) \, .$$

- The strong disorder renormalization group (Ma-Dasgupta-Hu, Fisher)



$$\tilde{J} = \frac{J_{i-1} J_{i+1}}{(1+\Delta_i) j_i} \, , \quad \tilde{\Delta} = \frac{(1+\Delta_i)}{2} \Delta_{i-1} \Delta_{i+1} \, .$$

- Continuing this procedure, we end up with a singlet distribution as an approximate ground state - random singlet phase, infinite disorder fixpoint



$$S_L = \frac{\ln 2}{3} \log L \; .$$

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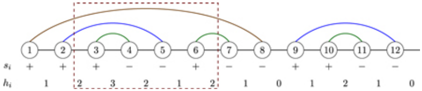
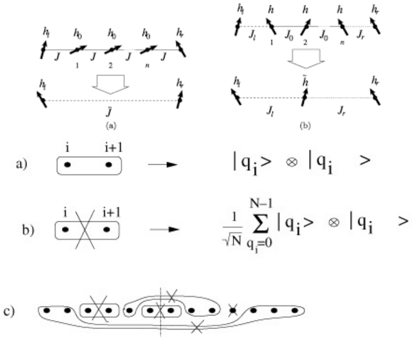


$$S_L = \frac{\ln 2}{3} \log L \,.$$

- The disordered transverse field Ising model

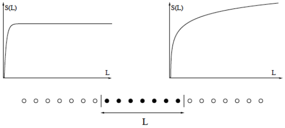
$$H = \sum_i J_i S_i^x S_{i+1}^x + h_i S_i^z \ .$$

- Two types of RG steps



$$S_L = \frac{\ln 2}{6} \log L \ .$$

- The entanglement entropy asymptotics of gapless models
  (C. Holzhey, F. Larsen, F. Wilczek, Nucl. Phys. B 424-443 (1994); Vidal et al. PRL 90, 227902 (2003); P. Calabrese and J. Cardy, JSTAT 06002 (2004); J. Eisert et al, Rev. Mod. Phys. 82, 277 (2010).)



$$S_L = \frac{c}{3} \log L + k$$

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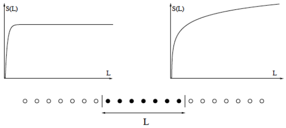

$$S_L = \frac{c}{3} \log L + k$$

Quasi-periodic models: the Professor-Student sequence

- Consider the substitution rule: $L \to LS$; $S \to L$.

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- Professors and students reproduce themselves as rabbits! Actually, the above sequence was originally constructed as a simple model for rabbit reproduction in 1202 by Leonardo of Pisa, also known as Fibonacci.

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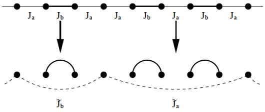
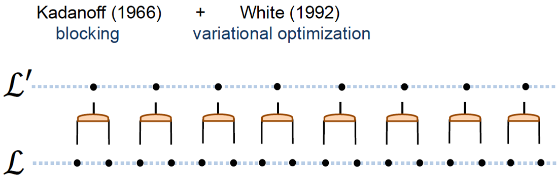
The Fibonacci XXZ chain

- The universality class of the model is not the infinite disorder fixpoint. This quasiperiodic modulation is only marginal.



$$S_L = \frac{\ln 2}{3 \ln 5} \log L \, .$$

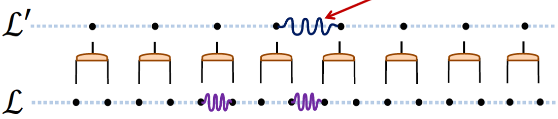
Kadanoff blocking RG in translation-invariant models



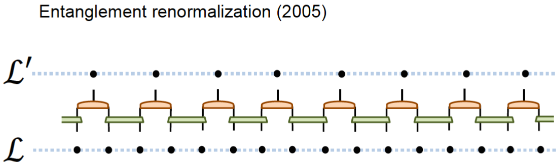
Kadanoff blocking RG

Kadanoff (1966) + White (1992)
blocking variational optimization

failure to remove *some* short-range entanglement !



Entanglement renormalization



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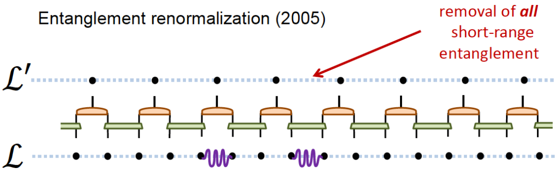
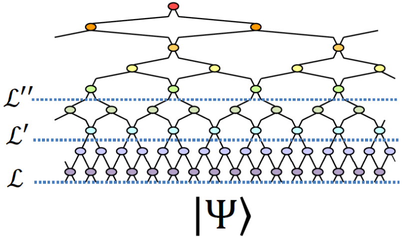


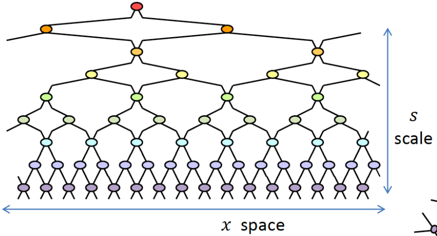
Multiscale Entanglement Renormalization Ansatz

MERA as a sequence of ground state wave-functions



$$|\Psi\rangle \to |\Psi'\rangle \to |\Psi''\rangle \to \cdots$$

Entanglement entropy and correlations



- entanglement entropy

$$S_L \approx \log(L)$$

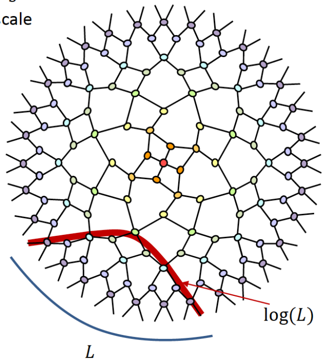
Computation of density matrix requires tracing out $\sim\log(L)$ indices

- two-point correlations

$$C(L) \approx L^{-2\Delta}$$

Geodesic distance $D \approx \log(L)$

$$C(L) \approx e^{-D} = e^{-2\Delta\log(L)} = L^{-2\Delta}$$



An interesting connection to AdS/CFT

Ryu-Takanayagi formula