Inhomogeneous vacuum in bilocal theories

Sándor Nagy, Janos Polonyi, Imola Steib

University of Debrecen, Department of Theoretical Physics, Hungary

Zalakaros, September 21st, 2017

Introduction, motivation

- The functional renormalization group treatment of the bilocal potential was performed
  - needs at least sixth order vertex to have a non-trivial tree-level evolution
  - unusual position of the Wilson-Fisher fixed point
  - momentum dependent couplings
- compare the phase structure of the local and the bilocal potential
- calculate the energy at semiclassical level
- investigate the ground state (or vacuum) of the phases

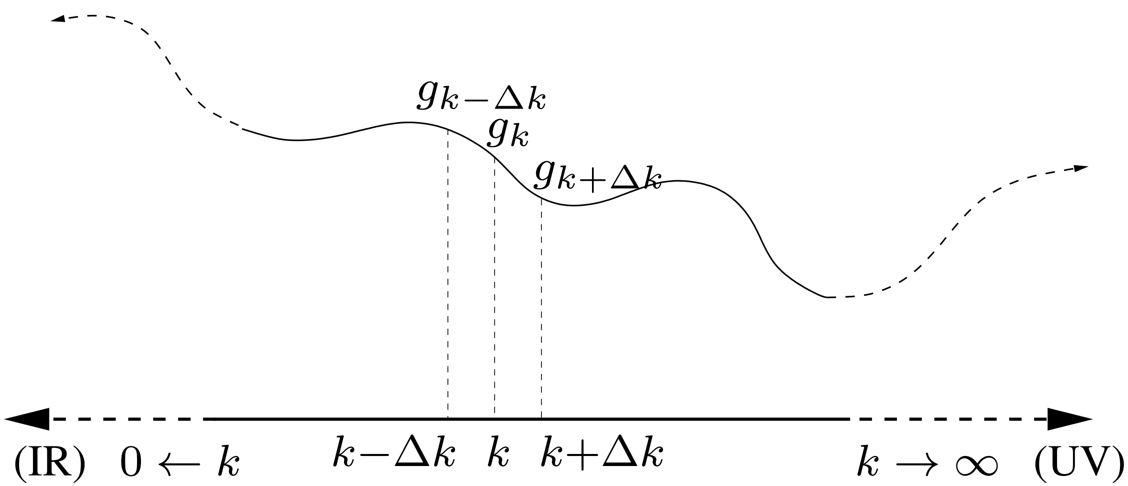
Renormalization group

- The functional renormalization group (RG) method is a non-perturbative method in quantum field theory.
- The RG method can elimininate the UV modes systematically and gives an IR description of the investigated model.

The evaluation of the path integral dresses up the values of the couplings with their corrections coming from the quantum fluctuations. The vacuum to vacuum transition amplitude is

$$Z[j] = \int \mathcal{D}\phi e^{-\frac{\hbar}{i}S_k - \frac{\hbar}{i}j\phi} \equiv \int d\phi_0 \dots d\phi_{k-\Delta k} d\phi_k d\phi_{k+\Delta k} \dots d\phi_\infty e^{-\frac{\hbar}{i}S_k - \frac{\hbar}{i}j\phi}$$

The path integral is performed by removing the modes one-by-one, which gives scale dependent couplings.



Bilocal model

- The Euclidean action for the scalar model with bilocal potential is

$$S = \frac{1}{2}\int_x \phi_x D_0^{-1} \phi_x + \int_x U(\phi_x) + \int_{xy} V_{x-y}(\phi_x, \phi_y)$$

- Local potential

$$U(\phi_x) = \sum_{n=0}^{\infty} \frac{g_n}{n!} \phi_x^n$$

- Bilocal potential

$$V_{x-y}(\phi_1, \phi_2) = \sum_{mn \geq 1} \frac{v_{x-ymn}}{m!n!} \phi_1^m \phi_2^n$$

- The bilocal couplings become momentum dependent

$$v_{x-ymn} \to v_{qmn}$$

Evolution equations

local
$$\begin{cases} \dot{g}_2 = -\alpha_3 k^3 \frac{g_4}{\omega_k^2} - 2\dot{v}_{011} \\ \dot{g}_4 = -\alpha_3 k^3 \frac{g_6}{\omega_k^2} + \alpha_3 k^3 \frac{3g_4^2}{\omega_k^4} - 6\dot{v}_{022} \\ \dot{g}_6 = \alpha_3 k^3 \frac{15 g_4 g_6}{\omega_k^4} - \alpha_3 k^3 \frac{30 g_4^3}{\omega_k^6} \end{cases}$$

bilocal tree-level
$$\left\{ \dot{v}_{q33} = \frac{k}{2\omega_k^2} g_4^2 \delta_{k,q} \right.$$

bilocal, fluctuations
$$\begin{cases} \dot{v}_{011} = -2\alpha_3 \frac{k^3}{\omega_k^2} v_{k22} \\ \dot{v}_{022} = -2\alpha_3 \frac{k^3}{\omega_k^2} v_{k33} \\ \dot{v}_{q11} = -\frac{\alpha_2}{\pi} \frac{k}{\omega_k^2} \int_p v_{p22} \\ \dot{v}_{q22} = -\frac{\alpha_2}{\pi} \frac{k}{\omega_k^2} \int_p v_{p33} \end{cases}$$

with $\omega_k^2 = k^2 + g_2 + 2v_{k11}, \;\; \alpha_d = \frac{\Omega_d}{2(2\pi)^2}$

Semiclassical energy

- In Euclidean spacetime we calculate system energy from the action at semiclassical level
- We consider a plane wave form of the field variable $\phi_x = \phi\cos(px^1)$ with $p = \ell\frac{2\pi}{L}$
- The classical action is

$$\begin{aligned} S &= \frac{1}{2}\int_x \phi_x D_0^{-1}\phi_x + \int_x U(\phi_x) + \int_{xy} V_{x-y}(\phi_x, \phi_y) \\ &= L^d\left[\frac{p^2\phi^2}{4} + s_1 + s_2\right], \end{aligned}$$

where

$$\begin{aligned} s_1 &= \sum_n \frac{g_{2n}\phi^{2n}}{2^{2n}(n!)^2} \\ s_2 &= \sum_{mn} \frac{\phi^{m+n}}{m!n!2^{m+n}} \sum_{j=\max(0,\frac{m-n}{2})}^{\min(m,\frac{m+n}{2})} \binom{m}{j}\binom{n}{\frac{m+n}{2}-j} V_{(2j-m)pmn} \end{aligned}$$

Semiclassical energy

- The action up to sixth order is

$$\begin{aligned} S &= \frac{\phi^2}{4}(q^2 + g_2 + 2v_{q11}) + \frac{\phi^4}{16}\left(\frac{g_4}{4} + \frac{v_{2q22}}{2} + v_{022}\right) \\ &= +\frac{\phi^6}{64}\left(\frac{g_6}{36} + \frac{v_{3q33}}{18} + \frac{v_{q33}}{4}\right) \\ &\equiv a\phi^2 + \frac{b}{2}\phi^4 + \frac{c}{3!}\phi^6 \end{aligned}$$

- The stability requires the condition $c > 0$
- Zeros

$$\begin{aligned} \phi_0 &= 0 \\ \phi_0 &= \frac{-b \pm \sqrt{b^2 - 2ac}}{c} \end{aligned}$$

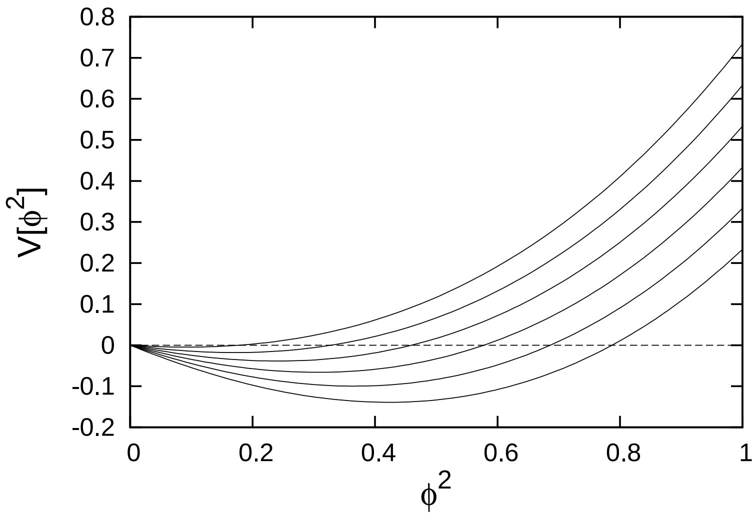
- single zero

$$\begin{aligned} a > 0, &\quad b > 0 \\ a < 0, &\quad b^2 < 2ac \end{aligned}$$

Zeros

one negative and one positive zeros:

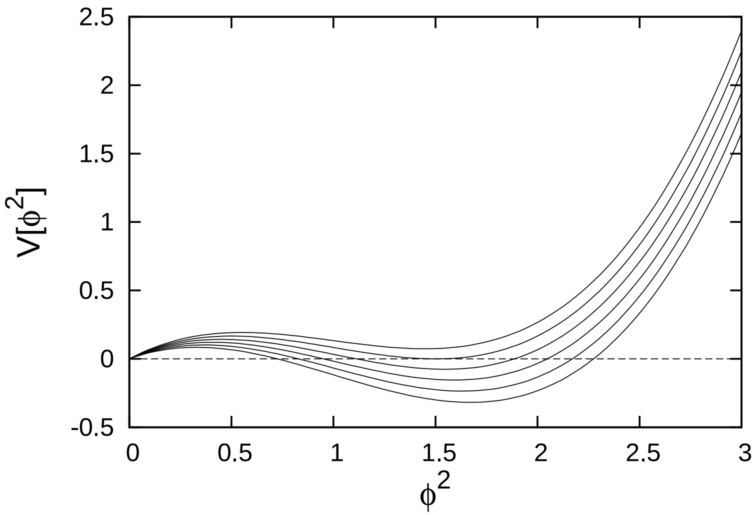
$$a < 0, \quad b^2 > 2ac$$



- the minimum of $V[\phi^2]$ continuously tends to zero
- **second order** phase transition

two positive zeros:

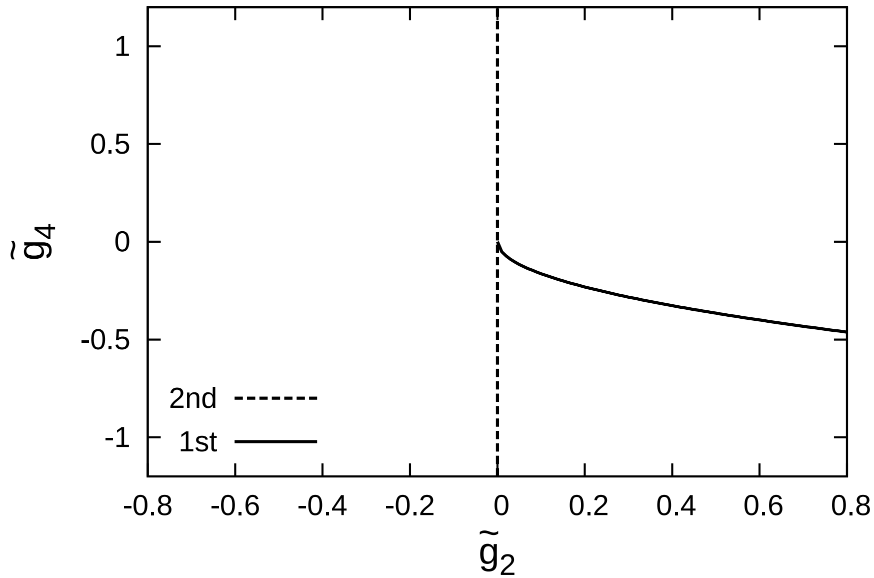
$$a > 0, \quad b < 0, \quad d \equiv b^2 - 8ac/3$$



- the minimum of $V[\phi^2]$ jumps to zero
- **first order** phase transition

Local potential

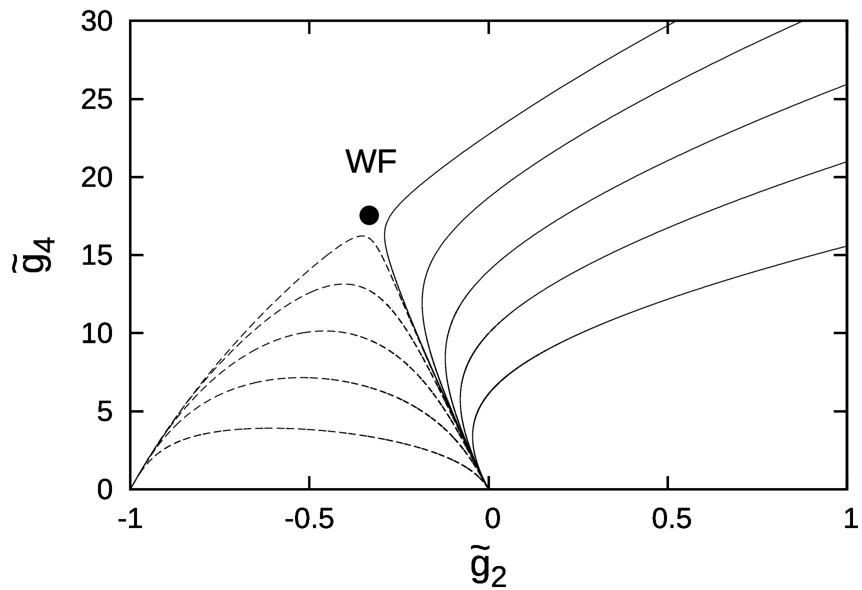
Tree level evolution



- 3d ϕ^4 model
- symmetric phase
- broken symmetric phase(s)
- tricritical point

Local potential

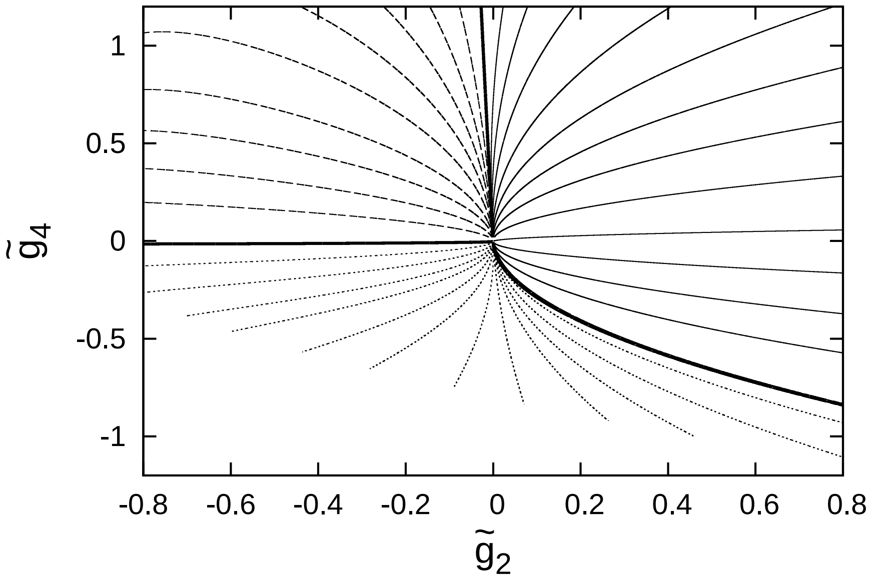
Finite initial values for $\tilde{g}_2(\Lambda)$ and for $\tilde{g}_4(\Lambda)$



- Wegner-Houghton equation (evolution of the Wilsonian action)
- symmetric phase
- broken symmetric phases, **singularity** in the IR $(k^2 + g_2 \to 0)$
- Gaussian, Wilson-Fisher fixed points

Local potential

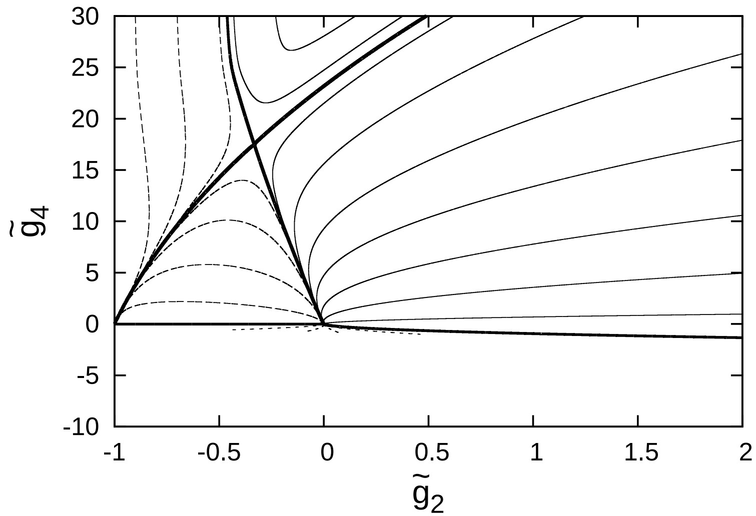
Trajectories around the origin

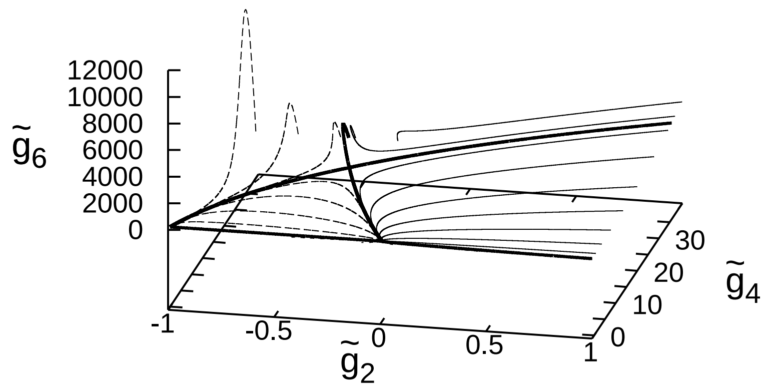


- solid line: symmetric phase
- dashed line: broken symmetric phase **singularity**
- dotted line: broken symmetric phase **instability** $(c < 0)$

Local potential

The phase space

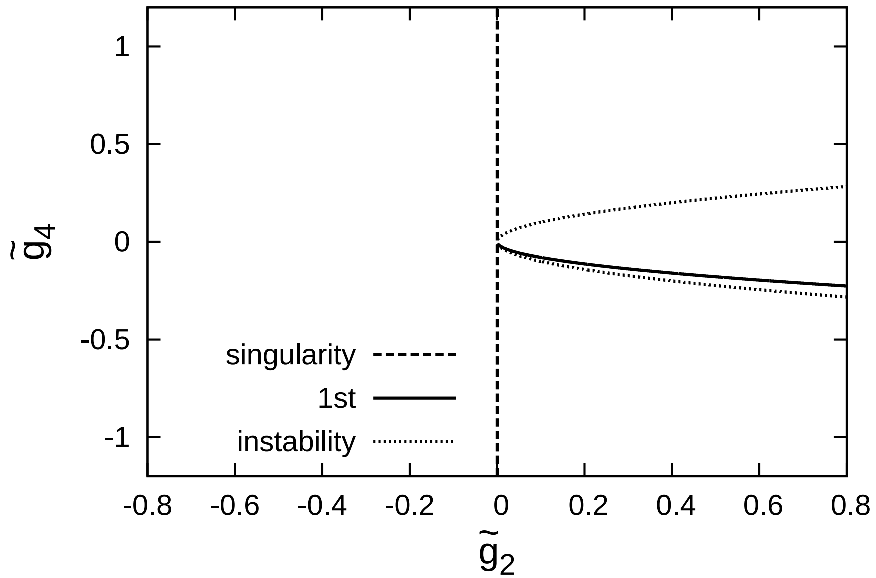




- symmetry
- singularity
- instability

Bilocal potential

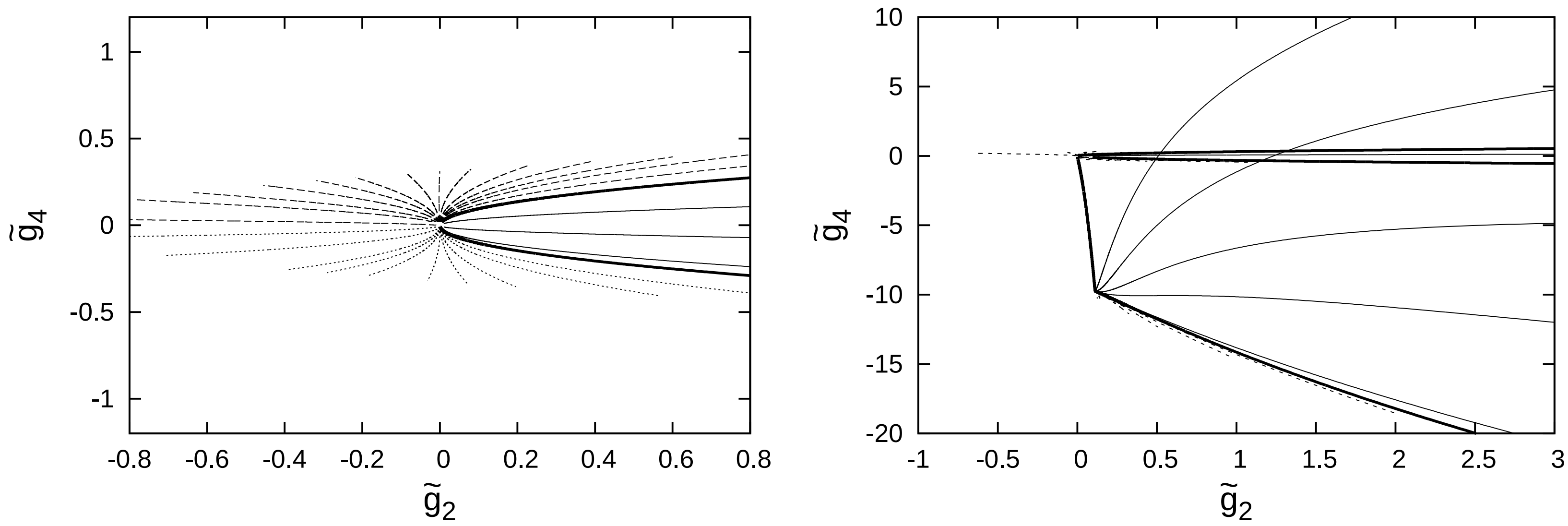
Tree level evolution



- dotted line: instability, $c = 0$
- dashed line: singularity, $a = 0$
- solid line: $d = 0$

Bilocal potential

The phase space



- the thick lines separate the symmetric and the broken symmetric phases ($c = 0$)
- instability, no singularity
- the Wilson-Fisher fixed point is situated around the instability

Inhomogeneous vacuum

- **trivial vacuum**

  symmetric phase: $\langle\phi\rangle = 0$

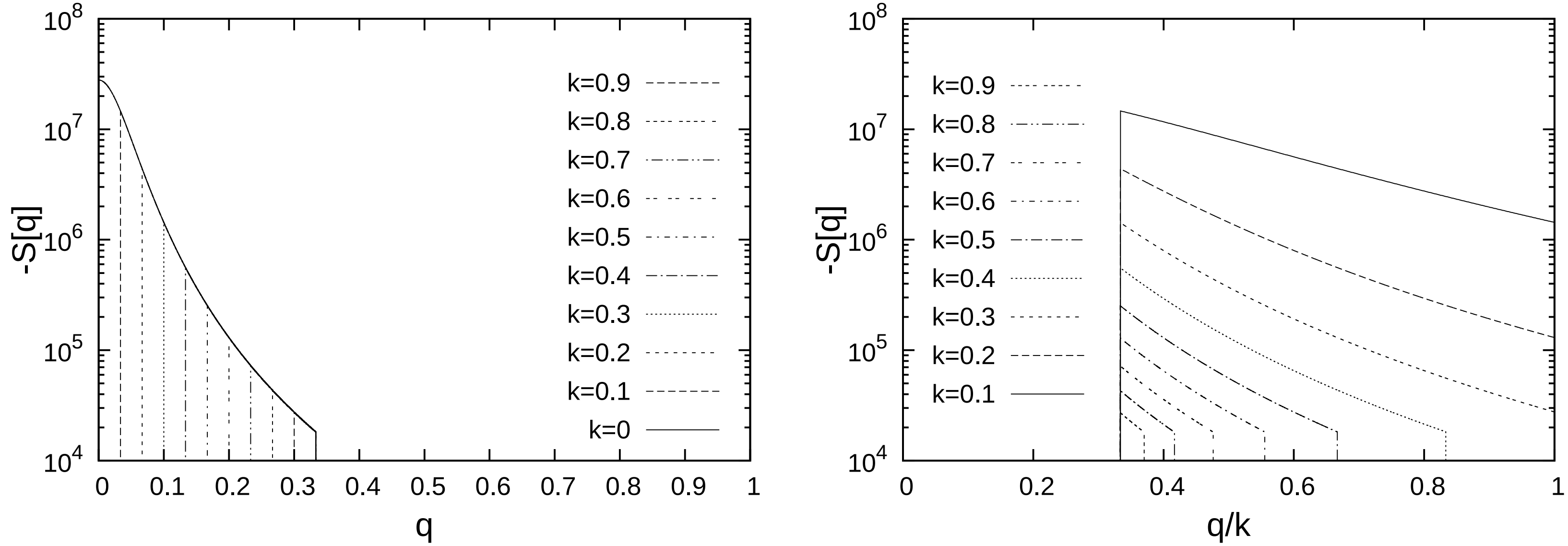
- **homogenenous vacuum**

  broken symmetric phase: $\langle\phi\rangle \neq 0$

  - singularity point: $\tilde{g}_2^* = -1$, $\tilde{g}_4^* = 0$, $\tilde{g}_6^* = 0$ (IR Landau pole)
  - tree level RG is needed to follow the evolution

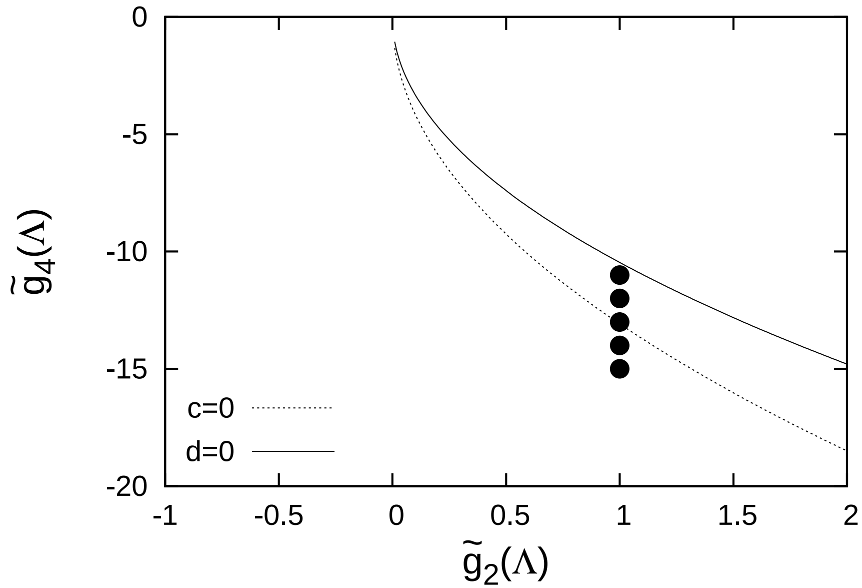
- **inhomogeneous vacuum**
  - when $d > 0$ (two positive zeros) then $S[\phi_0] < S[0]$
  - the energy is momentum dependent, $S[\phi_0(q)] \equiv S[q]$
  - how does it depend on q?
  - if it has nontrivial minimum at finite q
    $\rightarrow$ then we have an inhomogeneous vacuum

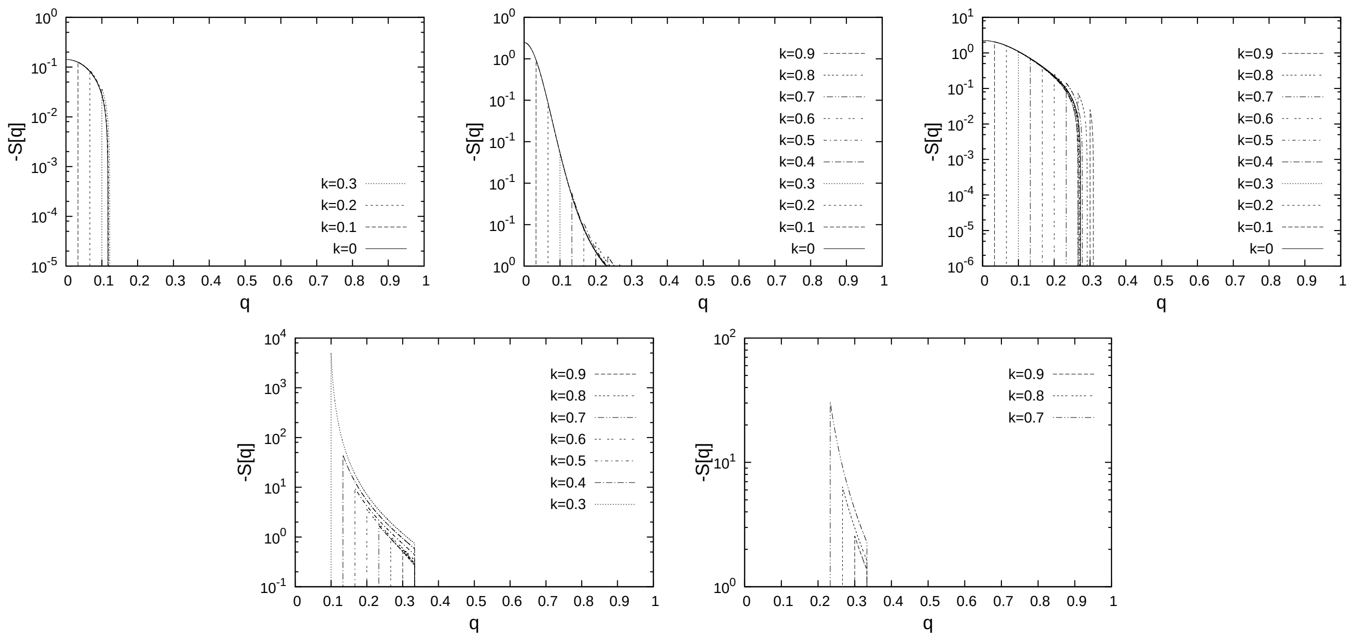
Inhomogeneous vacuum



- no instability: $c > 0$
- non-trivial energy minimum: $d > 0$
- the ratio q/k remains constant during the evolution

Inhomogeneous vacuum





Outlook

- map out the phase structure for the bilocal model with more couplings
- CTP RG equation beyond tree level

Acknowledgments

The financial support from the János Bolyai Programme of the Hungarian Academy of Sciences and the Hungarian National Research Fund OTKA (K112233) is acknowledged.