# The effect of bosonic quantum fluctuations on nuclear equation of state and neutron star observables

arXiv:1610.03674

(Submitted to Phys. Rev. C)


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Péter Pósfay

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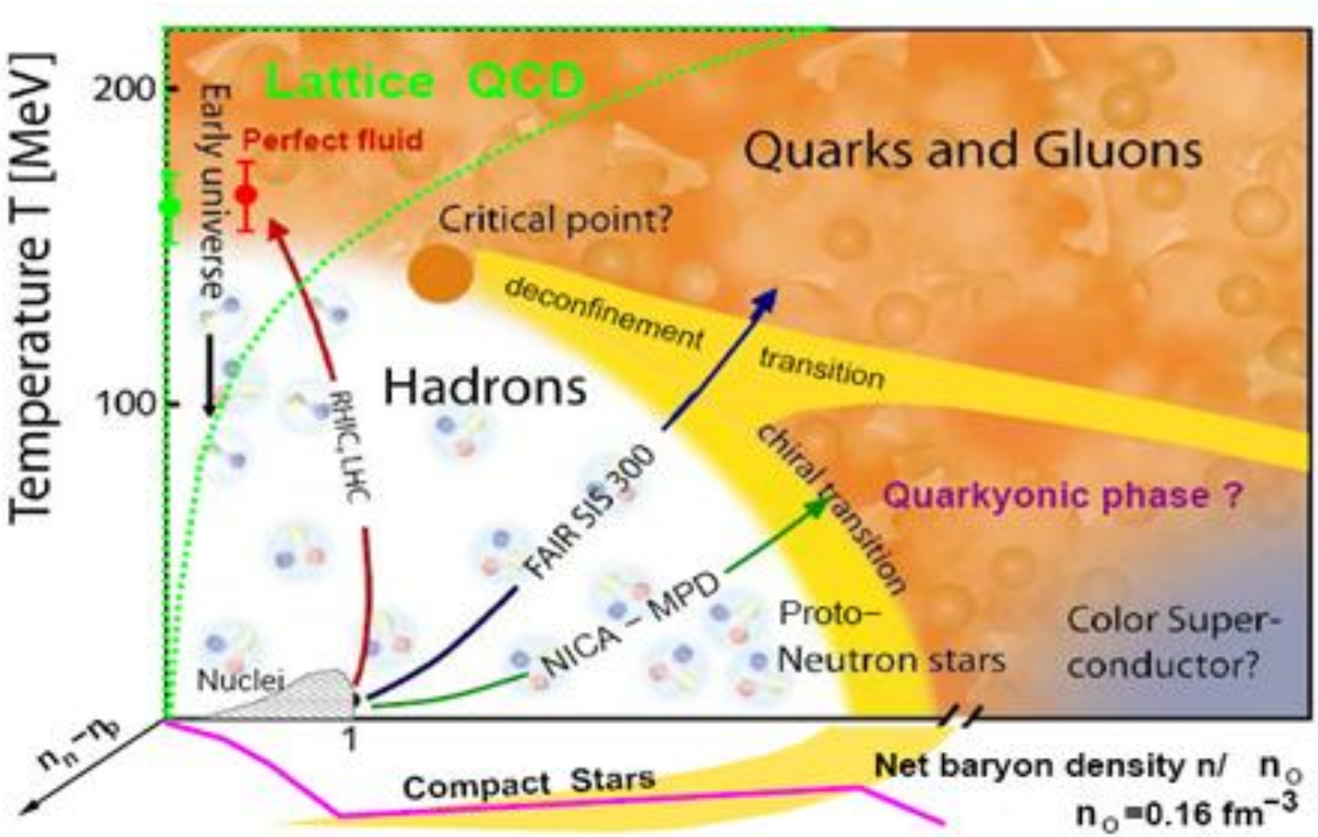
# Outline

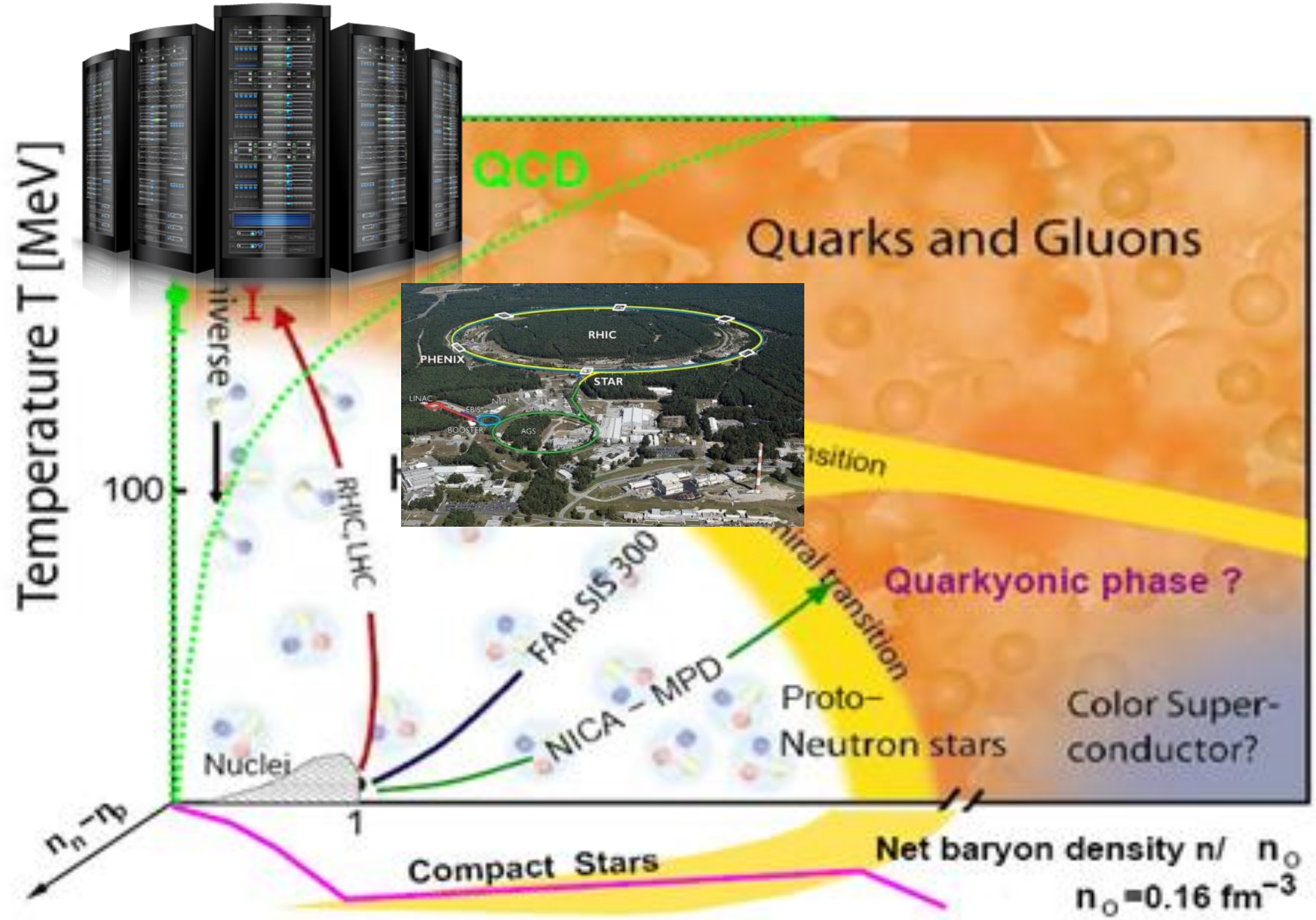
1. Motivation for using FRG in nuclear physics
2. Introduction to FRG
3. Technique for solving the FRG equations at finite temperature
4. Proof of concept: application for compact stars

# Motivation



Temperature T [MeV]
200
100
Lattice QCD
Early universe
Perfect fluid
Quarks and Gluons
Critical point?
deconfinement
transition
Hadrons
RHIC, LHC
FAIR SIS 300
chiral transition
Quarkyonic phase ?
NICA – MPD
Proto-
Neutron stars
Color Super-
conductor?
Nuclei
1
$n_n - n_p$
Compact Stars
Net baryon density n/ $n_o$
$n_o$=0.16 fm$^{-3}$

# Motivation



QCD
Quarks and Gluons
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RHIC,LHC
RHIC
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Net baryon density n/ $n_0$
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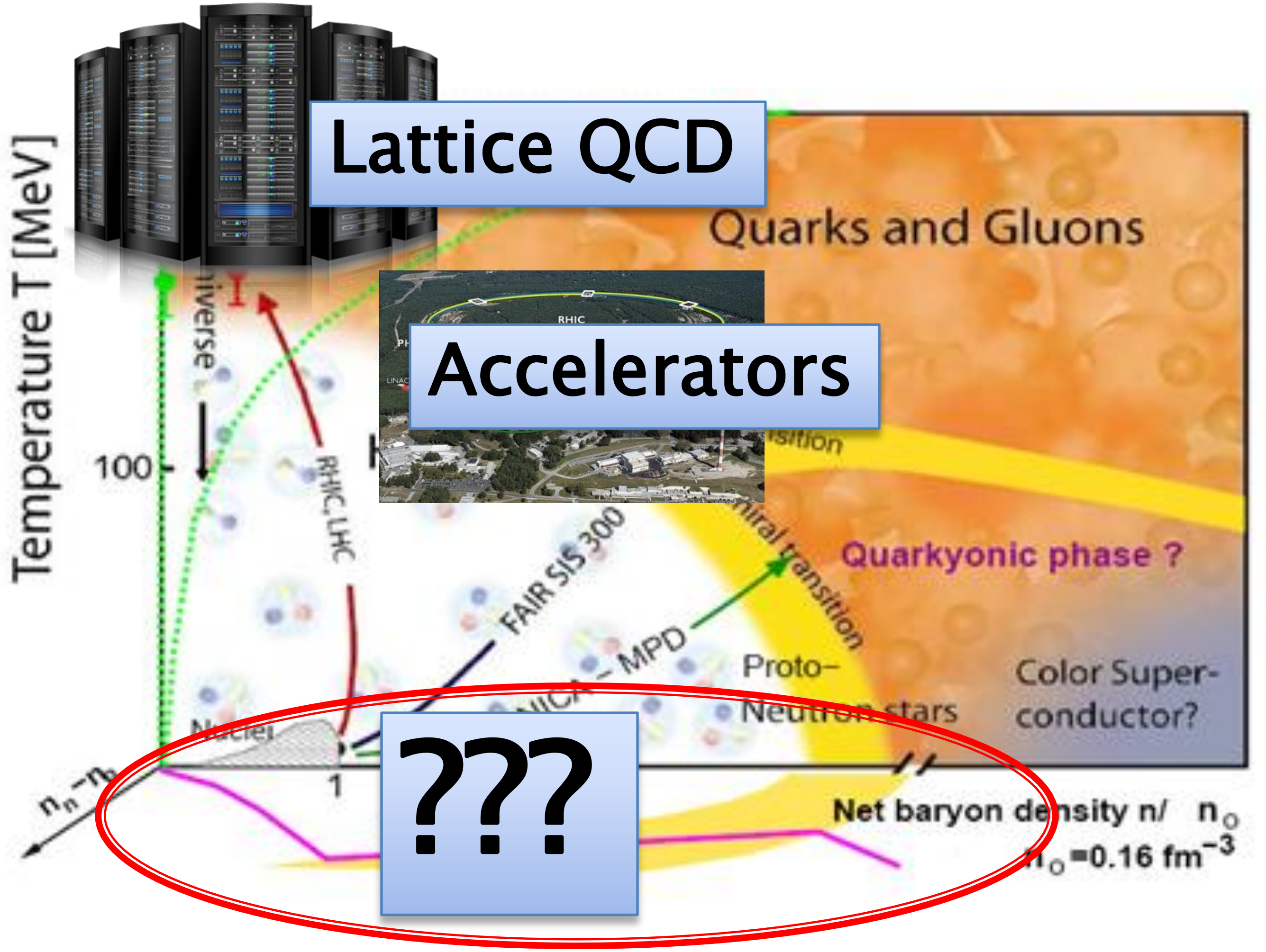
# Motivation


Lattice QCD
Accelerators
???
Quarks and Gluons
Quarkyonic phase ?
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MPD
RHIC,LHC
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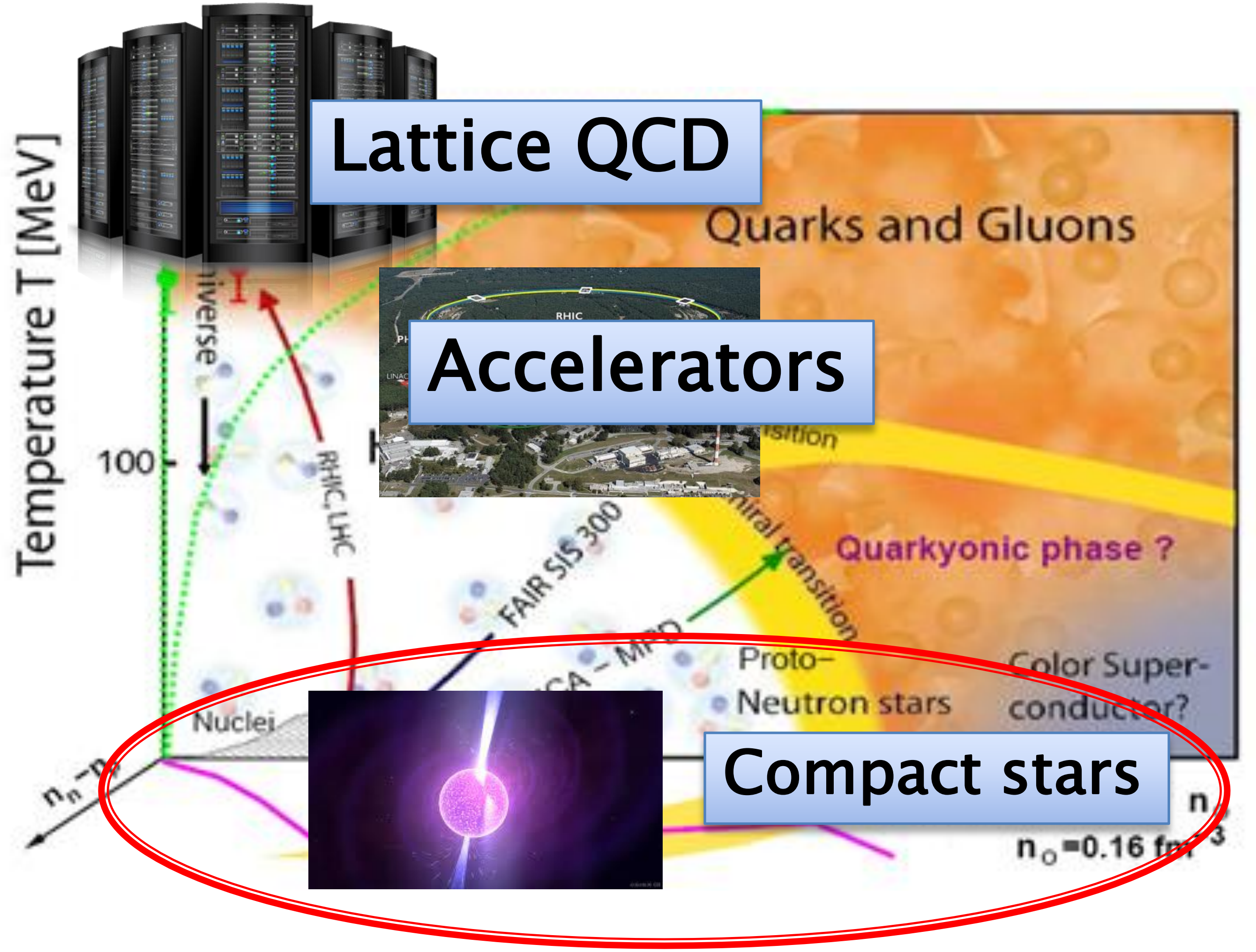
1
Net baryon density n/ n₀
n₀=0.16 fm⁻³

# Motivation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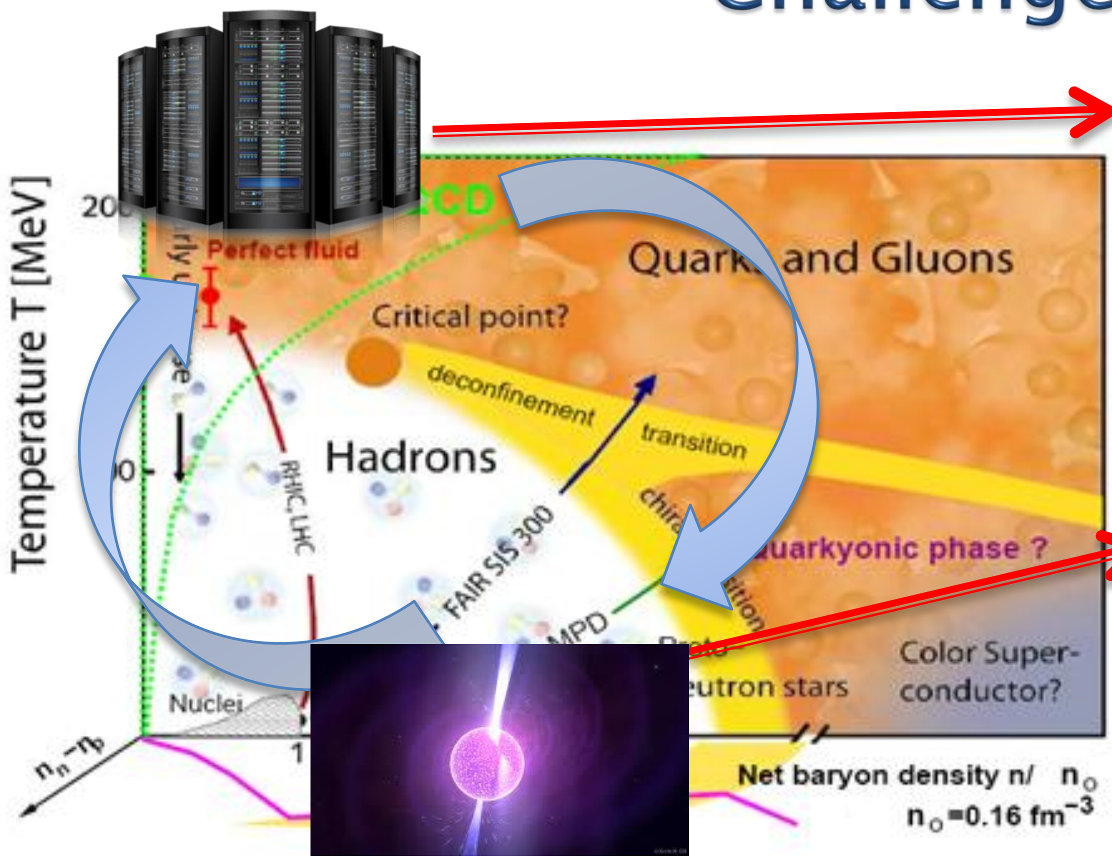
# Challenges




This region known:
Interaction, particles, degrees of freedom

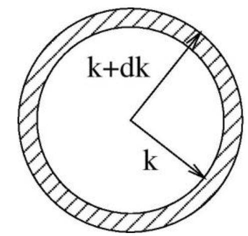
**Using this knowledge, how can we understand the behavior of matter in a different state?**

**Analogy:** to describe fluid flow, the knowledge of the quantum mechanics of fluid molecules is not needed. This separation makes hard to predict fluid flow based on laws describing the elementary particles of water.

# Introduction to FRG

# The recipe for FRG

- **Generating Functional+ Regulator**
  - The regualtos acts as a **mass term** and suppresses fluctuations below scale *k*
  - gradual momentum integration



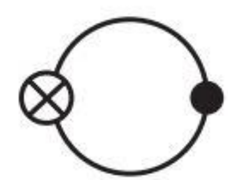

$$Z_k[J] = \int \left( \prod_a d\Psi_a \right) e^{-S[\Psi] - \frac{1}{2} R_{k,ab} \Psi_a \Psi_b + \Psi_a J_a}$$

- The **effective action** is the Legenrdre-transform of the Schwinger functional:

$$\Gamma_k[\psi] = \sup_J (\psi_a J_a - W[J]) - \frac{1}{2} R_{k,ab} \psi_a \psi_b$$

- The scale-dependece of the effective action is given by the **Wetterich-equation:**

$$\partial_k \Gamma_k = \frac{1}{2} Str \left[ (\partial_k R_k) \left( \Gamma_k^{(1,1)} + R_k \right)^{-1} \right]$$



# The Wetterich equation

- Exact equation for the effective action, but it is very hard to solve directly
  - **Scale dependent effective action (k scale parameter)**

$$\partial_k \Gamma_k = \frac{1}{2} \int dp^D \, STr \left[ \frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]$$

**Wetterich equation**

*$k=\Lambda$*
Classical action

Integration

*$k=0$*
*Quantum fluctuations included*

# The Wetterich equation

- Exact equation for the effective action, but it is very hard to solve directly
  - **Scale dependent effective action (k scale parameter)**

$$\partial_k \Gamma_k = \frac{1}{2} \int dp^D \; STr \left[ \frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]$$

We need an ansatz for the integration

**Not necessarily perturbative ansatz!**

$$\Gamma_k = \sum_{l=1}^{l=N} \frac{g_l(k)}{l!} \hat{O}_l$$

**Scale dependent coupling**

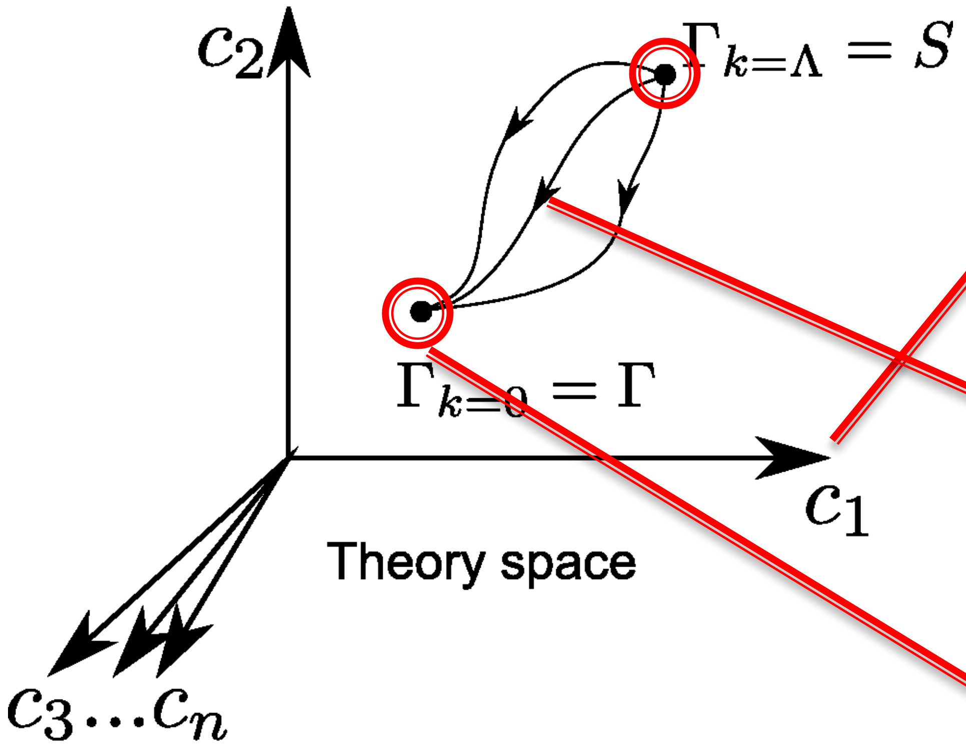
# The Wetterich equation

- Exact equation for the effective action, but it is very hard to solve directly
  - **Scale dependent effective action (k scale parameter)**

$$\partial_k \Gamma_k = \frac{1}{2} \int dp^D \, STr \left[ \frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]$$

**Regulator**:
- determines the modes present on scale *k*
- physics is regulator independent

# Theory space




**Operators**, relevant physical interactions particles etc…

**Operators change** between fixed points, some become less important some appear.

**Fixed Points**: Physical theories live near these points. They have given set of operators: physical quantities, particles etc.

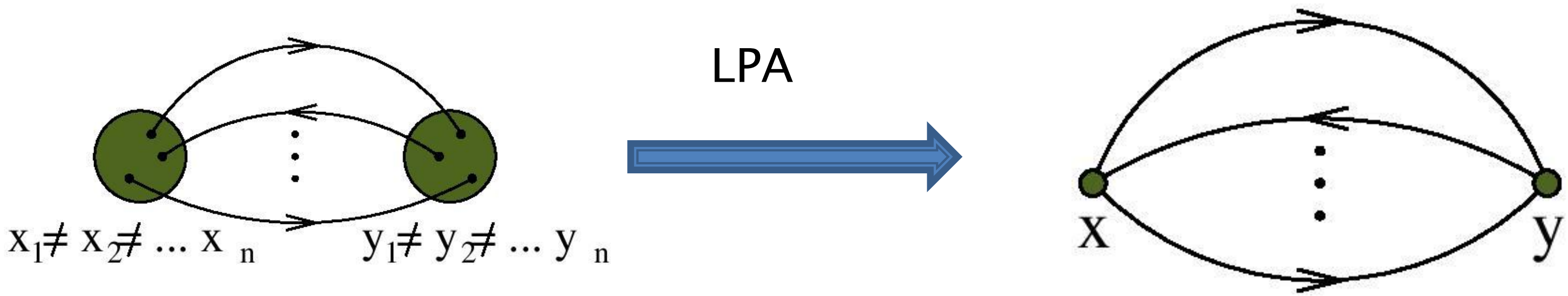
In a case where there are not any fixed points the FRG equaitons describe a cutoff theory

# Local Potential Approximation (LPA)

## What does the ansatz exactly mean ?

**LPA is based on the assumption that the contribution of these two diagrams are close.**

*(momentum dependence of the vertices is suppressed)*




This implies the following ansatz for the effective action:

$$\Gamma_k[\psi] = \int d^4x \left[\frac{1}{2}\psi_i K_{k,ij}\psi_j + U_k(\psi)\right]$$

# Demonstrative model

# Interacting Fermi-gas model

Ansatz for the effective action:

$$\Gamma_k[\varphi,\psi]=\int d^4x\left[\bar{\psi}\left(i\partial\!\!\!/-g\varphi\right)\psi+\frac{1}{2}\left(\partial_\mu\varphi\right)^2-U_k(\varphi)\right]$$

**Fermions** : *m=0*, **Yukawa-coupling** generates mass

**Bosons**: the **potential** contains self interaction terms

**We study the scale dependence of the potential only!!**

# Interacting Fermi-gas at finite temperature

Ansatz for the effective action:

$$\Gamma_k[\varphi,\psi] = \int d^4x \left[ \bar{\psi}\,(i\partial\!\!\!/ - g\varphi)\,\psi + \frac{1}{2}(\partial_\mu\varphi)^2 - U_k(\varphi) \right]$$

Wetterich -equation

$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[ \frac{1+2n_B(\omega_B)}{\omega_B} + 4\frac{-1+n_F(\omega_F-\mu)+n_F(\omega_F+\mu)}{\omega_F} \right]$$

Bosonic part

Fermionic part

$$U_\Lambda(\varphi) = \frac{m_0^2}{2}\varphi^2 + \frac{\lambda_0}{24}\varphi^4 \qquad \omega_F^2 = k^2 + g^2\varphi^2 \qquad \omega_B^2 = k^2 + \partial_\varphi^2 U \qquad n_{B/F}(\omega) = \frac{1}{1 \mp e^{-\beta\omega}}$$

# Solution at zero temperature $\mu=0$

# Demonstrative model at zero temperature

- The Wetterich equation depends on
  k (scale)
  gΦ (field vev)

- We use a taylor series expansion for U

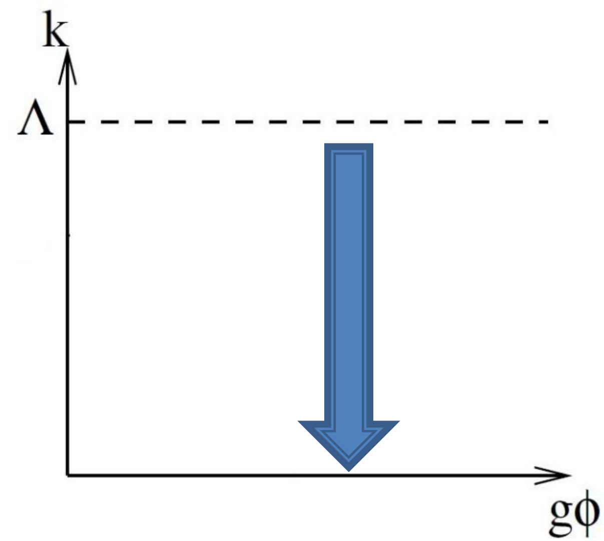
$$U_k(\phi) = \sum_i a_i(k)\,\phi^i$$

- Both sides are expanded as function of Φ and the coefficients are equated, which yields a DE for all coefficients

$$\partial_k\, a_i(k) = \beta_i(k)$$

$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[\frac{1}{\omega_B} - \frac{4}{\omega_F}\right]$$

$$\omega_F^2 = k^2 + g^2\varphi^2 \qquad \omega_B^2 = k^2 + \partial_\varphi^2 U$$

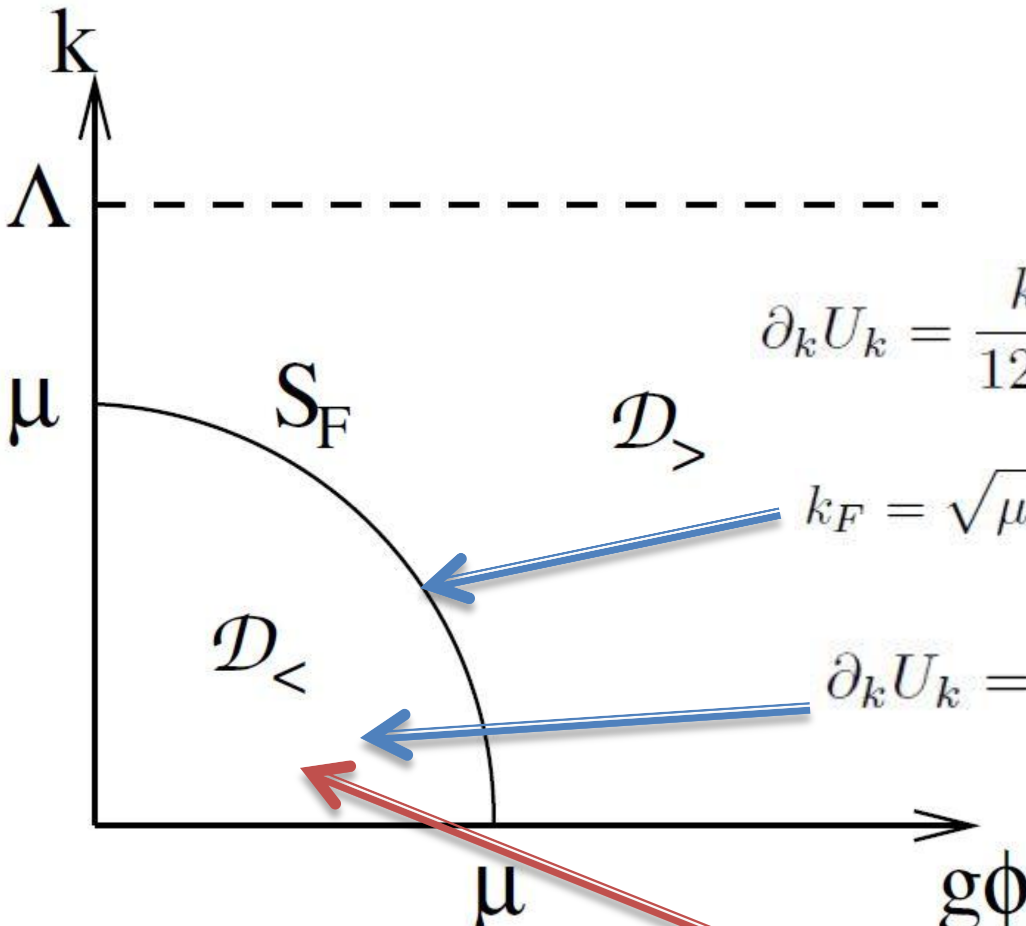


# Solution at zero temperature $\mu \neq 0$

# Demonstrative model at zero temperature

$T=0\,,\ \mu\neq 0 \;\Longrightarrow\; n_F(\omega)\rightarrow\Theta(-\omega)$

We have two equations for the two values of the step function each valid on different domain



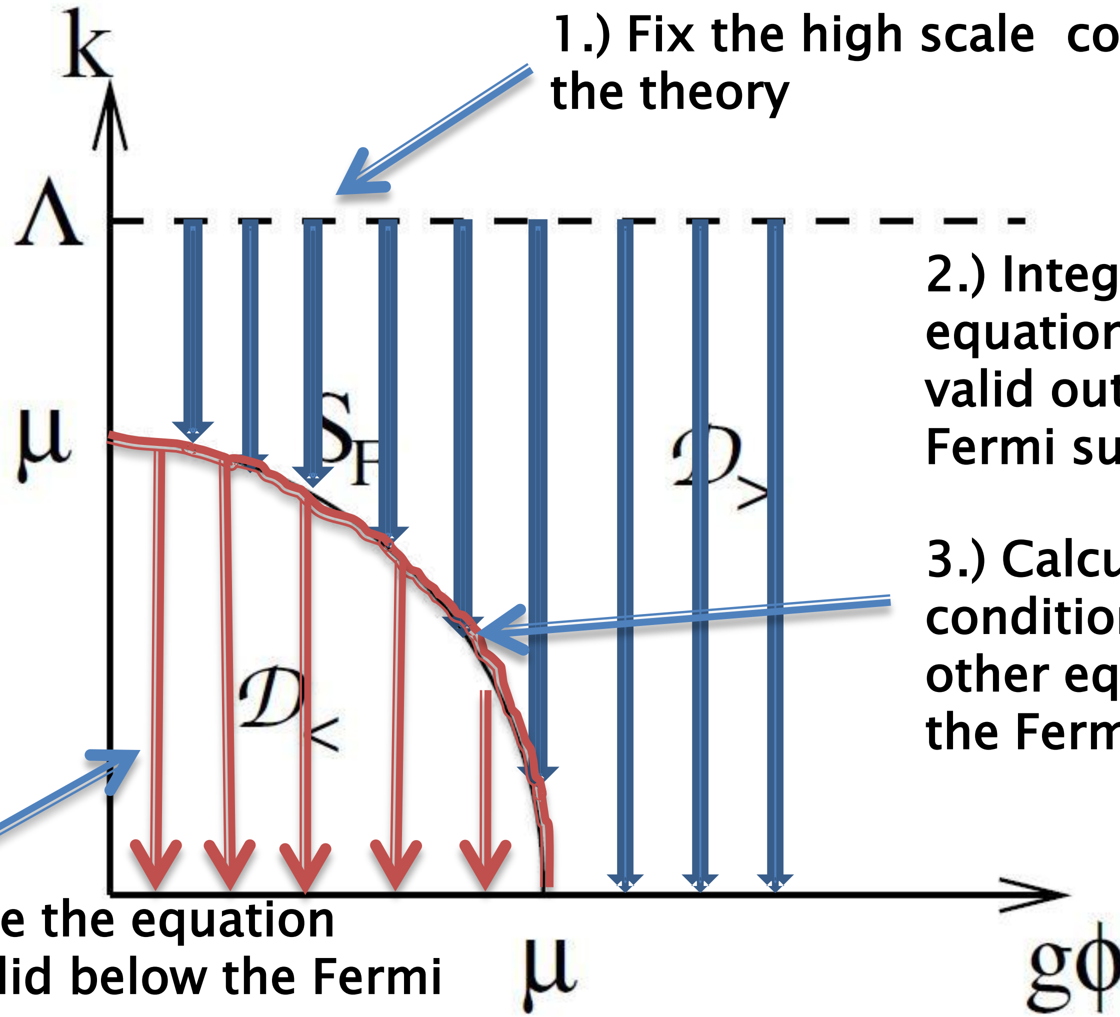

$$\partial_k U_k = \frac{k^4}{12\pi^2}\left[\frac{1}{\omega_B}-\frac{4}{\omega_F}\right]$$

$k_F=\sqrt{\mu^2-g^2\varphi^2}$; Fermi-surface

$$\partial_k U_k = \frac{k^4}{12\pi^2}\frac{1}{\omega_B}$$

Fermionic vacuum fluctuations and thermodynamic fluctuations cancel

# Integration of the Wetterich-equaiton



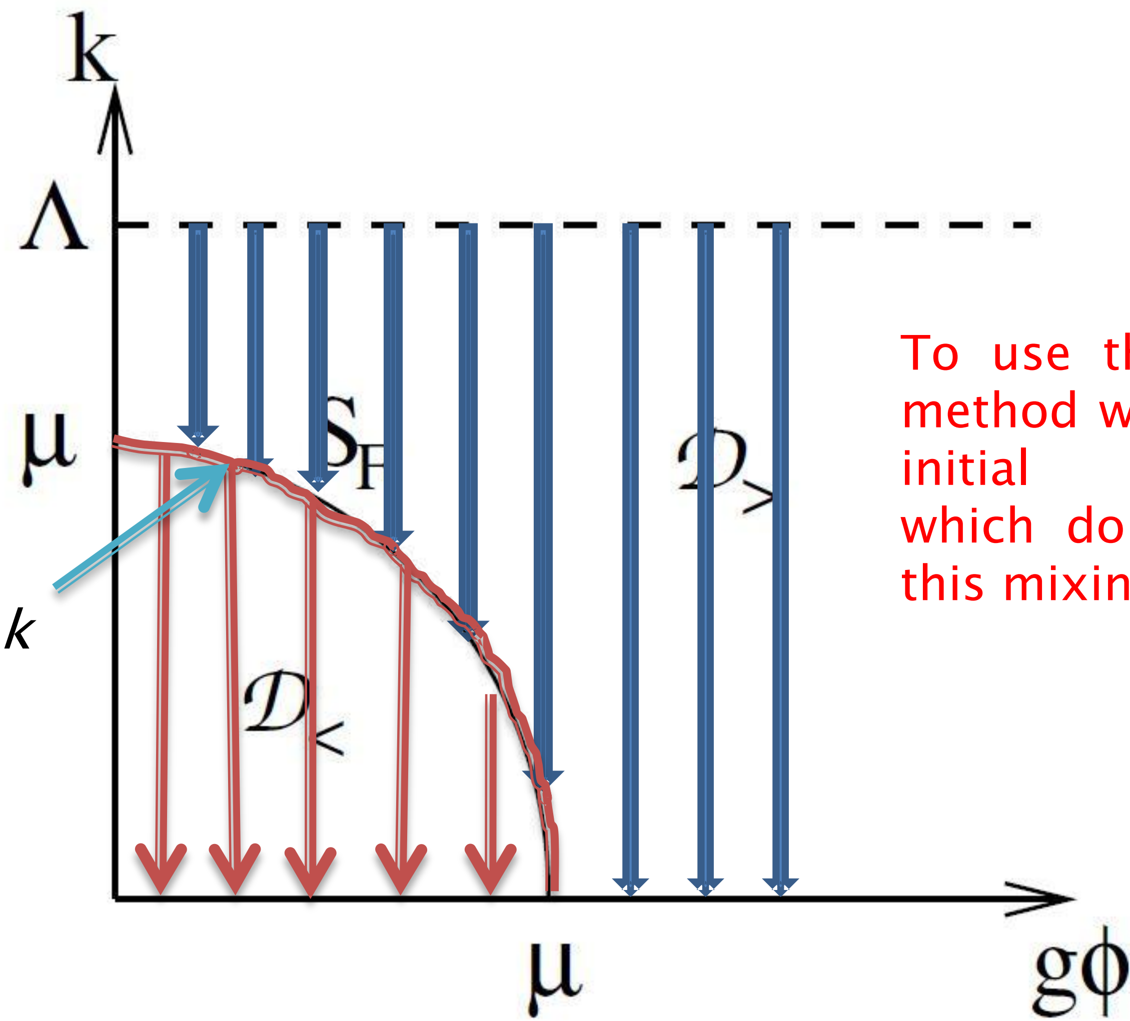

1.) Fix the high scale couplings in the theory

2.) Integrate the equation which is valid outside of the Fermi surface

3.) Calculate the initial conditions for the other equation inside the Fermi surface

4.) Integrate the equation which is valid below the Fermi surface

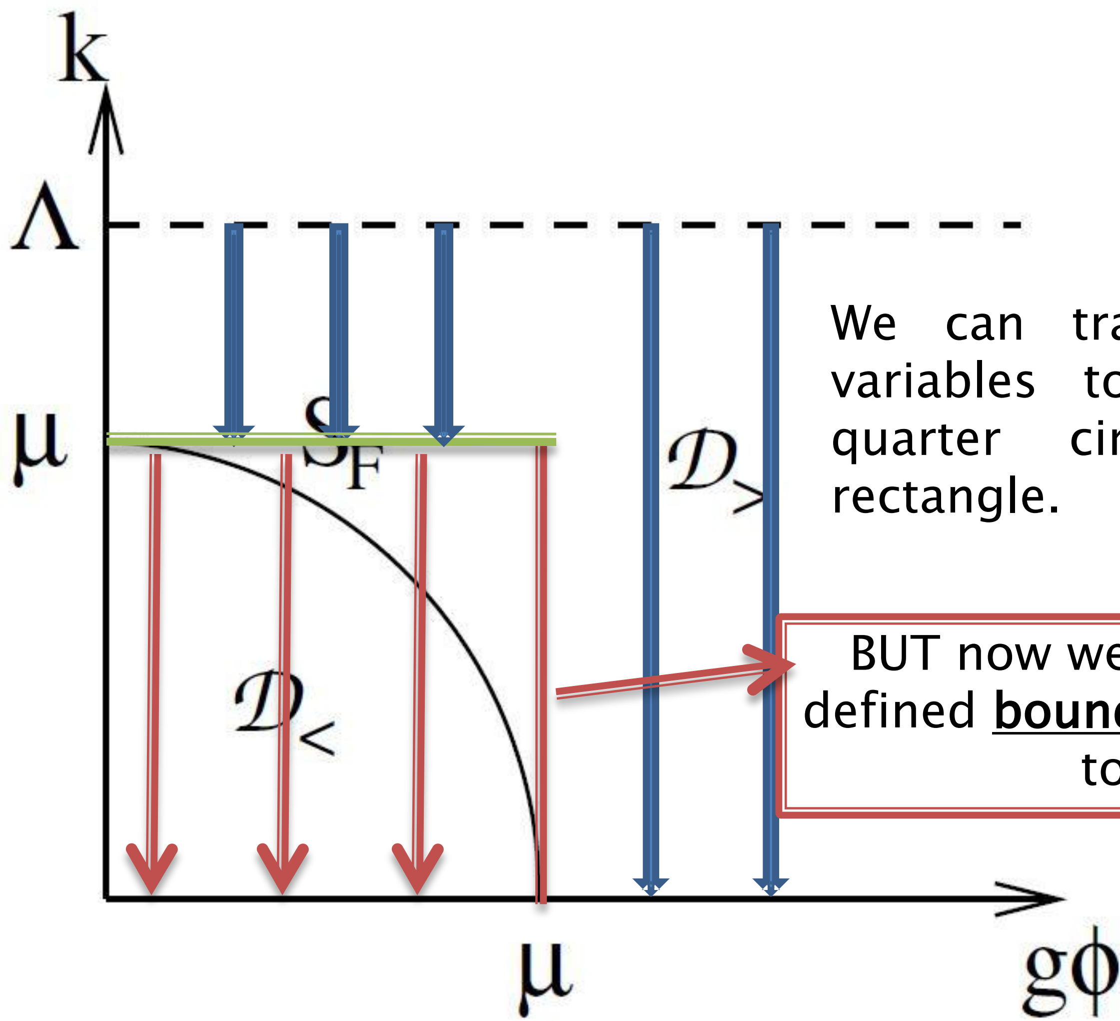
# BUT…




The boundary condition mix *k* and *gϕ*

To use the orginal method we need an initial condition which do not have this mixing

# Transform the variables




We can transform the variables to make the quarter cirle into a rectangle.

BUT now we have a well defined **boundary condition** too!

# Solution by orthogonal system

- Solution is expanded in an **orthogonal basis** to accomodate the strict boundary conditoin in the trasformed area

$$\tilde{u}(x,y) = \sum_{n=0}^{\infty} c_n(x) h_n(y) \quad h_n(1) = 0 \quad \int_0^1 dy\, h_n(y) h_m(y) = \delta_{nm}$$

- The **square root** in the Wetterich-equation is also expanded:

$$x c_n'(x) = \int_0^1 dy\, h_n(y) \left[ -x V_0' + y \partial_y \tilde{u} - \frac{g^2 (kx)^3}{12\pi^2} \sum_{p=0}^{\infty} \binom{-1/2}{p} \frac{(\partial_y^2 \tilde{u} - M^2)^p}{\omega^{2p+1}} \right]$$

Where: $\omega^2 = (kx)^2 + M^2$

Expanded square root

**We use harmonic base**

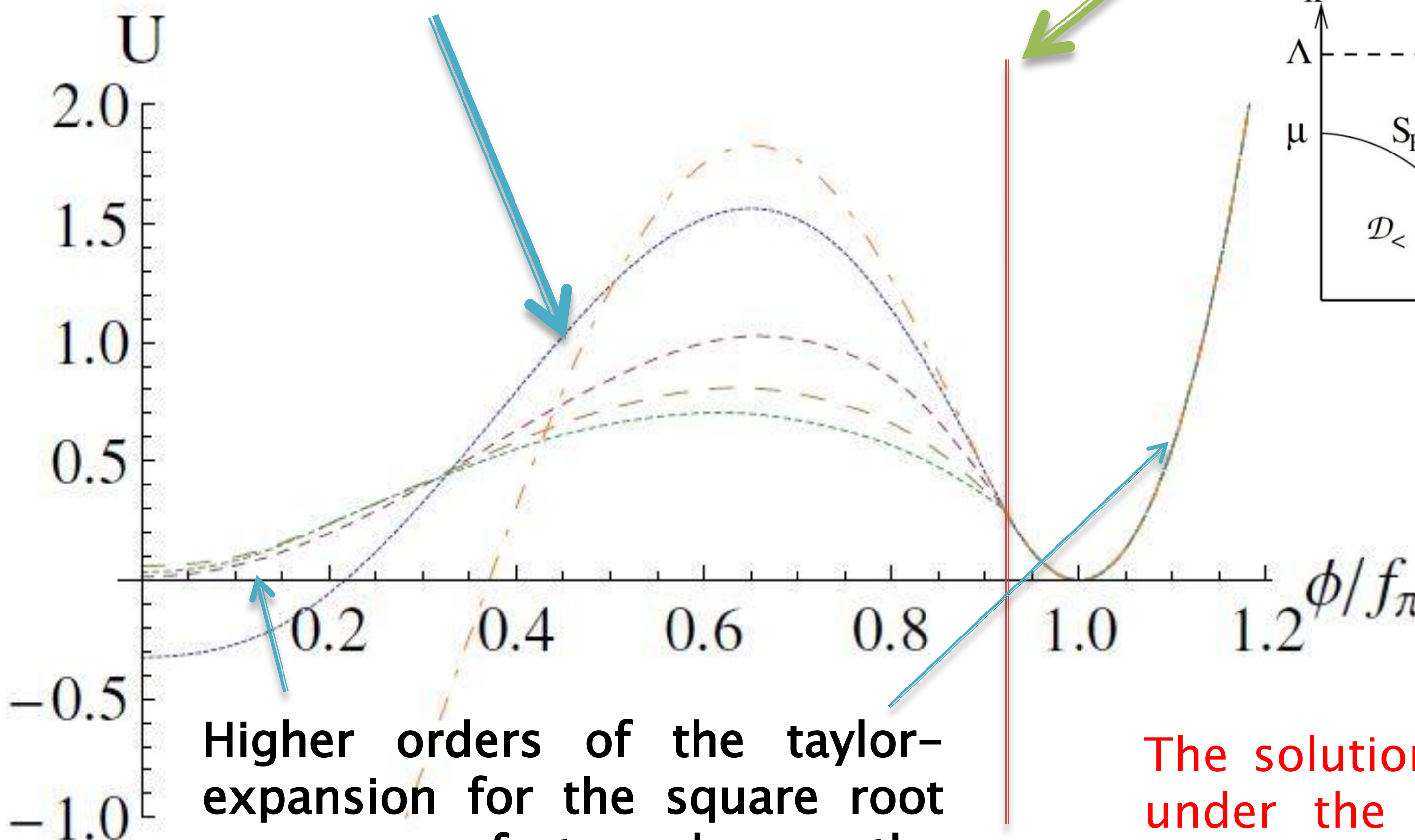
$$h_n(y) = \sqrt{2} \cos q_n y, \qquad q_n = (2n+1)\frac{\pi}{2}$$

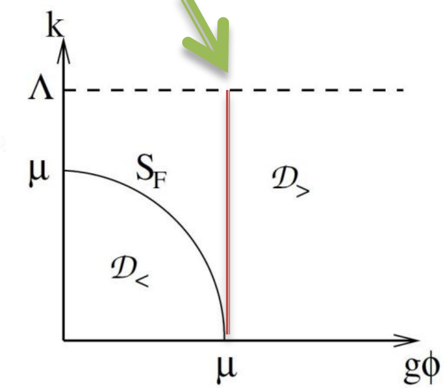
# The solution of the Wetterich equation

# Results

Potential in one-loop approximation

Fermi surface in the field variable



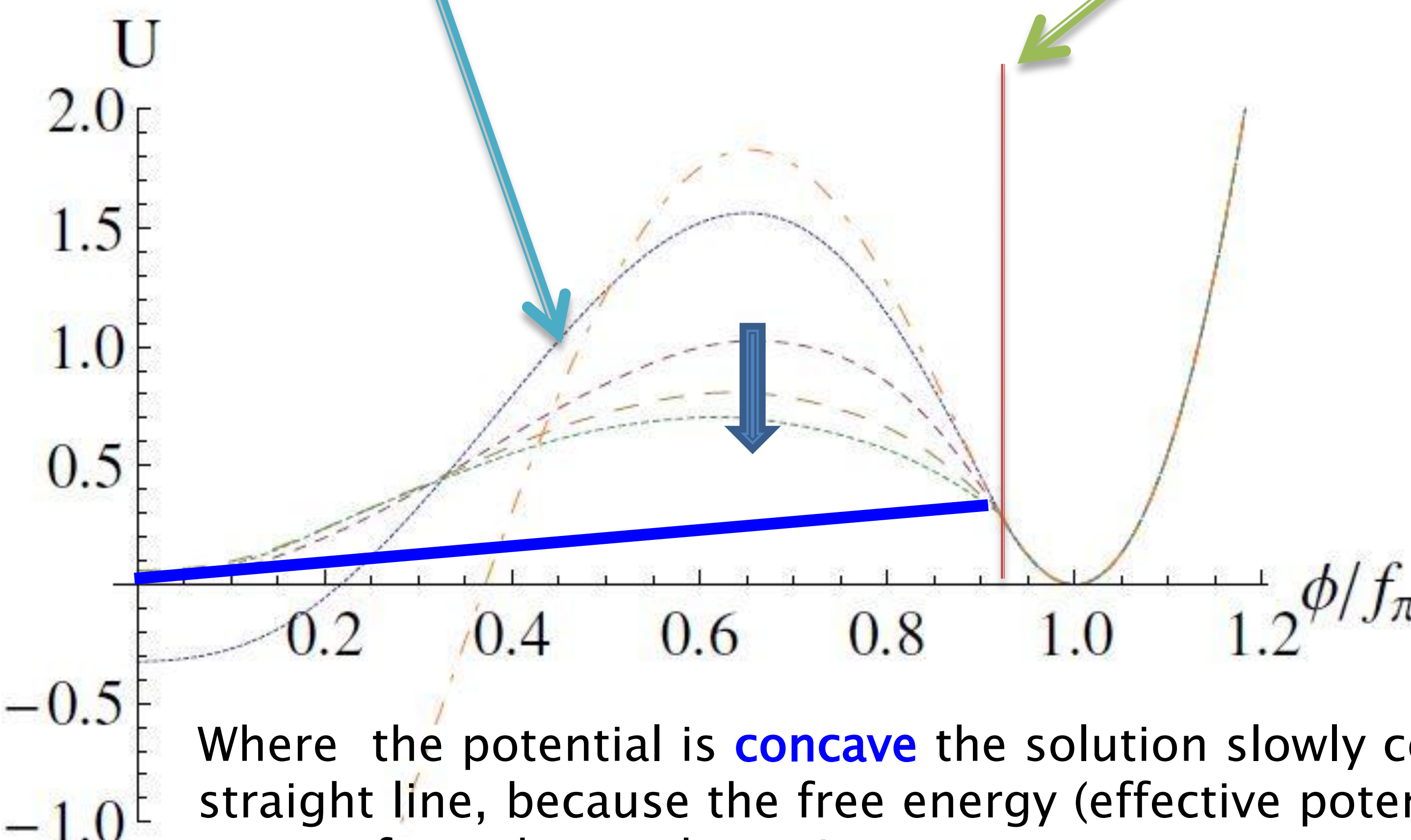

Higher orders of the taylor-expansion for the square root converge fast where the potential is convex

The solution changes only under the Fermi surface, because here we switch to the other equation

# Results

Potential in one-loop approximation

Fermi-surface in the field variable

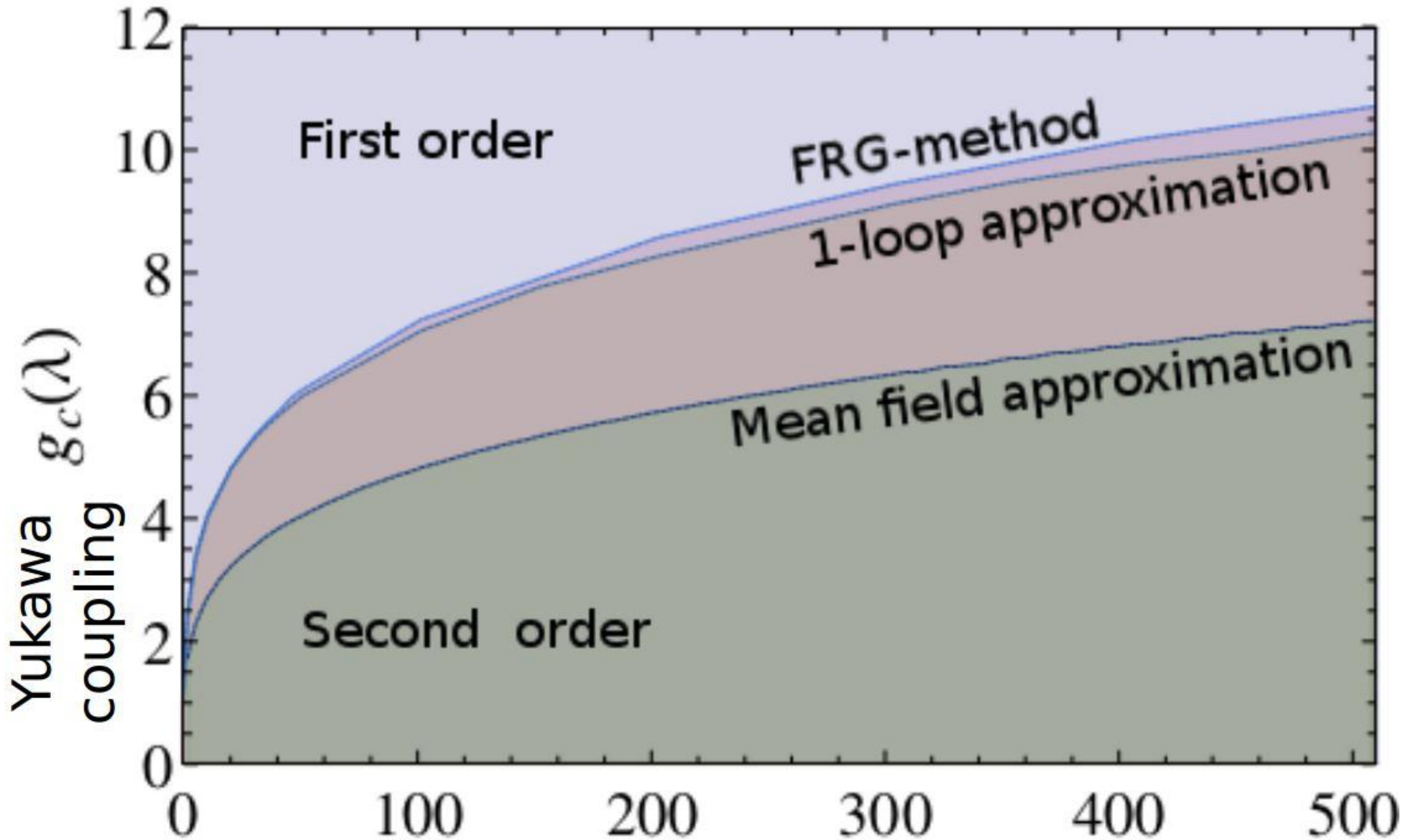



Where the potential is concave the solution slowly converges to a straight line, because the free energy (effective potential) must be convex from thermodynamics reasons.

This is the **Maxwell construction.**

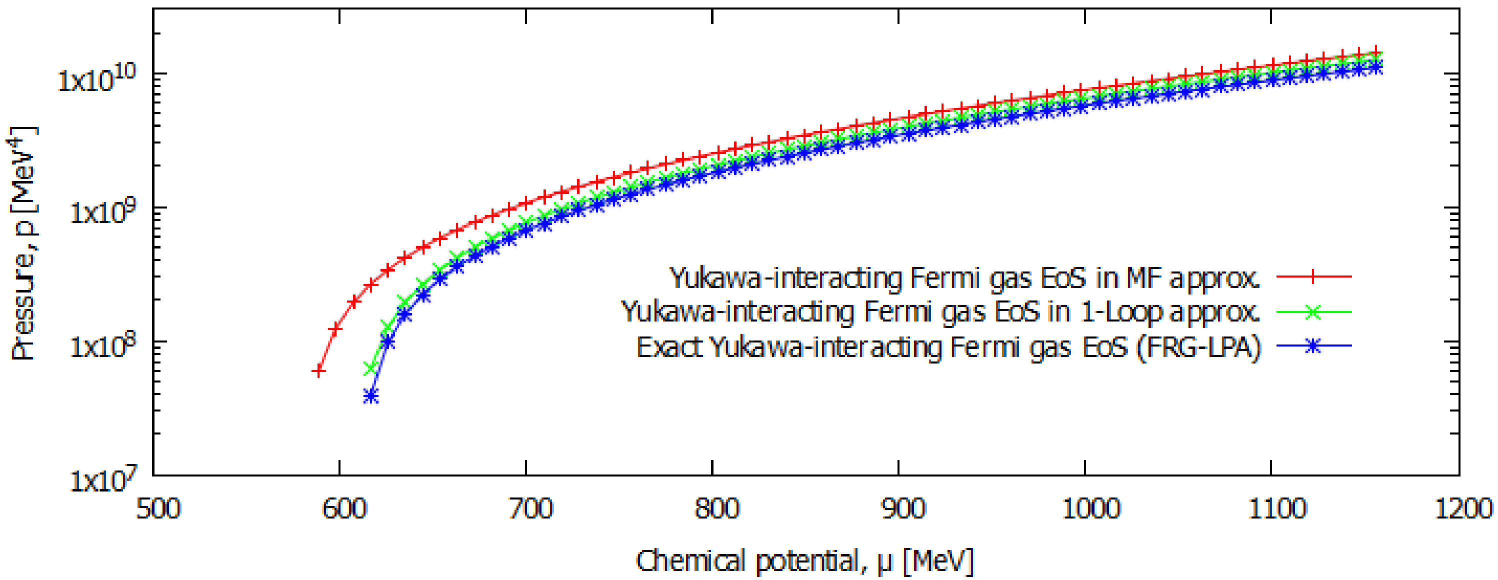
# Thermodynamical properties

# Phase structure




Scalar self interaction

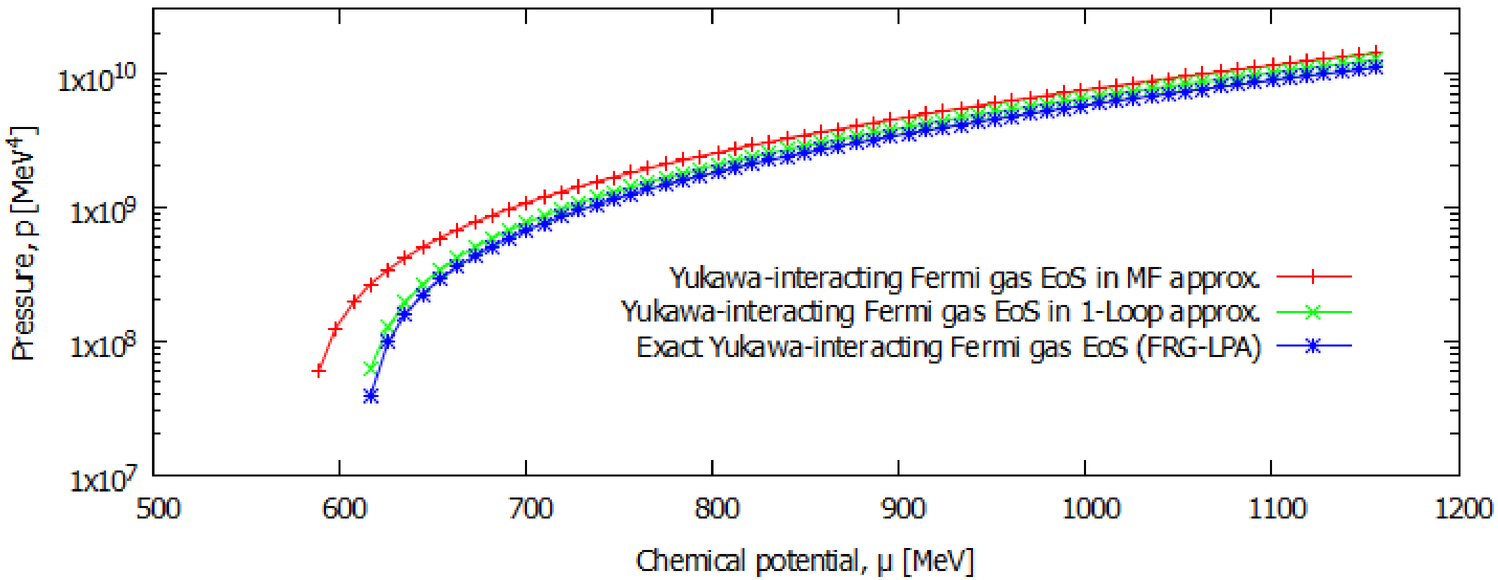
# The equation of state

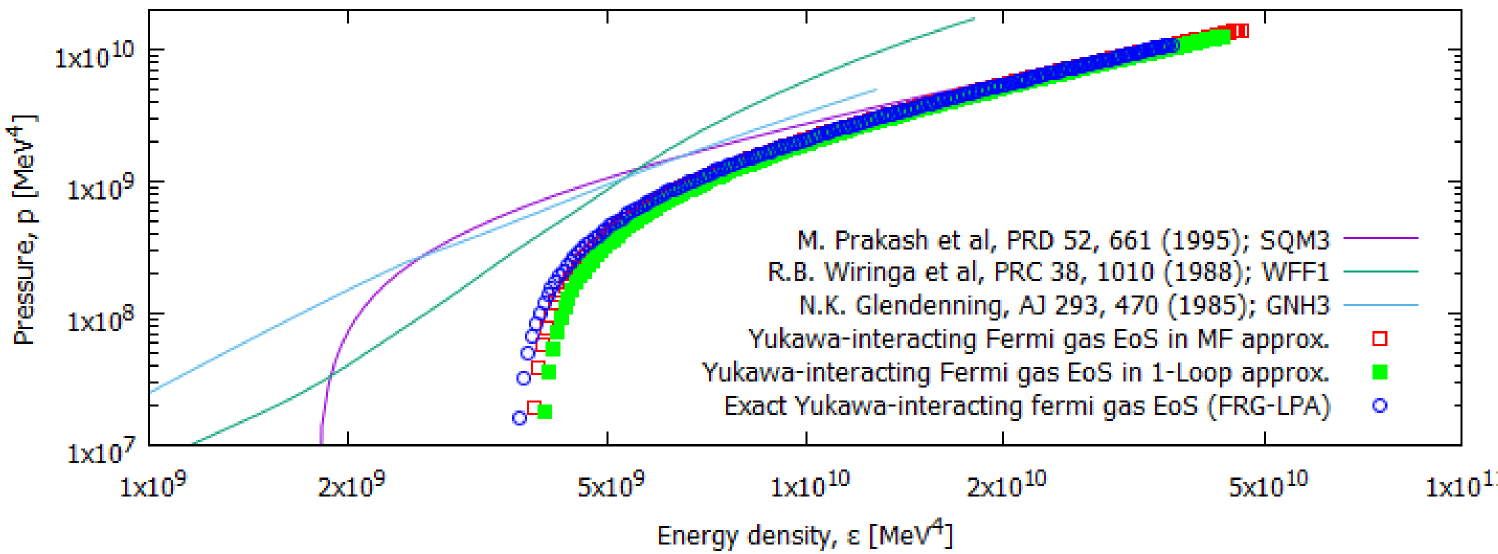



PRESSURE

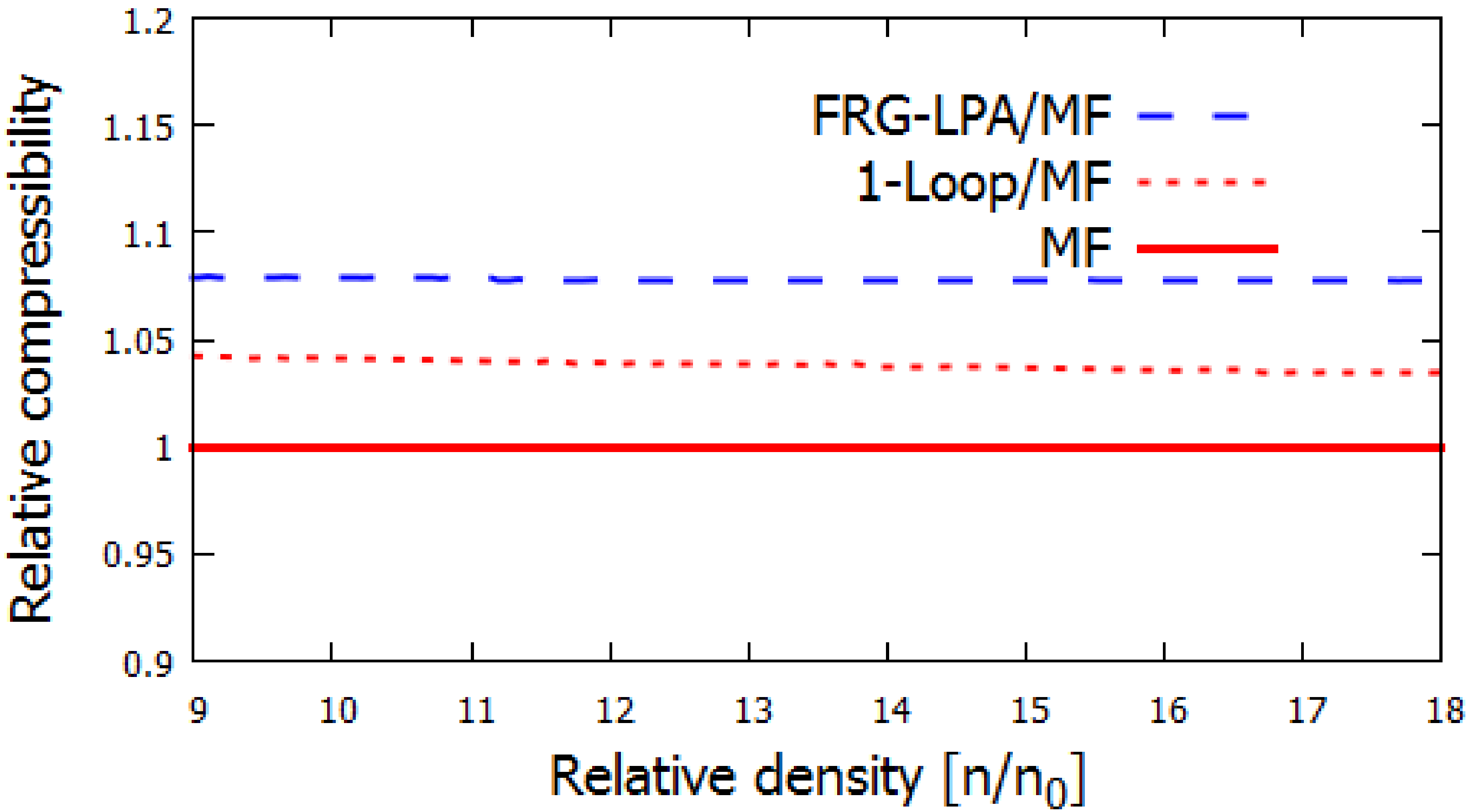
Mean Field > 1-LOOP > FRG

# The equation of state



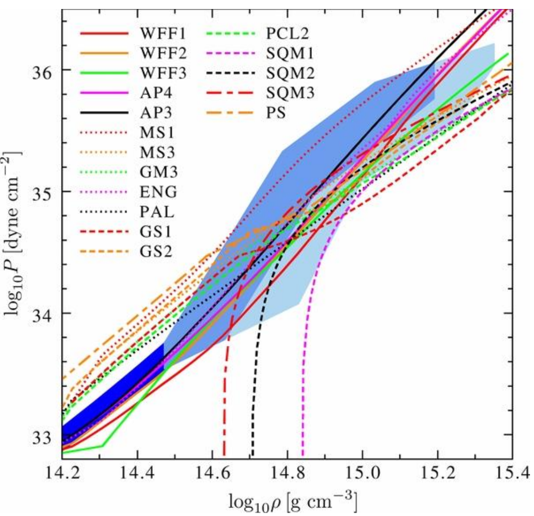



# Compressibility

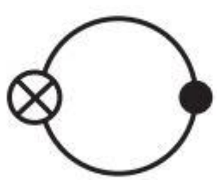


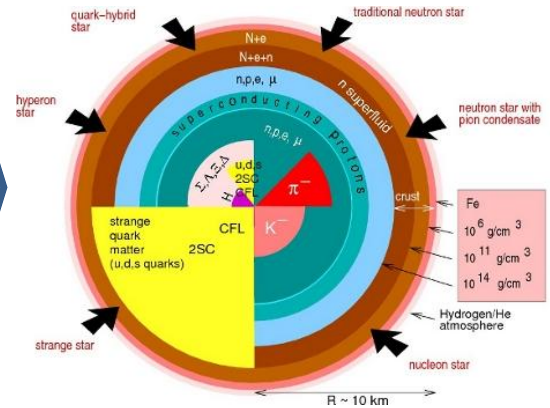
# From EoS to Compact Stars

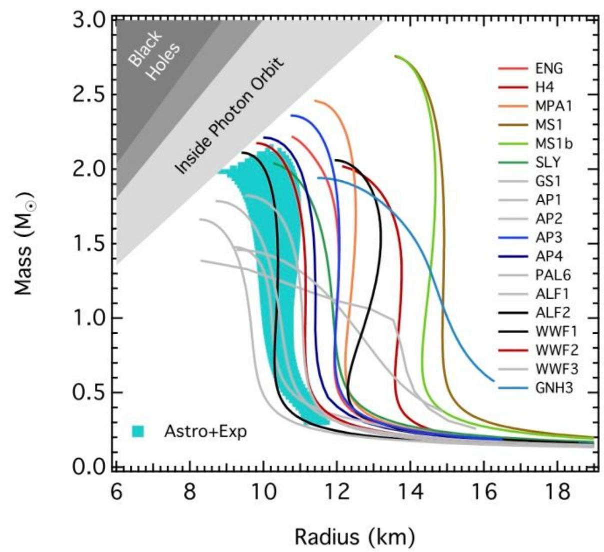
EOS

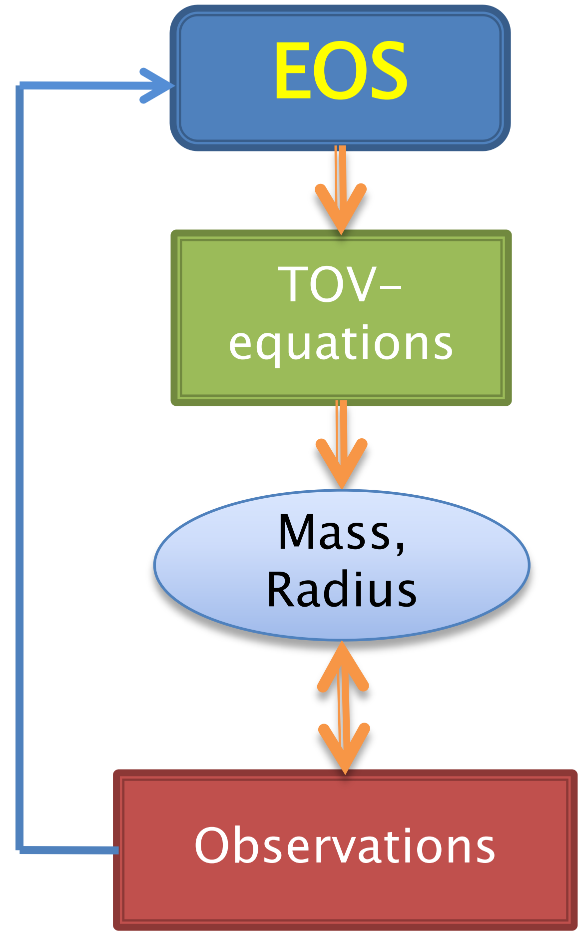



FRG





# Application for compact stars

The Fermi-gas is not a realistic model of a neutron star, this is just demonstrates that an apparently small change in the EoS, due to quantum fluctuations means a noticeable change in the solution of the TOV equations.

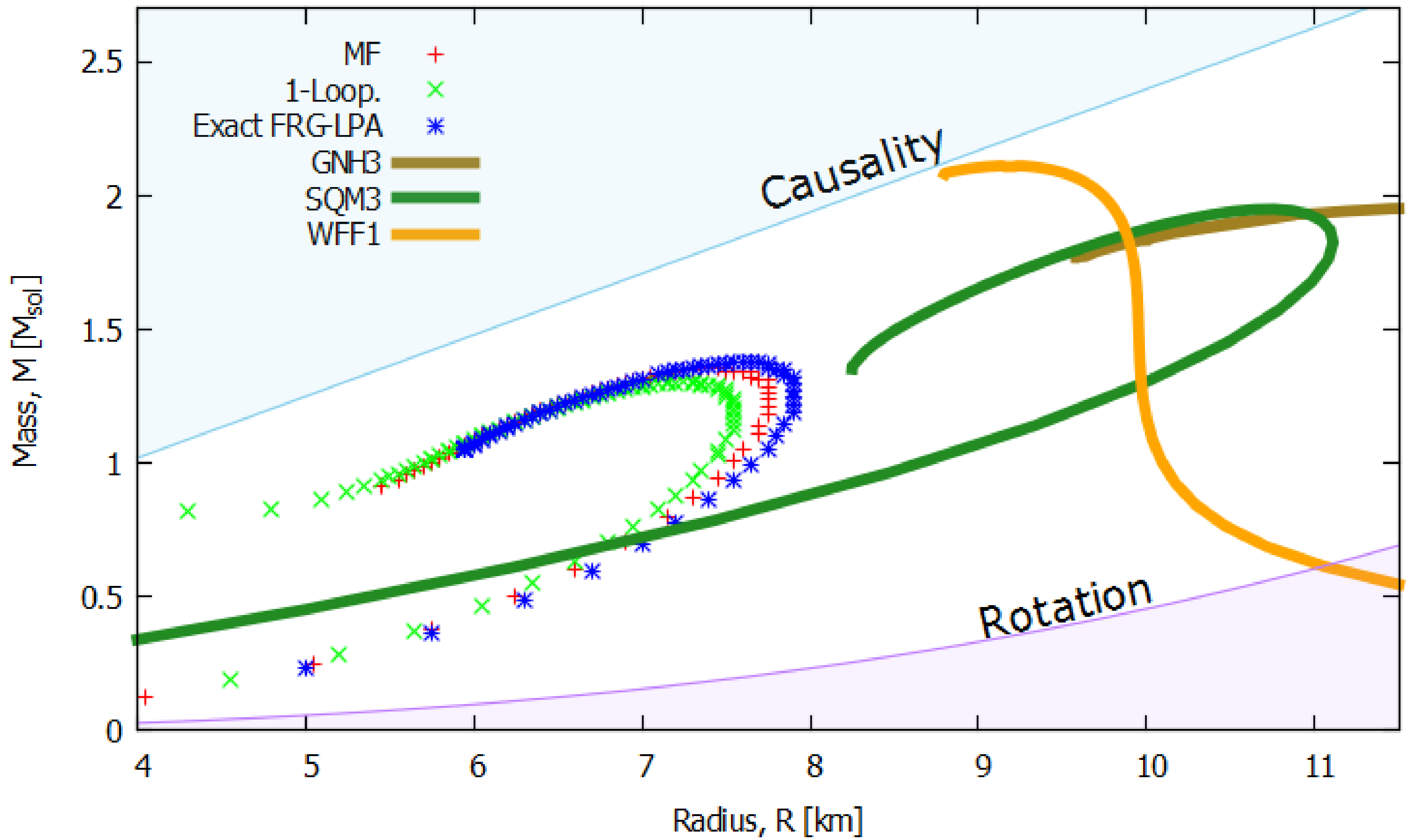
$M_{FRG}=1.377$

+1.5 %

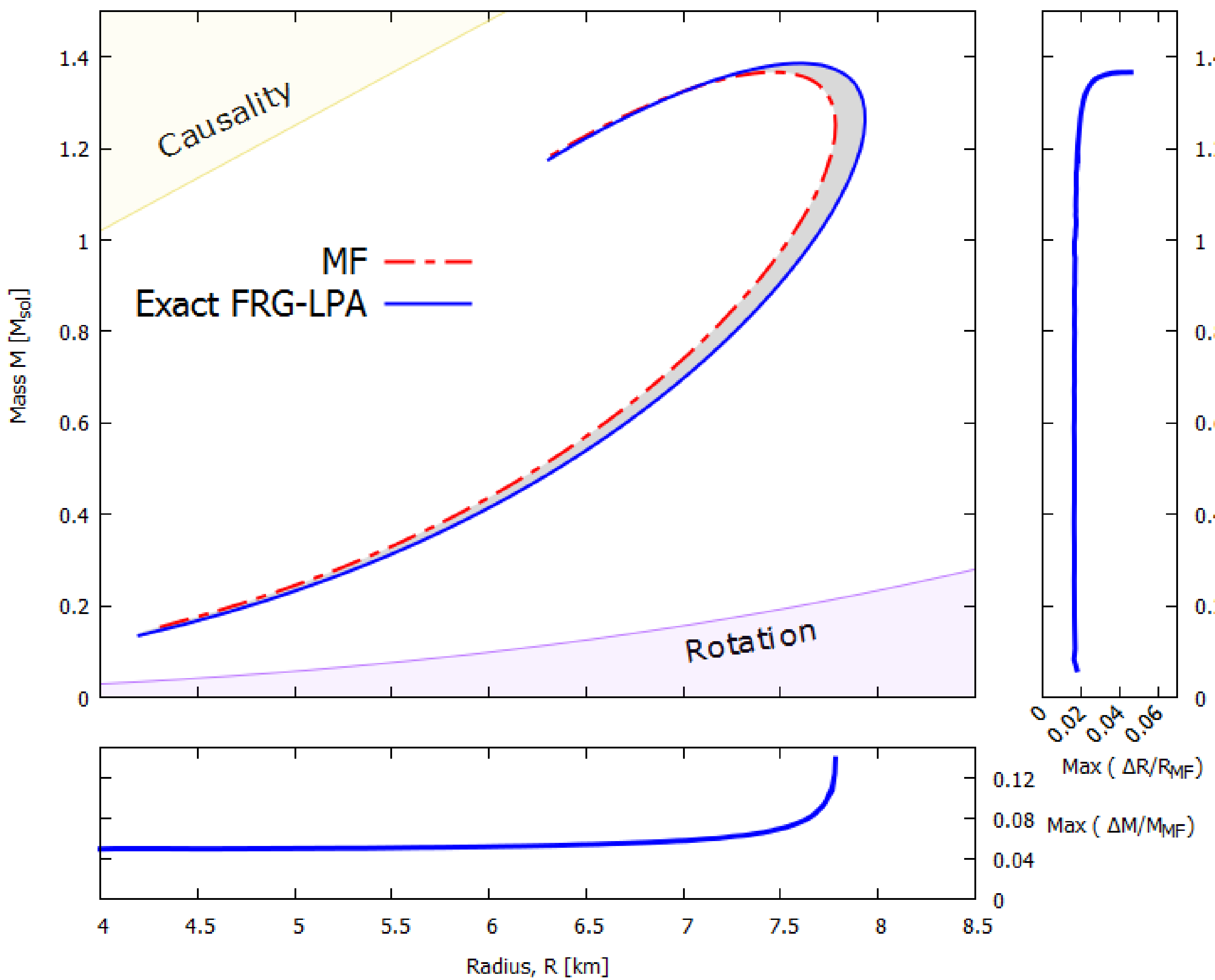
$M_{MF}=1.358$

-3.5 %

$M_{1L}=1.309$

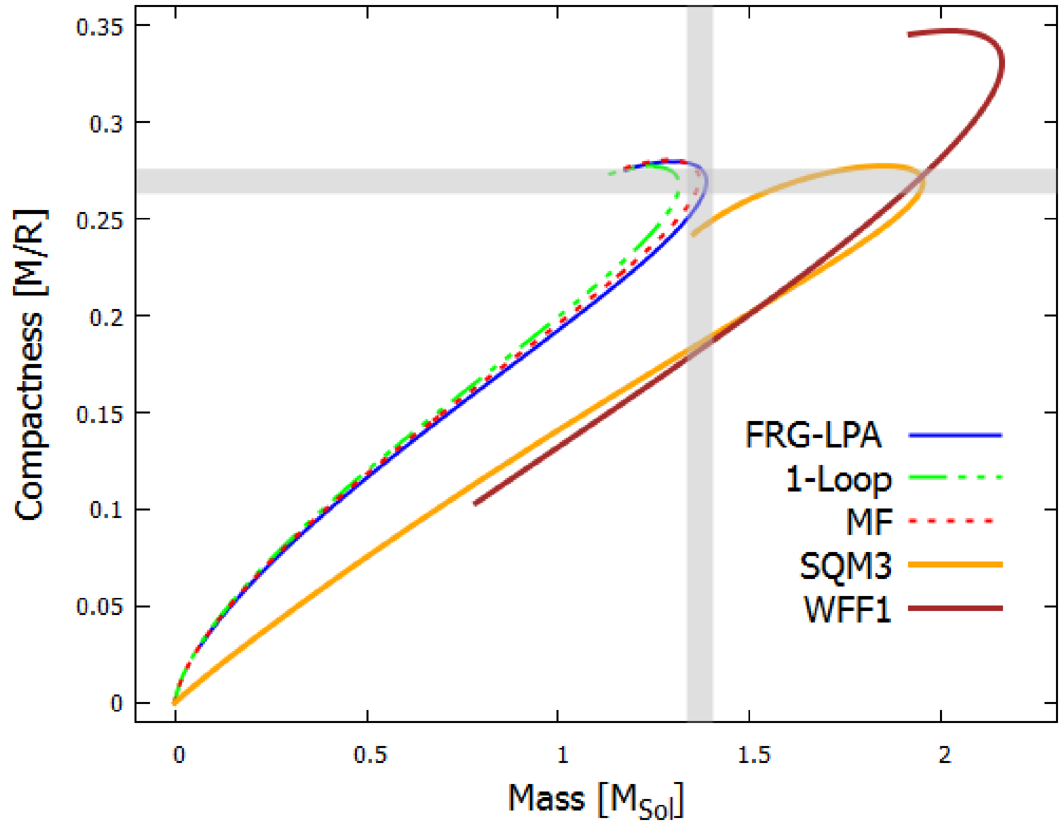


# Application for compact stars



# Gravity waves and compactness

- The compactness (M/R) of the neutron stars can be extracted from gravity waves and pulsar timing measurements.
- The NICER experiment will be able to determine compactness with error less than 10%
- This is a new method to distinguish between models.

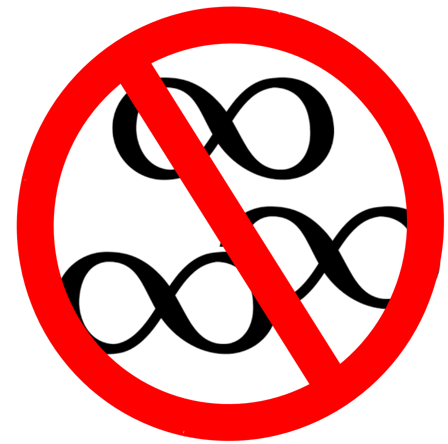


# Thank you for the attention !

**If you have an FRG Problem**

**[http://pospet.web.elte.hu/](http://pospet.web.elte.hu/)**

**(Contact and related materials)**



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