Renormalization of bilocal potentials

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TRADITIONAL RG

- The functional renormalization group technique (RG) is used to find the relevant interactions and describe the phase structure of various models.
- This method enables us to remove the degrees of freedom of a physical system successively.
- The traditional RG technique
  - It is based on the scale invariance of the transition amplitude between an in and an out **(pure) states**.
  - The blocking transformation takes into account only the contributions of the **pure states**.

Motivation, CTP (Schwinger-Keldysh) formalism

- We can take into account the contribution of the **mixed states** → Closed time path (CTP, Schwinger-Keldysh) formalism
- **CTP**:
  - generating functional:

$$Z[j^+, j^-] = \mathrm{Tr}[U(t_f, t_i; j^+)\rho_i U^\dagger(t_f, t_i; -j^-)]$$

  - we can consider expectation values
  - initial state →final state (reflected)→ initial state
  - reflection → closed path and nontrivial connection between the two time axes
  - interaction between the time axes → **non-local potential**

    → **Bilocal potential**

Motivation and the bilocal potential

- bilocality → two well separated points → more general treatment
- the difficulty: momentum dependent bilocal couplings
- motivation: CTP Minkowski formalism containing bilocal potential
- **STP bilocality** has not been investigated yet.
- Our main **goal** is to get the evolution equation for the bilocal potential and investigate the phase structure.
- We use Euclidean formalism.

Investigation of the bilocal potential in traditional RG

- STP bilocality
  - nontrivial saddle point evolution
  - separation of the evolution of the bilocal potential into a saddle point and the loop contributions
  - find a self-consistent system of flow equation for a closed set of the couplings
  - momentum dependent bilocal couplings
  - use Wegner-Houghton equation
  - determine the evolution of the bilocal potential beyond the tree-level approximation
  - investigate the phase structure

Bilocal potential in traditional RG

- The model: 3d ϕ^4

- The Euclidean action

$$S = \frac{1}{2}\int_x \phi_x D_0^{-1}\phi_x + \int_x U(\phi_x) + \int_{xy} V_{x-y}(\phi_x,\phi_y)$$

Local potential:

$$U(\phi) = \sum_{n=0}^{\infty} \frac{g_n}{n!}\phi^n$$

Bilocal potential

$$V_{x-y}(\phi_1,\phi_2) = \sum_{mn\geqslant 1} \frac{v_{x-ymn}}{m!n!}\phi_1^m\phi_2^n$$

$$V_{x-y}(\phi_1,\phi_2) = V_{y-x}(\phi_2,\phi_1)$$

WH EQUATION

- We use Wegner-Houghton equation
- ϕ is separated into two terms: $\phi \to \phi + \varphi \to$ sharp cutoff
  - ϕ denotes the IR component , that is non-vanishing for $0 < |p| < k - \Delta k$
  - φ stands for the UV term , that is non-vanishing for $k - \Delta k < |p| < k$
- The elimination of the UV modes

$$e^{-\frac{1}{\hbar}S_{k-\Delta k}(\phi)} = \int D[\varphi]e^{-\frac{1}{\hbar}S_k[\phi+\varphi]}$$

- Evolution equation

$$S_{k-\Delta k}(\phi) = S_k[\phi + \varphi_0] + \frac{\hbar}{2}\mathrm{Tr}\ln \frac{\delta^2 S_k[\phi + \varphi_0 + \varphi]}{\delta\varphi\delta\varphi}_{|\varphi=0}$$

WH EQUATION

- We consider the evolution equation at $\phi_x = \Phi + \chi_x$ where Φ and χ_x denote a homogeneous and a generic, infinitesimal, inhomogeneous IR field.
- The form of the action up to $\mathcal{O}(\varphi_x^2)$ term

$$S[\varphi_x^2] = \frac{1}{2}\int_{xy} \varphi_x D^{-1}_{x-y}\varphi_y + \int_x L_x\varphi_x,$$

- where the inverse propagator on the inhomogeneous IR field is

$$D^{-1}_{xy} = D^{-1}_{0x-y} + \delta_{xy}[U''(\chi) + 2\partial_1^2 V_{x-y}(\chi_x,\chi_y)] + 2\partial_1\partial_2 V_{x-y}(\chi_x,\chi_y)$$

- and

$$L_x = U'(\chi_x) + 2\int_y \partial_1 V_{x-y}(\chi_x,\chi_y)$$

we set $\Phi = 0$

WH EQUATION

- $\mathcal{O}(\chi_x^0)$ tree-level contributions
- $\mathcal{O}(\chi_x^2)$ fluctuations
- φ_{0x} denotes the saddle point,

$$\varphi_{0x} = -\int_y D_{xy} L_y$$

- The corresponding tree-level change of the action is

$$\Delta S^t = -\frac{1}{2}\int_{xy} L_x D_{xy} L_y$$

BILOCALITY AND TREE-LEVEL

- **Bilocal potential** and tree-level:
  - STP euclidean: there is a tree-level evolution
    → there is a non trivial saddle point
    → it is bilocal, we can follow its evolution. The evolution of the saddle point is determined by the tree level evolution.

- **Local potential** → we have tree-level evolution → We can **not** follow the evolution of the saddle point.
- **Bilocal potential** → we have tree-level evolution → we can follow the evolution of the saddle point.

- RG equation: Wegner-Houghton equation

Wegner-Hougton equation

- sharp cutoff
- we can linearize the evolution equation→saddle point
  →we have tree-level evolution
  →we can determine the saddle point

Wetterich equation

- smooth cutoff
- we can **not** linearize the evolution equation
  →difficult numerical problem

Evolution equations:

Local couplings

$$\dot{g}_2 = -\alpha_3 k^3 \frac{g_4}{\omega_k^2} - 2\dot{v}_{011}$$

$$\dot{g}_4 = -\alpha_3 k^3 \frac{g_6}{\omega_k^2} + \alpha_3 k^3 \frac{3g_4^2}{\omega_k^4} - 6\dot{v}_{022}$$

$$\dot{g}_6 = \alpha_3 k^3 \frac{15 g_4 g_6}{\omega_k^4} - \alpha_3 k^3 \frac{30 g_4^3}{\omega_k^6}$$

where

$\omega_k^2 = k^2 + g_2 + 2v_{k11}, \ \ \alpha_d = \frac{\Omega_d}{2(2\pi)^2}$

Bilocal couplings

- Tree-level evolution:
  $\dot{v}_{q33} = \frac{k}{2\omega_k^2} g_4^2 \delta_{k,q}$
- Loop evolution:

$$\dot{v}_{011} = -2\alpha_3 \frac{k^3}{\omega_k^2} v_{k22}$$

$$\dot{v}_{022} = -2\alpha_3 \frac{k^3}{\omega_k^2} v_{k33}$$

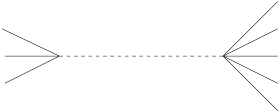
$$\dot{v}_{q11} = -\frac{\alpha_2}{\pi} \frac{k}{\omega_k^2} \int_p v_{p22}$$

$$\dot{v}_{q22} = -\frac{\alpha_2}{\pi} \frac{k}{\omega_k^2} \int_p v_{p33}$$

Couplings

- Momentum dependent tree-level bilocal coupling:

$$\dot{v}_{q33} = \frac{k}{2\omega_k^2} g_4^2 \delta_{k,q}$$

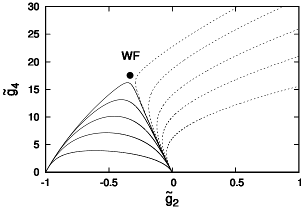


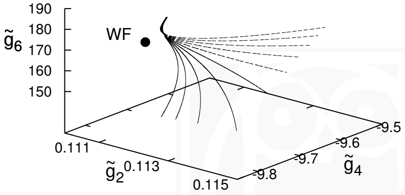
- $v_{q\mathbf{33}}$ → we should include the evolution of $g_{\mathbf{6}}$
- local potential → quartic coupling
- bilocal potential → at least the sixth order couplings needed to get the evolution
- closed system of couplings

- Phase diagram:

Local potential

Bilocal potential





  - 2 fixed points: Gaussian and Wilson-Fisher fixed points
  - 2 phases: the symmetric and the broken symmetric phases

- **Fixed points**
  - Local potential:
    - Gaussian fixed point:

$$\tilde{g}_2^* = 0, \quad \tilde{g}_4^* = 0, \quad \tilde{g}_6^* = 0$$

    - Wilson-Fisher fixed point:

$$\tilde{g}_2^* = -\frac{1}{3}, \quad \hbar_l \tilde{g}_4^* = \frac{16\pi^2}{9}, \quad \hbar_l^2 \tilde{g}_6^* = \frac{256\pi^4}{27}$$

  - Bilocal case: (tree-level) $(h = \hbar_b/\hbar_l)$
    - Gaussi fixed point:

$$\tilde{g}_2^* = 0, \quad \tilde{g}_4^* = 0, \quad \tilde{g}_6^* = 0$$

    - Wilson-Fisher fixed point:

$$\tilde{g}_2^* = -\frac{1}{3-12h}, \quad \hbar_l \tilde{g}_4^* = \frac{16\pi^2(1-6h)}{9(1-4h)^2}, \quad \hbar_l^2 \tilde{g}_6^* = \frac{256\pi^4(1-6h)}{27(1-4h)^3}$$

  - significant difference between the local and bilocal cases
  - non-continuous transition $(0 \leqslant h \leqslant 1)$

Outlook

- S. Nagy, J. Polonyi, and I. Steib, *Bilocal Euclidean scalar field theory*, in prep.
- semiclassical vacuum
- CTP loop corrections

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Thank you for your attention!