Scenario for the Possible Role of Potentially Observable Out-of-Equilibrium Effects on the Evolution of Heavy Stars

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Abstract

To study out-of-equilibrium effects on the evolution of heavy stars, we propose the following scenario: In the small volume somewhere within the low metallicity star we assume the species P, D, He_4, γ to be in a local equilibrium characterized by the (high) temperature T and chemical potentials μ_P, μ_D, and μ_{He_4}. The nuclear fusion reactions release hard (5MeV) photons: $P + D \rightarrow He_3 + \gamma$. Photons are produced with reaction rate R_γ and destroyed by pair production damping rate (by using one photon from heat bath) R_D. The "balance" is reached at number density of hard photons R_γ / R_D. The hard photons significantly enhance the pressure above the equilibrium values. We try to answer the question: At which temperatures and particle densities does the above pressure enhancement produce observable effects? Could it influence the evolution and destiny of stars?

It is generally believed that, inspite of the dramatical events in the life of the star, the most of the processes could be described as almost equilibrium processes, with the nonequilibrium features unimportant, or washed out by the next stages of evolution of stars, with a litlle or no chance to detect the traces. In this study we try to show that there are the exceptions from this rule.

General Scenario

In the small wolume somewhere within the star we assume a number of nuclear species A_1, A_2, ..., A_n, photons, electrons to be in a local equilibrium characterized by the temperature T and chemical potentials μ_1, μ_2, ...,μ_n and μ_e. When these parameters approach some treshold values The nuclear fusion reactions could be possible, with the release/apsorption of the energy in the form of, say photons:

$A_1 + A_2 \rightarrow A_3 + \gamma$

In such a medium it should not be expected that the additional photon (and as well the species A_3) be equilibrated. It is usually high energy photon capable of inverting the above process

$A_3 + \gamma \rightarrow A_1 + A_2,$

The activity of such photon is disturbed by other processes in the medium: production of e^+, e^- pairs in the scattering with photons in the medium $\gamma+\gamma' \rightarrow e^+ + e^-$, and in the scattering with nuclear species $\gamma + A_i \rightarrow e^+ e^- + A_i$, degradation of its energy by elastic scaterring with e^+, e^- and A_i . When the photon looses enough of energy, the inverse process stops and the photon is further "degraded" towards equilibrium thermal distribution. Our interest here is to find at which conditions this interplay becomes significant and to describe it quantitatively.

Now, if it happens that the fusion process is slow, slow enough that the processes degrading the hard photon, are comparable or faster than it, photon desintegration processes fail partly , or even completely. The result will be the increased growth of free energy and the nuclear species A_3. Tese are reflected in the evolution of a star.

This Paper Scenario

In the small wolume somewhere within the low metality star we assume nuclear species P, D, He_3, (there should a 2 5 % of He_4 as well but it will be, more or less ,passive) , photons, and electrons (positrons) to be in a local equilibrium characterized by the temperature T and chemical potentials μ_P, μ_D, μ_{He_3} and μ_e. When these parameters approach some treshold values The nuclear fusion reactions could be possible, with the release/apsorption of the energy in the form of, say photons:

$P + D \to He_3 + \gamma$

In such a medium it should not be expected that the additional photon (and as well the species He_3) be equilibrated. It is usually high energy photon capable of inverting the above process

$He_3 + \gamma \to P + D,$

The activity of such photon is disturbed by other processes in the medium:

production of e^+, e^- pairs in the scattering with photons in the medium $\gamma + \gamma' \to e^+ + e^-$,

scattering with nuclear species $\gamma + A_i \to e^+e^- + A_i$, Where A_i are p, D, and He_3

degradation of its energy by elastic scaterring with e^+, e^- and A_i . When the photon looses enough of energy, the inverse process stops and the photon is further "degraded" towards equilibrium thermal distribution. Our interest here is to find at which conditions this interplay becomes significant and to describe it quantitatively.

The total rate of photons produced is

$R_{PH} = R_{He_3} - R_{pd,He_3}$

and damped:

$D_{PH} = D_{pd,He_3} + D_{pd,D} + D_{el,e^-} + D_{el,p} + D_{el,D} + D_{el,He_3} + D_{pair,f} + D_{pair,e}$

The photons ($n(p_0, 0)$) existing at $t = 0$ are damped by $e^{-tD_{PH,p_0}}$,

those produced at time $\delta t'$, ($\delta t' R_{PH,p_0}$) are damped by $e^{-(t-t')D_{PH,p_0}}$,

Number of surviving energetic photons is

$n(p_0, t) = n(p_0, 0)e^{-tD_{PH}(p_0)} + \int_0^t dt' R_{PH}(p_0) e^{-(t-t')D_{PH}(p_0)}$,

$n(p_0, t) = [n(p_0, 0) - \frac{R_{PH}(p_0)}{D_{PH}(p_0)}]e^{-tD_{PH}(p_0)} + \frac{R_{PH}(p_0)}{D_{PH}(p_0)}$,

Very soon ($t_0 \approx 1/D_{PH}(p_0)$) photon density (at p_0 energy) reaches constant value $\bar{n}(p_0) = \frac{R_{PH}(p_0)}{D_{PH}(p_0)}$. It produces additional pressure not included in EOS (equation of state)

$\Delta P = \int dp_0 \frac{p_0 c}{3} \bar{n}(p_0) = \int dp_0 \frac{p_0 c}{3} \frac{R_{PH}(p_0)}{D_{PH}(p_0)}$

Or in our approximation where $R_{PH}(p_0) = R_{PH}(\bar{p}_0)\delta(p_o - \bar{p}_0)$

$\Delta P = \frac{\bar{p}_0 c}{3} \frac{R_{PH}(\bar{p}_0)}{D_{PH}(\bar{p}_0)}$

It should be compared to the total pressure of the gas, or (only for simplicity) to the pressure of ideal gas:

$P = NRT/V$

It is low if $R_{PH}(p_0) \leq D_{PH}(p_0)$ In this case photons depose all energy into a medium. Depending on heat transfer capacity, the temperature of medium is stayng constant (high transfer capacity) or grows (low transfer capacity).

The case $R_{PH}(p_0) >> D_{PH}(p_0)$ is particularly interesting as in such a case number of energetic photons is achieving a stacionary value $\frac{R_{PH}(p_0)}{D_{PH}}(p_0)$ weakly depending on t, it can greatly enlarge the preasure in medium, and

Thus the destiny of project depends on the ratio $\frac{R_{PH}(p_0)}{D_{PH}}(p_0)$. The crucial quantity appears to be $D_{pd,D}$ - photon desintegration of D - damping rate.

Other quantities are of QED origin and we may expect them to be slower than nuclear processes. In addition elastic scattering on p, D and He_3 is almost not slowing the energetic photon (in fact maybe slowing it just enough to prevent it to desintegrate He_3) and could be ignored in the simplest estimates.

1 Fussion Rate - Adelberger II

$$r_{12} = \frac{n_1 n_2}{1+\delta_{12}} < \sigma v >_{12} \tag{1}$$

n_1, n_2 are number densities
δ_{12} prevents double counting for identical particles

$$< \sigma v >= 1.3005 \times 10^{-15} [\frac{Z_1 Z_2}{A T_6^2}]^{1/3} f_0 S_{eff} \times exp(-\tau) cm^3 s^{-1}.$$
$$A = \frac{A_1 A_2}{A_1 + A_2}$$
$$E_0 = [(\pi \alpha Z_1 Z_2 kT)^2 (mAc^2/2)]^{1/3}$$
$$= 1.2204 (Z_1^2 Z_2^2 A T_9^{-1})^{1/3} keV.$$
$$\tau = \frac{3E_0}{kT} = 42.487 (Z_1^2 Z_2^2 A T_9^{-1})^{1/3}$$
$$f_0 = \exp(\frac{Z_1 Z_2 \alpha}{r_D kT}), \ r_D \ Debay \ radius \tag{2}$$

T_6 is expressed in $10^6 K$, E_0 - Gamow energy, the most probable energy at which the reaction occurres.
$D(p,\gamma)^3He$
$S_{eff} = S(0)[1 + \frac{5\tau}{108}] + S'(0) E_0 [1 + \frac{35\tau}{108}]$
$+ S''(0) E_0^2 [1 + \frac{89\tau}{108}]$
$S(0) = 2.14^{.17}_{-16} \times 10^{-4} keV b$
$S'(0) = 5.56^{+.18}_{-.20} \times 10^{-6} b$
$S''(0) = 9.3^{+3.9}_{-3.4} \times 10^{-9} b/keV$
$\pm 7.1\%$ Gamow peak uncertainty
f_0 effective electronic screening on nuclear reactions in the solar plasma

2 Pair Production off the Thermal Photon - Akhiezer-Beresteckij

Calculate damping rate - described by two Feynman diagrams

$$|\bar{u}Qv|^2 = \frac{1}{4}TrF$$
$$-\frac{1}{8}TrF = 4[\frac{1}{\kappa_1}+\frac{1}{\kappa_2}]^2 - 4[\frac{1}{\kappa_1}+\frac{1}{\kappa_2}] - [\frac{\kappa_2}{\kappa_1}+\frac{\kappa_1}{\kappa_2}],$$
$$q_1 = -p_+ + k = p_- - k',\ q_2 = -p_+ + k' = p_- - k$$

$$m^2\kappa_1 = m^2 - q_1^2,\ \ m^2\kappa_2 = m^2 - q_2^2$$

$$F_{D,pair} = [f(\omega_{k'})(1-f(\omega_{p_-})(1-f(\omega_{p_+}))$$

$$-(1+f(\omega_{k'}))f(\omega_{p_-}))f(\omega_{p_+})],$$

$$d\sigma = \frac{e4}{(2\pi)^2}|\bar{u}Qv|^2\frac{d^3p_+d^3p_-}{4p_{0+}p_{0-}4J}\delta(k_0+k_0'-p_{0+}-p_{0-})\delta^3(\vec{k}+\vec{k}'-\vec{p}_+-\vec{p}_-),$$
$$J = 2\omega_0^2,\ SCM. \qquad (3)$$

This expression is further integrated over e_- and e_+ momenta to obtain

$$\sigma(\omega_0) = \Theta(\omega_0 - m)\frac{\pi r_0^2 m^2}{\omega_0^2}[(2+\frac{2m^2}{\omega_0^2}-\frac{m^4}{\omega_0^4})ln[\frac{\omega_0}{m}+[\frac{\omega_0^2}{m^2}-1]^{1/2}] - [1-\frac{m^2}{\omega_0^2}]^{1/2}[1+\frac{m^2}{\omega_0^2}]]$$
$$r_0 = \frac{e^2}{4\pi m}. \qquad (4)$$

To be compared to the expressions for production of a pair in the electirc field of nucleus with Z protons without and with screening:

$$\sigma_{eff.ph.} = \frac{28Z^2r_0^2}{9\times 137}[ln\frac{2\omega_0}{m}-\frac{109}{42}-f(Z)],$$
$$\sigma_{eff.ph.scr.} = \frac{28Z^2r_0^2}{9\times 137}[ln(183Z^{-1/3})-\frac{1}{42}-f(Z)]$$

$$f(Z) = (Z\alpha)^2 \sum_{n=1}^{\infty} \frac{1}{n^2 + (Z\alpha)^2}. \tag{5}$$

For $Z = 1, 2$ (i.e. from $e_-, p, {}^3He, {}^4He$) not competitive with thermal photon contribution.

Damping by Pair Production

$$R_D = 4 \int (2\pi)^4 \delta^4(p_\gamma + p_{\gamma T} - p_- - p_+)|M|^2$$
$$\frac{d^3 p_{\gamma T}}{(2\pi)^3} \frac{d^3 p_-}{(2\pi)^3} \frac{d^3 p_+}{(2\pi)^3} F_D \tag{6}$$

Factor 4 stands for sum over initial photon polarisations. By assuming that there are very few e_- and e_+ in the medium, Damping rate for pair production is obtained by (4)

$$J\sigma(\omega_0) = \int |M|^2 \frac{d^3 p_-}{(2\pi)^3} \frac{d^3 p_+}{(2\pi)^3}$$
$$J = 2\omega_0^2, \quad C.M., \quad J = (p_\gamma + p_T)^2/2, \quad anyframe$$
$$F_D \approx f(\omega_{p_{\gamma T}, \gamma})$$
$$R_D = 4 \int \frac{d^3 p_{\gamma T}}{(2\pi)^3} 2\omega_0^2 \sigma(\omega_0) f(\omega_{p_{\gamma T}, \gamma})$$
$$= \frac{4}{(2\pi)^2} \int_{-1}^{1} dz \int_{\omega_{T,m}}^{\infty} d\omega_T \omega_T^2 2\omega_0^2 \sigma(\omega_0) \frac{1}{e^{\omega_T/T} + 1}$$
$$\omega_0^2 = p_\gamma p_{\gamma'}/2 = \omega_T \omega (1 - z)/2$$
$$\omega_0 - m > 0 \to 1 - \frac{2m^2}{\omega\omega_T} > z$$
$$\to \omega_T > \omega_{T,m} = \frac{2m^2}{\omega(1 - z)} \tag{7}$$

3 Results

$T\ [MeV]$	$R_D\ [MeV^3]$	$< \sigma v >_{12}\ [\frac{cm^3}{s}]$	$\frac{<\sigma v>_{12}}{R_D}\ [\frac{cm^3}{MeV^3 s}]$		
0.1	1.76008×10^{-7}	$1.59 \times 10{-}22$	9.03×10^{-16}		
0.2	2.04471×10^{-6}				
0.3	8.13441×10^{-6}				
0.4	2.13377×10^{-5}	$3.08 \times 10{-}22$	1.44×10^{-17}		
0.5	4.47807×10^{-5}				
0.6	8.17647×10^{-5}	$3.20 \times 10{-}22$	3.91×10^{-18} 7	0.7	1.35731×10^{-4}
0.8	2.10239×10^{-4}	$3.84 \times 10{-}22$	1.82×10^{-18}		
0.9	3.08949×10^{-4}				
1.0	4.35608×10^{-4}	$4.26 \times 10{-}22$	9.98×10^{-19}		

4 Conclusions (preliminary!)

Scenario with the enhancement of pressure within the heavy star, by assuming that hard photons could survive long enough, is verified on simple $D(p,\gamma)^3He$ process. We find

1. number of hard photons (i.e.) R_γ/R_D ratio is rising with the densities of P and D
2. It is falling with the rise of temperature
3. The effect on heavy star evolution is doomed by (assumed) low N_D/N_P ratio (26×10^{-6}) after primordial nucleo synthesis.
4. Try with other more abundant constituents with low Z_1 and Z_2!

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