Exclusive hard processes with mesons

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Outline

EXCLUSIVE REACTIONS AT LARGE MOMENTUM TRANSFER

(hard exclusive reactions)

↓

HARD-SCATTERING PICTURE

(Brodsky, Lepage; Efremov, Radyushkin; . . . ('80))

factorization of short and long distance dynamics

↙ ↘

(elementary) hard-scattering amplitude

hadron distribution amplitudes

$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}$

hard-scattering amplitude

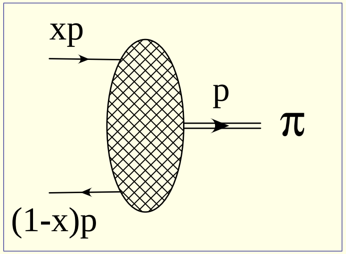
STANDARD APPROXIMATIONS:

(on an example of flavour-nonsinglet meson: π)

- $|\pi\rangle \longrightarrow |q\bar{q}\rangle + |q\bar{q}g\rangle + \cdots$
- collinear approximation:

$$p_q = x\,p, \quad p_{\bar{q}} = (1-x)\,p$$

($0 < x < 1 \longrightarrow$ longitudinal momentum fraction)



- $m_q = m_{\bar{q}} = 0\,, m_\pi = 0$

CONVOLUTION FORMULA:

$$\mathcal{M}(Q^2) = \int_0^1 [dx] \; T_H(x_j, Q^2, \mu_F^2) \prod_{h_i} \Phi_{h_i}(x_j, \mu_F^2)$$

$$[dx] = \prod_{j=1}^{n_{h_i}} dx_j \, \delta(1 - \sum_{k=1}^{n_{h_i}} x_k)$$

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Example: PHOTON-TO-PION TRANSITION FORM FACTOR $F_{\pi\gamma^{(*)}}$

$$\gamma^*(q_1, \mu) \quad \gamma^{(*)}(q_2, \nu) \quad \longrightarrow \quad \pi(p) \ ,$$

$$-q_1^2 = Q^2 \gg$$

$$\Gamma^\mu = i\, e^2 \ F_{\pi\gamma}(Q^2) \ \varepsilon^{\mu\nu\alpha\beta} \ q_{1\alpha} q_{2\beta} \ \epsilon_\nu(q_2)$$

in the standard hard-scattering picture:

$$F_{\pi\gamma}(Q^2) = T_H(x, Q^2, \mu_F^2) \otimes \Phi(x, \mu_F^2)$$

$$A(x) \otimes B(x) = \int_0^1 dx A(x) B(x)$$

$\mu_F^2 \cdots$ factorization scale

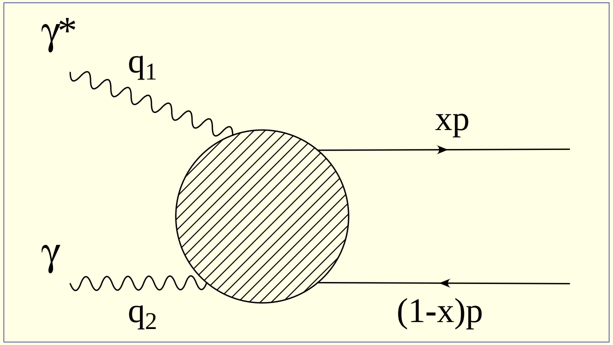
T_H ... *process–dependent*

(ELEMENTARY) HARD SCATTERING AMPLITUDE (HSA)

⇑

PQCD

$\gamma^* \gamma \longrightarrow q\bar{q}$



$$T_H(x,Q^2) = T_H^{(0)}(x,Q^2) + \frac{\alpha_S(\mu_R^2)}{4\pi} T_H^{(1)}(x,Q^2,\mu_F^2) + \frac{\alpha_S^2(\mu_R^2)}{(4\pi)^2} T_H^{(2)}(x,Q^2,\mu_F^2,\mu_R^2) + \cdots$$

μ_R^2 ... renormalization scale

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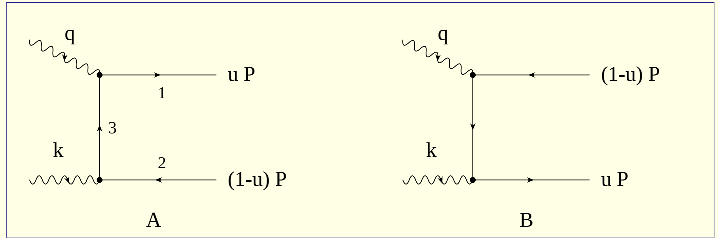
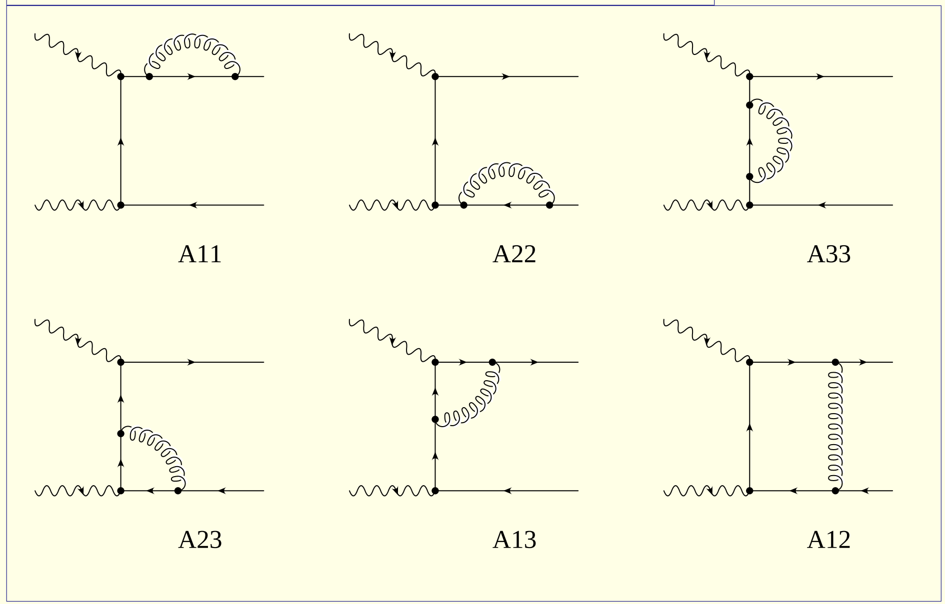
leading order (LO)



next-to-leading order (NLO)



↙

UV singularities $\longrightarrow$ *coupling constant* (α_S) *renormalization* $\Rightarrow \boldsymbol{\mu_R^2}$

collinear singularities $\longrightarrow$ *factorization* $\Rightarrow \boldsymbol{\mu_F^2}$

Φ ... *process–independent* pion

DISTRIBUTION AMPLITUDE (DA)

defined in terms of the matrix elements of composite operators: $\langle 0|\Psi(-z)\gamma^{+}\gamma_5\Omega\Psi(z)|\pi\rangle$

$\Uparrow$

form: (nonperturbative) input at scale $\mu_0^2 \longrightarrow \Phi(x, \mu_0^2)$

evolution to scale μ_F^2: PQCD $\Rightarrow \Phi(x, \mu_F^2)$

$\downarrow$

$$\Phi(x, \mu_F^2) = \underbrace{\phi_V(x, y, \mu_F^2, \mu_0^2)} \otimes \Phi(y, \mu_0^2)$$

$\downarrow$ *(resummation of* $(\alpha_S \ln(\mu_F^2/\mu_0^2))^n$ *terms)*

$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \phi_V = V \otimes \phi_V \cdots$ evolution equation

evolution kernel: $V = \frac{\alpha_S(\mu_F^2)}{4\pi} V_1 + \frac{\alpha_S^2(\mu_F^2)}{(4\pi)} V_2 + \cdots$

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Solution of the DA evolution equation:

$$\Phi = \frac{f_\pi}{2\sqrt{2N_c}}\,\phi$$

$f_\pi = 0.131$ GeV ... pion decay constant

$\int_0^1 dx\; \phi(x,\mu_F^2) = 1$

$$\phi(x,\mu_F^2) = 6x(1-x)\left[1 + \sum_{n=2}^{\infty}{}' B_n(\mu_F^2)\; C_n^{3/2}(2x-1)\right]$$

$C_n^{3/2}$ ... Gegenbauer polynomials

$\longrightarrow$ eigenfunctions of the LO evolution equation

$$B_n(\mu_F^2) = B_n^{LO}(\mu_F^2) + \frac{\alpha_S(\mu_F^2)}{4\pi} B_n^{NLO}(\mu_F^2) + \cdots$$

$$B_n^{LO}(\mu_F^2) \;=\; B_n \left(\frac{\alpha_S(\mu_0^2)}{\alpha_S(\mu_F^2)}\right)^{\gamma_n/\beta_0} \qquad (\leq B_n)$$

Exclusive processes at higher-orders

Explicitly calculated higher-order corrections to exclusive processes:

(dimensional regularization, $\overline{MS}$ renormalization scheme)

♦ PHOTON-TO-π (η, η') TRANSITION FORM FACTOR

$$\gamma^* \quad \gamma \quad \rightarrow \quad \pi^0(\eta, \eta')$$

$$F_{\pi\gamma}(Q^2) = F_{\pi\gamma}^{(0)}(Q^2) + \frac{\alpha_S(\mu_R^2)}{4\pi} F_{\pi\gamma}^{(1)}(Q^2) + \frac{\alpha_S^2(\mu_R^2)}{(4\pi)^2} \left[\beta_0 \, F_{\pi\gamma}^{(2,\beta_0)}(Q^2, \mu_R^2) + \cdots \right] +$$

LO: (2 diagrams)

NLO: (12 one-loop diagrams)
Aguila, Chase (1981); Braaten (1983); Kadantseva, Mikhailov, Radyushkin (1986); Kroll, Passek-Kumerički (2003) [η, η': two-gluon states – 6 more diagrams]

β_0-proportional NNLO: (12 two-loop diagrams)
Melić, Nižić, Passek (2002)

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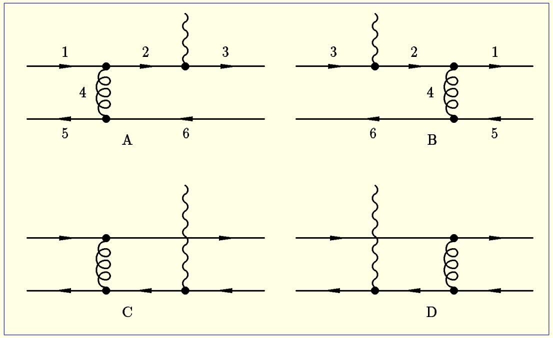
♦ PION ELECTROMAGNETIC FORM FACTOR

$$\gamma^* \quad \pi^{+(-)} \longrightarrow \quad \pi^{+(-)}$$

$$F_\pi(Q^2) = \frac{\alpha_S(\mu_R^2)}{4\pi} F_\pi^{(1)}(Q^2) + \frac{\alpha_S^2(\mu_R^2)}{(4\pi)^2} F_\pi^{(2)}(Q^2, \mu_R^2) + \cdots$$

LO:

(4 diagrams)



$\gamma^*(q_1\bar{q}_2) \rightarrow (q_1\bar{q}_2)$

NLO:

(62 one-loop diagrams)

Field, Gupta, Otto, Chang (1981); Dittes, Radyushkin (1981); Sarmadi (1982); Khalmuradov, Radyushkin (1985); Bratten (1987); Kadantseva, Mikhailov, Radyushkin (1986); Melić, Nižić, Passek (1999)

♦ PION PAIR PRODUCTION

$\gamma \quad \gamma \rightarrow \quad \pi^+\pi^-$

$$\mathcal{M}(s,t) = \frac{\alpha_S(\mu_R^2)}{4\pi}\mathcal{M}^{(1)}(s,t) + \frac{\alpha_S^2(\mu_R^2)}{(4\pi)^2}\mathcal{M}^{(2)}(s,t,\mu_R^2) + \cdots$$

LO: (20 diagrams)

NLO: (454 one-loop diagrams)

Nižić (1987), Duplančić, Nižić (2006)

$\gamma\gamma \rightarrow (q_1\bar{q}_2)(q_2\bar{q}_1)$

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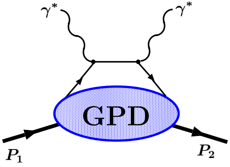
NOTE: related (sub)processes with nucleons

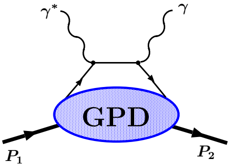
GPD... generalized parton distribution

Deeply virtual Compton scattering (DVCS)



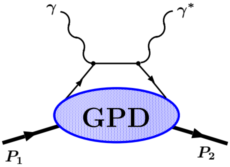
(double) DVCS

$\gamma^* p \to \gamma^* p$



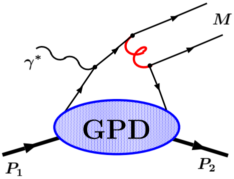
spacelike DVCS

$\gamma^* p \to \gamma p$



timelike DVCS

$\gamma p \to \gamma^* p$



Deeply virtual production of mesons (DVMP)

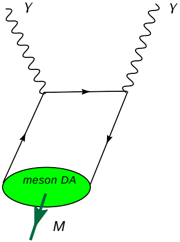
more difficult, but access to flavours

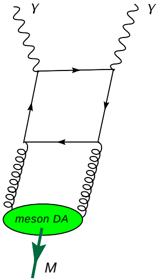
$\gamma^* p \to Mp$

Hard-scattering amplitudes (meson form factors vs. deeply virtual processes on nucleons)

Meson transition form factor

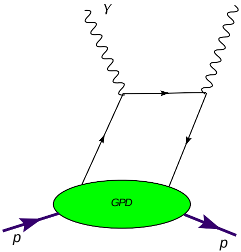
$$\gamma^*\gamma \to (q\bar{q}),\ \gamma^*\gamma \to (gg)$$

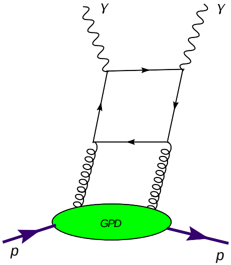




DVCS

$$\gamma^* q \to \gamma q,\ \gamma^* g \to \gamma g$$

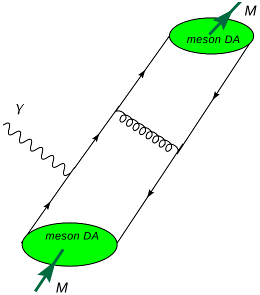




Hard-scattering amplitudes (meson form factors vs. deeply virtual processes on nucleons)

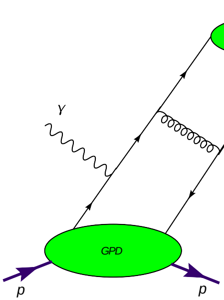
Meson em form factor

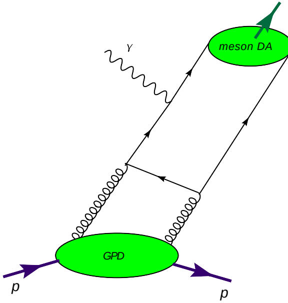
$$\gamma^*(q\bar{q}) \to (q\bar{q})$$



DVMP

$$\gamma^* q \to (q\bar{q})q,\ \gamma^* g \to (q\bar{q})g$$





Pion transition form factor

$$F_{\pi\gamma}(Q^2) = F^{(0)}_{\pi\gamma}(Q^2) + \frac{\alpha_S(\mu_R^2)}{4\pi} F^{(1)}_{\pi\gamma}(Q^2) + \cdots$$

$\mu_R^2 = \mu_R^2(Q^2) \ldots$ renormalization scale

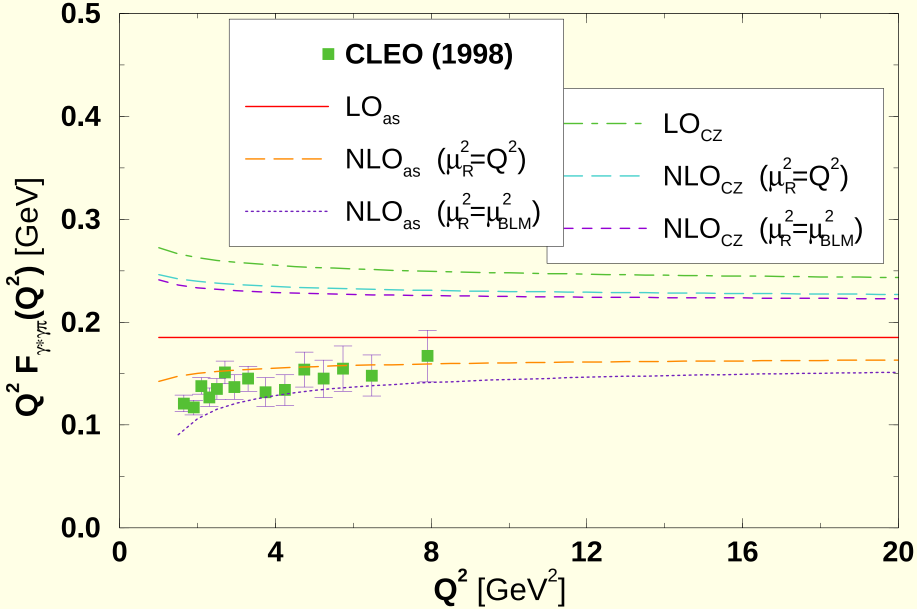
- power law $(1/Q^2)$ and logarithmic corrections

$$F_{\pi\gamma}(Q^2 \to \infty) = \frac{\sqrt{2} f_\pi}{Q^2} \quad \Leftarrow \quad \text{perturbative QCD}$$

$$F_{\pi\gamma}(Q^2 \to 0) = \frac{\sqrt{2}}{(4\pi^2) f_\pi} \quad \Leftarrow \quad \Gamma(\pi^0 \to \gamma\gamma), \text{axial anomaly}$$

$f_\pi = 0.131$ GeV

Numerical predictions for $F_{\pi\gamma}(Q^2)$



$(\mu^2_{BLM})^{as} \approx Q^2/9\,,$ $\alpha_S \leq 0.5$ for $Q^2 > 4$ GeV^2 !

$(\mu^2_V)^{as} \approx Q^2/2$

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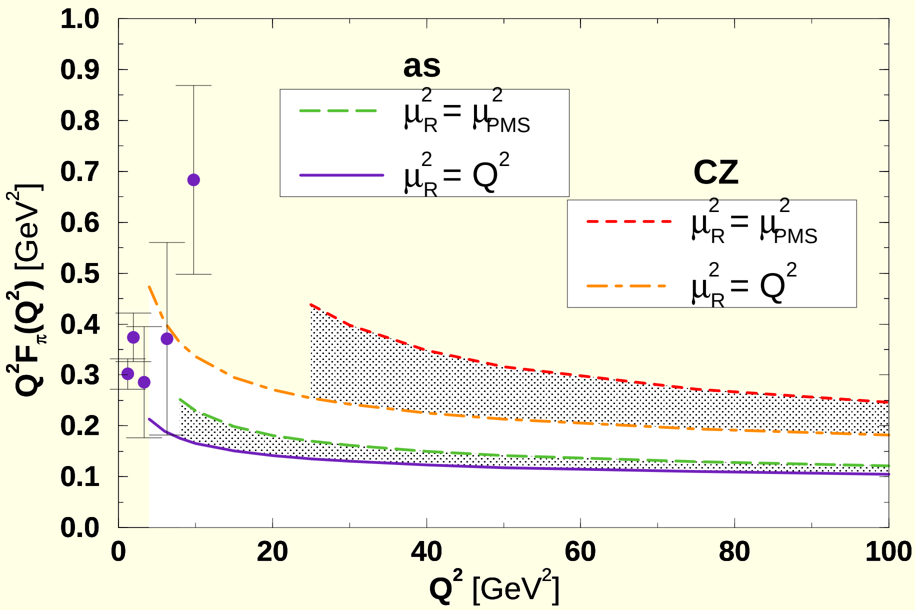
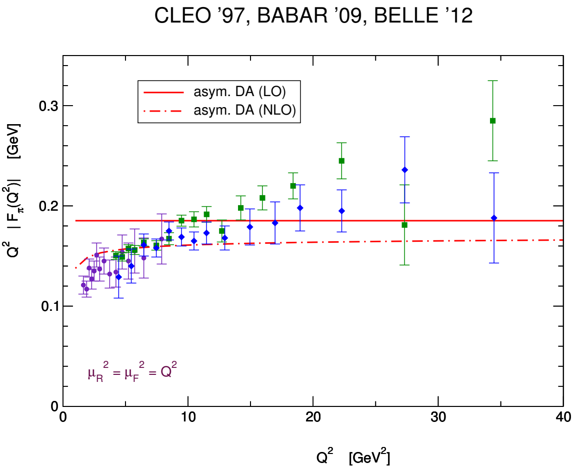
Numerical predictions for $F_\pi(Q^2)$



- $\mu_R^2 = Q^2$: NLO corrections large ($<30(50)\%$ for $Q^2 > 500(10)$ GeV^2)
- $\mu_R^2 = (\mu_{BLM}^2)^{as} \approx Q^2/106$: very small scale ! $\Rightarrow \alpha_S$ large

Experimental situation

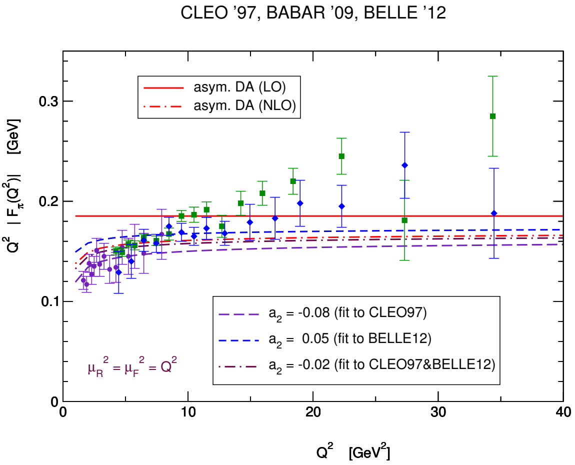


CLEO '97 (purple)

BABAR '09 (green)

BELLE '12 (blue)

Fits

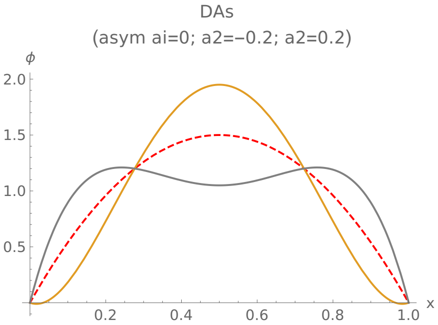


CLEO '97 (purple)

BABAR '09 (green)

BELLE '12 (blue)

DAs



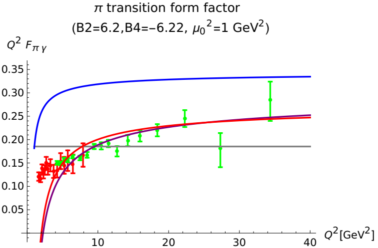
$a_i = 0$ (asym.) ... red

$a_2(1\text{GeV}^2) = -0.2$ ... brown

$a_2(1\text{GeV}^2) = 0.2$ ... gray

What about BaBar '09 data?

Example:

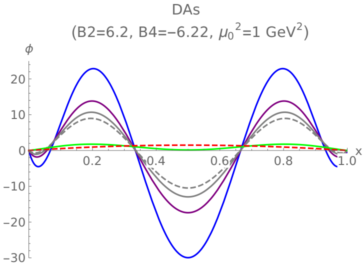


$B_2(1\text{GeV}^2) = 6.2$

$B_4(1\text{GeV}^2) = -6.22$

LO (blue)

NLO (red, purple)



$\phi(1\text{GeV}^2)\ldots$ blue

$\phi(10\text{GeV}^2)\ldots$ purple

$\phi(100\text{GeV}^2)\ldots$ grey

$\phi(1000\text{GeV}^2)\ldots$ dashed grey

$\phi(Q^2 \gg)\ldots$ green

$\phi(Q^2 \to \infty) = \phi_{asy}\ldots$ dashed red

On fits, on literature...

How much can the fits tell us?

- at most two coefficients to be determined

$$\Uparrow$$

tff contribution is fractional polynomial(s) in $t = \alpha_S(Q^2)$ variable with the range $0.2 < t < 0.4$, large correlations

Impact of tff results on literature:

- Round 1: a number of papers trying to accommodate BABAR '09 results, eg. flat DA [Radyuskin '09, Polyakov '09] ...
- Round 2: no definitive proof (neither from experimental nor theoretical side) but BELLE '12 results favoured in the literature

HARD EXCLUSIVE REACTIONS involving η and η'

Valence FOCK components of $M = \eta, \eta'$:

$$|q\bar{q}_8\rangle = |(u\overline{u} + d\overline{d} - 2s\overline{s})/\sqrt{6}\rangle \quad \text{(flavour-octet)}$$

$$\left\{ \begin{array}{l} |q\bar{q}_1\rangle = |(u\overline{u} + d\overline{d} + s\overline{s})/\sqrt{3}\rangle \quad \text{(flavour-singlet)} \\ \\ |gg\rangle \end{array} \right.$$

Novel features:

- flavour-mixing

  ($\Leftarrow$ SU(3)$_F$ broken, U(1)$_A$ anomaly)

(review: *Feldmann (2000)*)

We adopt:

$$\phi_{Mi} = \phi_i$$

$\Rightarrow$ the particle dependence and the flavour-mixing is solely embedded into the decay constants f_M^i

The decay constants are parameterized as

$$f_\eta^8 = f_8 \cos\theta_8 \,, \qquad f_\eta^1 = -f_1 \sin\theta_1$$
$$f_{\eta'}^8 = f_8 \sin\theta_8 \,, \qquad f_{\eta'}^1 = f_1 \cos\theta_1$$

(Leutwyler (1998), Feldmann, Kroll, Stech (1998,'99))

Novel features:

- flavour-mixing

  ($\Leftarrow$ $SU(3)_F$ broken, $U(1)_A$ anomaly)

  (review: *Feldmann (2000)*)

- contribution of the $|gg\rangle$ states and

  mixing of singlet and gluon DAs under evolution

  singlet $(\Phi_{M1} \equiv \Phi_{Mq})$, gluon DAs (Φ_{Mg})

$$\downarrow$$

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \begin{pmatrix} \Phi_{Mq} \\ \Phi_{Mg} \end{pmatrix} = \begin{pmatrix} V_{qq} & V_{qg} \\ V_{gq} & V_{gg} \end{pmatrix} \otimes \begin{pmatrix} \Phi_{Mq} \\ \Phi_{Mg} \end{pmatrix}$$

$$\Phi_{M1} \equiv \Phi_{Mq} = \frac{f_{M1}}{2\sqrt{2N_C}} \phi_{M1}, \qquad \Phi_{Mg} = \frac{f_{M1}}{2\sqrt{2N_C}} \phi_{Mg}$$

$$\int_0^1 dx\ \phi_{M1}(x, \mu_F^2) = 1 \quad \Leftarrow \quad \phi_{M1}(x, \mu_F^2) = \phi_{M1}(1-x, \mu_F^2)$$

$$\int_0^1 dx\ \phi_{Mg}(x, \mu_F^2) = 0 \quad \Leftarrow \quad \phi_{Mg}(x, \mu_F^2) = -\phi_{Mg}(1-x, \mu_F^2)$$

$$\phi_{M1}(x, \mu_F^2) = 6x(1-x) \left[1 + \sum_{n=2}^{\infty}{}' B_{Mn}^{1}(\mu_F^2)\ C_n^{3/2}(2x-1) \right]$$

$$\phi_{Mg}(x, \mu_F^2) = x^2(1-x)^2 \sum_{n=2}^{\infty}{}' B_{Mn}^{g}(\mu_F^2)\ C_{n-1}^{5/2}(2x-1)$$

$$B_{Mn}^{1}(\mu_F^2) = f\left(\underline{B_{Mn}^{1}(\mu_0^2)}, \underline{B_{Mn}^{g}(\mu_0^2)};\ \alpha_S(\mu_F^2),\ \gamma_n^{ij}\right)$$

$$B_{Mn}^{g}(\mu_F^2) = g\left(\underline{B_{Mn}^{g}(\mu_0^2)}, \underline{B_{Mn}^{1}(\mu_0^2)};\ \alpha_S(\mu_F^2),\ \gamma_n^{ij}\right)$$

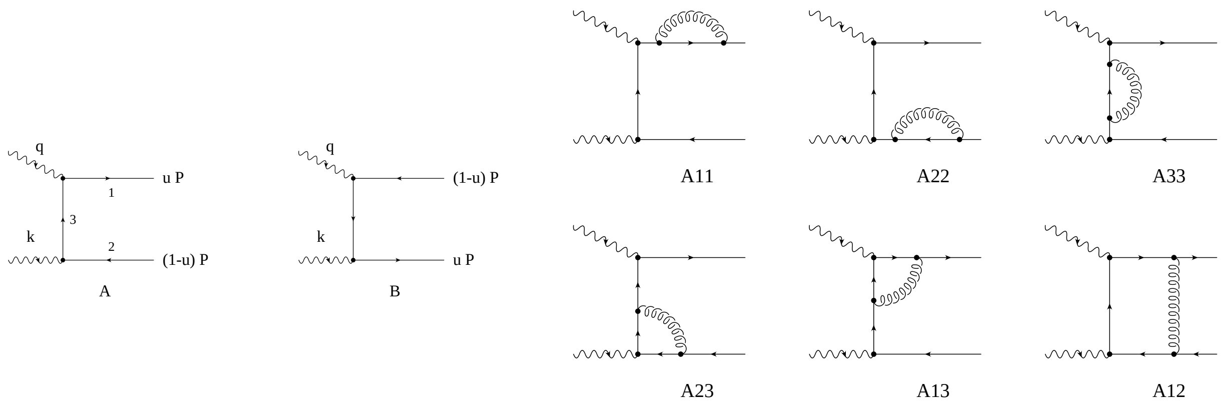
$\mu_0^2 \ldots$ initial scale

$$\gamma^*\gamma \to q\bar{q}_1 \quad \Rightarrow \quad T_{q\bar{q}}$$

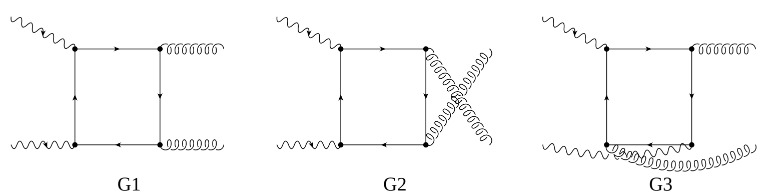
$$\gamma^*\gamma \to gg \quad \Rightarrow \quad T_{gg}$$

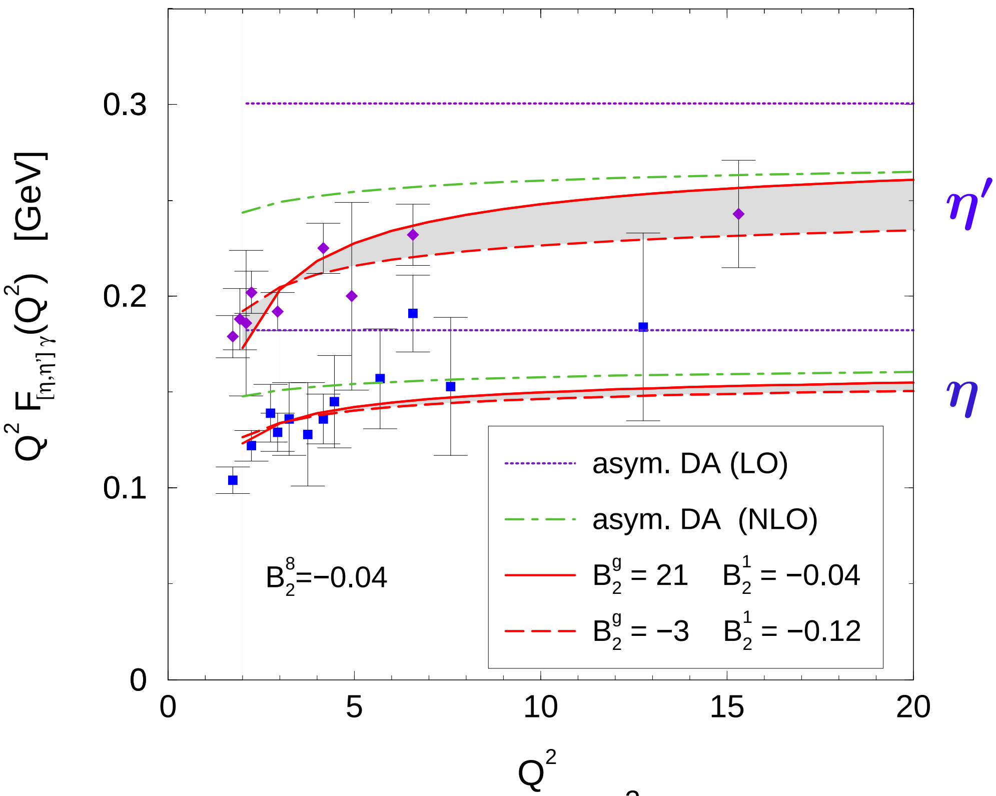
$T_{q\bar{q}}$

LO NLO



T_{gg}



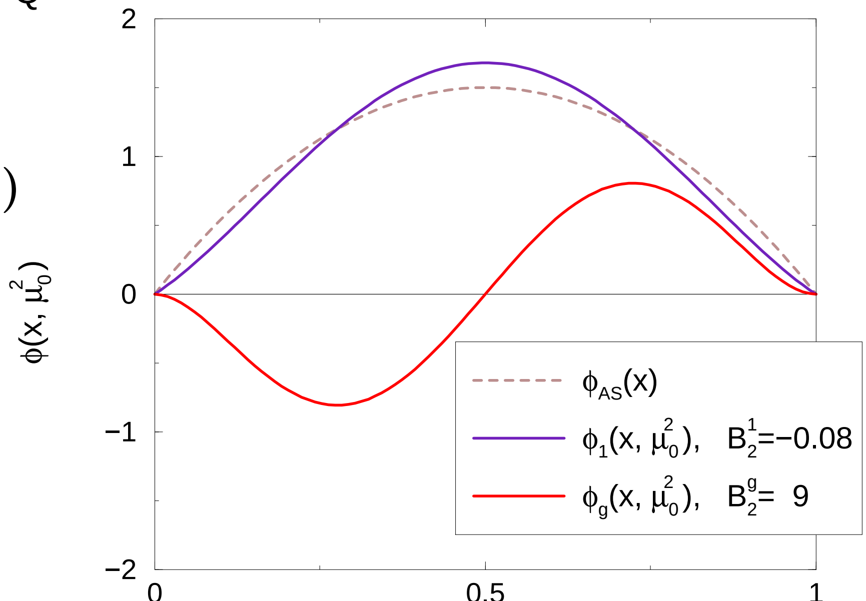


(exp:
CLEO (1998),
L3 (1998))

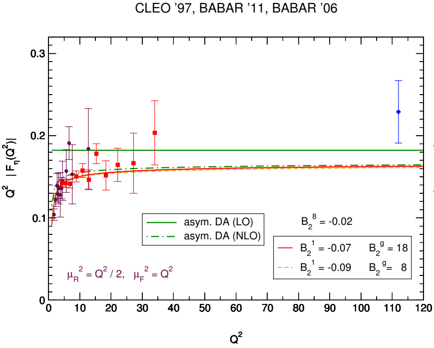
$$\mu_F^2 \to \infty:$$

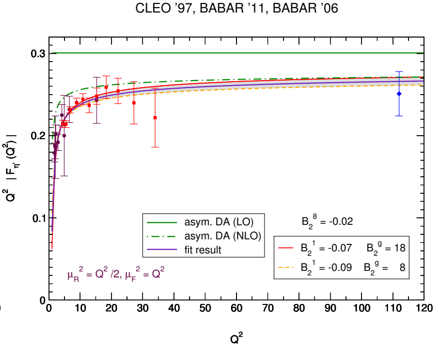
$$\phi_8, \phi_1 \;\to\; \phi_{AS} = 6x(1-x)$$

$$\phi_g \;\to\; 0$$



Experiments and fits



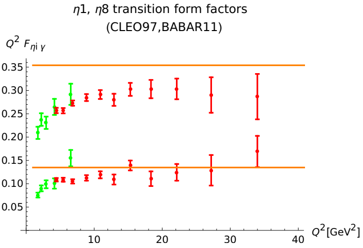


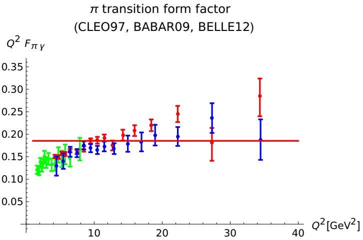
CLEO '97 (purple)

BABAR '11 (red)

BABAR '06 (blue) . . . timelike! (not used in fits)

Flavour singlet and flavour nonsinglet analysis

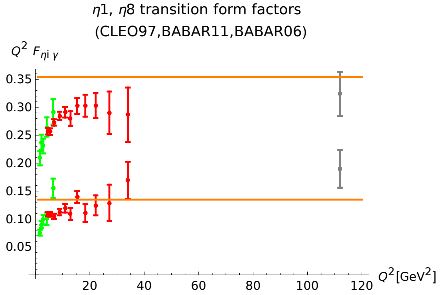




CLEO '97 (green)

BABAR '09,
BABAR '11 (red)

BELLE '12 (blue)



CLEO '97 (green)

BABAR '11 (red)

BELLE '06 (gray)

. . . timelike!

PSEUDOSCALAR CASE:

$$\begin{cases} \phi_{P1}(x,\mu_F^2) = 6x(1-x)\left[1 + \sum_{n=2}^{\infty}{}' B^1_{Pn}(\mu_F^2)\, C_n^{3/2}(2x-1)\right] \\ \phi_{Pg}(x,\mu_F^2) = x^2(1-x)^2 \sum_{n=2}^{\infty}{}' B^g_{Pn}(\mu_F^2)\, C_{n-1}^{5/2}(2x-1) \end{cases}$$

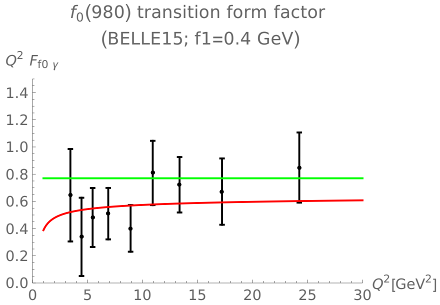
$$\begin{matrix} \int_0^1 dx\ \phi_{P1}(x,\mu_F^2) = 1 \\ \int_0^1 dx\ \phi_{Pg}(x,\mu_F^2) = 0 \end{matrix}, \quad \mu_F^2 \to \infty: \begin{matrix} \phi_{P1} \to 6x(1-x) \\ \phi_{Pg} \to 0 \end{matrix}$$

SCALAR CASE:

$$\begin{cases} \phi_{S1}(x,\mu_F^2) = x(1-x) \sum_{n=2}^{\infty}{}' B^1_{Sn}(\mu_F^2)\, C_{n-1}^{3/2}(2x-1) \\ \phi_{Sg}(x,\mu_F^2) = 30x^2(1-x)^2\left[1 + \sum_{n=2}^{\infty}{}' B^g_{Sn}(\mu_F^2)\, C_n^{5/2}(2x-1)\right] \end{cases}$$

$$\begin{matrix} \int_0^1 dx\ \phi_{S1}(x,\mu_F^2) = 0 \\ \int_0^1 dx\ \phi_{Sg}(x,\mu_F^2) = 1 \end{matrix}, \quad \mu_F^2 \to \infty: \begin{matrix} \phi_{S1} \to 0 \\ \phi_{Sg} \to 30x^2(1-x)^2 \end{matrix}$$

$f_0(980)$ transition form factor (experiment and fits)



BELLE '15 (black)

LO (green)

NLO (red)

Conclusions and outlook

- at the moment two contradicting sets of experimental data exist for the simplest exclusive process (pion transition form factor)
- the question of the form of pion DA is thus still open (more than before)
- fits inconclusive and cannot be improved significantly but should be tested on other processes (pion em form factor)
- η, η' transition form factor reexamined and first results on the scalar meson f_0^8 obtained
- a number of recent proposals for pion DA in the literature
- BELLE II data (2018?) expected to shed more light

Thank you! Köszönöm!