Aspects of Schwinger-Dyson model description of pseudoscalar mesons at $T > 0$

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Hrvatska zaklada za znanost

Introduction

- Dyson-Schwinger (DS) approach: ranges from solving DS equations for Green's functions of non-perturbative QCD *ab initio*, to very high degrees of phenomenological modeling, esp. in applications including $T, \mu > 0$.

  e.g., [Alkofer, v.Smekal Phys. Rept. 353 (2001) 281], and [Roberts, Schmidt Prog.Part.Nucl.Phys. 45 (2000)S1]

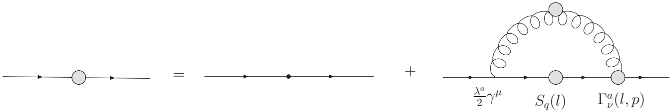
- In any case, DS approach to quark-hadron physics = nonpertubative, covariant bound state approach with strong connections with QCD.
- e.g., understanding chiral symmetry (ChS) & its breaking (esp. dynamical, DChSB), is crucial for understanding QCD ground state and its excitations
- $\rightarrow$ It is important that DS approach has chiral behavior as in QCD: light pseudoscalar octet mesons = both $q\bar{q}'$ composites and almost-Goldstones: $M^2_{q\bar{q}'} = \mathrm{const}\,(m_q + m_{q'})$ for consistent truncations, e.g., R-L
- develop a model that works at $T = 0$

  $\rightarrow$ extrapolate it to $T \geq 0$ (and $\mu > 0$).

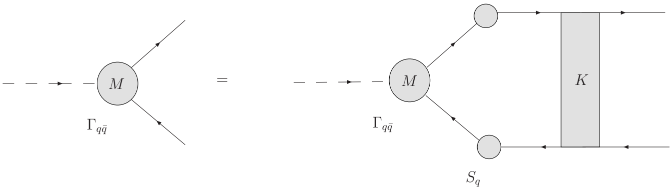
  We need models easily applicable to different phenomena and **as simple as possible – but not simpler!** (Einstein)

Dyson-Schwinger approach to quark-hadron physics

- Gap equation for propagator S_q of dressed quark q



- Homogeneous Bethe-Salpeter (BS) equation for a *M*eson $q\bar{q}$ bound state vertex $\Gamma_{q\bar{q}}$



Gap and BS equations in the rainbow-ladder truncation

$$S_q(p)^{-1} = i\gamma \cdot p + \widetilde{m}_q + \frac{4}{3} \int \frac{d^4\ell}{(2\pi)^4}\, g^2 G^{\rm eff}_{\mu\nu}(p-\ell)\gamma_\mu S_q(\ell)\gamma_\nu$$

$$\to S_q(p) = \frac{1}{i\not{p}A_q(p^2) + B_q(p^2)} = \frac{-i\not{p}A_q(p^2) + B_q(p^2)}{p^2 A_q(p^2)^2 + B_q(p^2)^2} = \frac{1}{A_q(p^2)}\,\frac{-i\not{p} + m_q(p^2)}{p^2 + m_q(p^2)^2}$$

$$\Gamma_{q\bar{q}'}(p,P) = -\frac{4}{3} \int \frac{d^4\ell}{(2\pi)^4} g^2 G^{\rm eff}_{\mu\nu}(p-\ell)\gamma_\mu S_q(\ell + \frac{P}{2})\Gamma_{q\bar{q}'}(\ell,P) S_q(\ell - \frac{P}{2})\gamma_\nu$$

- Euclidean space: $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$, $\gamma_\mu^\dagger = \gamma_\mu$, $a \cdot b = \sum_{i=1}^4 a_i b_i$
- P is the total momentum, $M^2 = -P^2$ meson mass^2
- $G^{\rm eff}_{\mu\nu}(k)$ an "effective gluon propagator" - modeled !

Renormalization-group improved interactions

$$\text{Landau gauge gluon propagator}: \; g^2 G^{\text{eff}}_{\mu\nu}(k) = G(-k^2)(-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}),$$

$$G(Q^2) \equiv 4\pi \frac{\alpha_s^{\text{eff}}(Q^2)}{Q^2} = G_{\text{UV}}(Q^2) + G_{\text{IR}}(Q^2), \qquad Q^2 \equiv -k^2 \; .$$

$$G_{\text{UV}}(Q^2) = 4\pi \frac{\alpha_s^{\text{pert}}(Q^2)}{Q^2} \approx \frac{4\pi^2 d}{Q^2 \ln(x_0 + \frac{Q^2}{\Lambda^2_{\text{QCD}}})} \{1 + b \, \frac{\ln[\ln(x_0 + \frac{Q^2}{\Lambda^2_{\text{QCD}}})]}{\ln(x_0 + \frac{Q^2}{\Lambda^2_{\text{QCD}}})}\} \; ,$$

- but modeled non-perturbative part, e.g., Jain & Munczek:

$$G_{\text{IR}}(Q^2) = G_{\text{non-pert}}(Q^2) = 4\pi^2 \, a \, Q^2 \, \exp(-\mu Q^2) \quad (\text{similar}: \; \text{Maris, Roberts...})$$

- or, the dressed propagator with dim. 2 gluon condensate $\langle A^2 \rangle$-induced dynamical gluon mass (Kekez & Klabučar):

$$G(Q^2) = 4\pi \frac{\alpha_s^{\text{pert}}(Q^2)}{Q^2} \left(\frac{Q^2}{Q^2 - M^2_{\text{gluon}} + \frac{c_{\text{ghost}}}{Q^2}} \right)^2 \frac{Q^2}{Q^2 + M^2_{\text{gluon}} + \frac{c_{\text{gluon}}}{Q^2}} \; .$$

Separable model = phenom. successful, + easier at $T > 0$

- Simplifying **separable Ansatz**: $G^{\rm eff}_{\mu\nu}(p-q) \to \delta_{\mu\nu}\, G(p^2, q^2, p\cdot q)$

  where $$G(p^2, q^2, p\cdot q) = D_0\; f_0(p^2) f_0(q^2) + D_1\; f_1(p^2)(p\cdot q) f_1(q^2)$$

- If the **both** form factors $f_0(p^2), f_1(p^2)$ and their respective strength parameters D_0, D_1 are **nonvanishing**, this is a **rank-2 model.**
  If only $f_0(p^2) \neq 0 \neq D_0$, this is a **rank-1 model.**

- In the separable model, the gap equation yields

$$\begin{aligned} B_f(p^2) &= \widetilde{m}_f + \frac{16}{3}\int \frac{d^4q}{(2\pi)^4} G(p^2, q^2, p\cdot q)\frac{B_f(q^2)}{q^2 A_f^2(q^2) + B_f^2(q^2)} \\ \Big[A_f(p^2) - 1\Big]\, p^2 &= \frac{8}{3}\int \frac{d^4q}{(2\pi)^4} G(p^2, q^2, p\cdot q)\frac{(p\cdot q) A_f(q^2)}{q^2 A_f^2(q^2) + B_f^2(q^2)}\,. \end{aligned}$$

- This gives $B_f(p^2) = \widetilde{m}_f + b_f\; f_0(p^2)$ and $A_f(p^2) = 1 + a_f\; f_1(p^2)$, reducing to nonlinear equations for constants b_f and a_f. For rank-1, $A_f(p^2) = 1$.

- Due to loss of $O(4)$ symmetry at $T > 0$,
  $i\not{p} A_f(p^2) \longrightarrow i\vec{\gamma}\cdot\mathbf{p}\, A_f({p_n}^2) + i\gamma_4\omega_n C_f({p_n}^2)$, where $p_n = (\omega_n, \vec{p})$ and $\omega_n = (2n+1)\pi T$, implying $a_f \to \{a_f, c_f\}$ in the separable models.

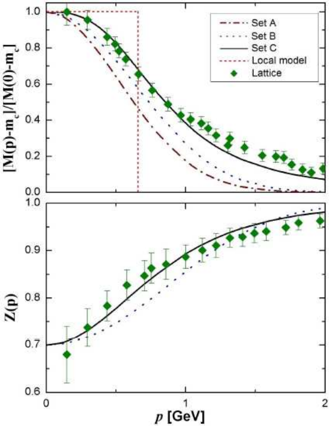
Non-local DSE models are much closer to QCD than NJL is:

- in NJL - severe divergences (which must be regularized with very low cutoffs $\Lambda \sim 1$ GeV or less).

Not so in non-local mod's!

- in NJL – no confinement! $M_q =$ const, so quarks can come on mass shell $p^2 = -M_q^2$ even at low E.
- in NJL $Z_q = 1$
- but **in QCD**, $M_q = = M_q(p^2) = B(p^2)/A(p^2)$, $Z_q = Z_q(p^2) = 1/A(p^2)$, **also in non-local models!**

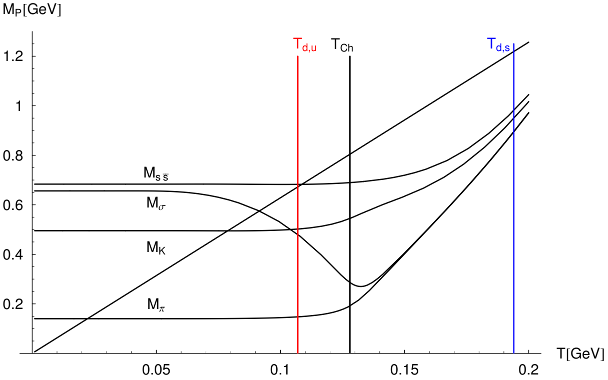
$M_q(p^2)^2 \neq -p^2 \Rightarrow$ no free quarks
Fit non-local DSE models to lattice to mimic QCD!



pictures taken from [Noguera, Scoccola, PRD78 (2008) 114002]

Dynamical chiral symmetry breaking (DChSB) and its restoration at high T

- DChSB dresses light ($q = u, d, s$) current quarks and so creates much more massive constituent quarks, and QCD vacuum condensates $\langle q\bar{q}\rangle$, and (very light) pseudoscalar mesons as (almost-) Goldstone bosons:



- ... while the rank-1 model gives $T_{d,u} = 126$ MeV, $T_{\rm Ch} = 128$ MeV and $T_{d,s} = 166$ MeV – better, but still not good in the view of lattice results!
- very different deconfinement temperatures $T_{d,u}$, $T_{d,s}$ ... can be synchronized with $T_{\rm Ch}(= T_{cri})$ by Polyakov loop

Polyakov loop

- low T: gluons "confined" in "flux tubes", high T gluons quasi–free
- Wilson line in (imaginary, $t \to i\beta$) time direction

$$\Phi(\mathbf{x}) = \frac{1}{N_c} \mathrm{Tr} \left[\exp \left(ig \int_0^\beta A_4(\mathbf{x}, \tau) d\tau \right) \right]$$

- order parameter for confinement of static color sources: $\langle \Phi(\mathbf{x}) \rangle = e^{-\beta F_q(\mathbf{x})}$, expect. value of PL measures F_q = free energy of an external static quark
- confinement: $F_q \to \infty$, $\Phi \to 0$ an isolated quark would cost ∞ energy
- deconfinement: $F_q \neq \infty$, $\Phi \neq 0$ states with a single quark possible
- Mean field approximation, $\langle \Phi(\mathbf{x}) \rangle = \Phi$. At $\mu = 0$: $\Phi^* = \Phi$.
- the simplest case: if the only PL background field is ϕ_3:

$$\Phi = \frac{1}{N_c} \left(1 + 2 \cos \frac{\phi_3}{T} \right) , \quad \phi_3 = g \frac{A_4^3}{2}$$

Coupling to quarks - by modifying their Matsubaras (α = quark colors):

$$\omega_n \to \omega_n^\alpha = (2n+1)\pi T + \alpha \phi_3 + (3|\alpha| - 2) \frac{\phi_8}{\sqrt{3}} , \quad (\alpha = -1, 0, 1)$$

(green denotes a possible refinement of the PL contribution)

Effective potential for PL

- due to inputs from lattice

i) Logarithmic [Roesner, Ratti, Weise, PRD75 (2007) 034007]

$$\frac{\mathcal{U}}{T^4} = -\frac{1}{2}a\Phi^*\Phi + b\log[1 - 6\Phi^*\Phi + 4(\Phi^3 + \Phi^{*3}) - 3(\Phi^*\Phi)^2],$$

$$a(T) = a_0 + a_1\frac{T_0}{T} + a_2\left(\frac{T_0}{T}\right)^2 \qquad b(T) = b_3\left(\frac{T_0}{T}\right)^3,$$

$a_0 = 3.51,\ a_1 = -2.47,\ a_2 = 15.22,\ b_3 = -1.75$

ii) Polynomial [Ratti, Thaler, Weise, PRD73:014019,2006] is an alternative

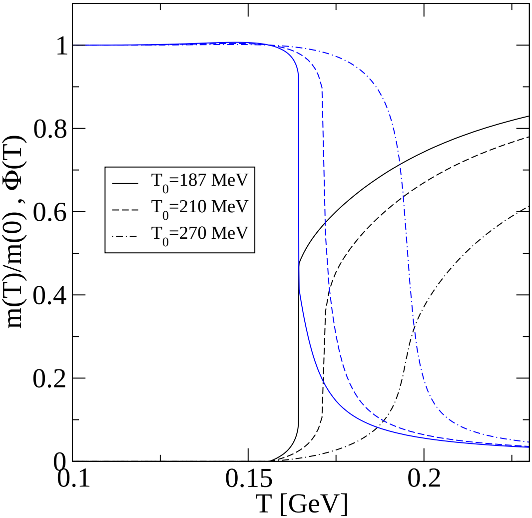
$$\frac{\mathcal{U}}{T^4} = -\frac{b_2}{2}(|\Phi|^2 + |\Phi^*|^2) - \frac{b_3}{6}(\Phi^3 + (\Phi^*)^3) + \frac{b_4}{16}(|\Phi|^2 + |\Phi^*|^2)^2$$

$$b_2(T) = a_0 + a_1\frac{T_0}{T} + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3$$

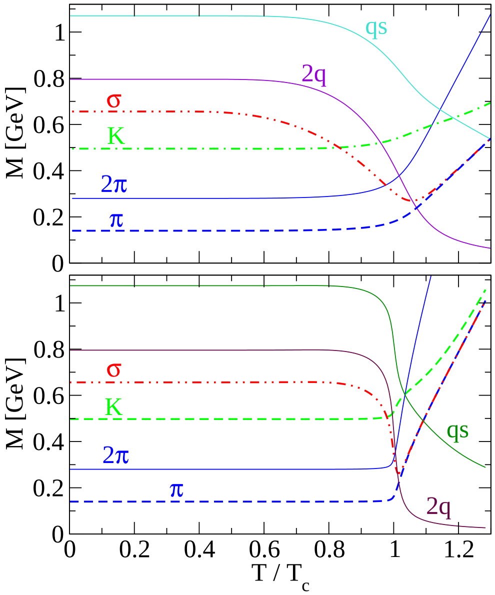
$a_0 = 6.75,\ a_1 = -1.95,\ a_2 = 2.625,\ a_3 = -7.44,\ b_3 = 0.75$ and $b_4 = 7.5.$

- in pure Yang-Mills: $T_0 = 270$ MeV, otherwise lower $\Rightarrow$ pushes T_c lower

Synchronization of T_{Ch} and T_d by Polyakov loop



Relative T/T_c-dependence of meson masses without and with PL



But what about η and η' ? (our major interest)

$U_A(1)$ symmetry breaking is why $\eta_0 \approx \eta'$ has an anomalous piece of mass since $U_A(1)$ symmetry is broken by nonabelian ("gluon") axial anomaly: even in the chiral limit (ChLim, where $m_q \to 0$),

$$\partial_\alpha \bar{\psi}(x)\gamma^\alpha\gamma_5 \frac{\lambda^0}{2}\psi(x) \propto F^a(x)\cdot \widetilde{F}^a(x) \equiv \epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu}(x) F^a_{\rho\sigma}(x) \neq 0 \,.$$

This breaks the $U_A(1)$ symmetry of QCD and precludes the 9^{th} Goldstone pseudoscalar meson $\Rightarrow$ very massive η': even in ChLim, where $m_\pi, m_K, m_\eta \to 0$, still ('ChLim WVR')

$$0 \neq \Delta M^2_{\eta_0} = \Delta M^2_{\eta'} = \frac{(A = \mathrm{qty.dim.mass})^4}{(\text{“}f_{\eta'}\text{”})^2} = \frac{6\,\chi_{\mathrm{YM}}}{f_\pi^2} + O(\frac{1}{N_c})$$

$$\text{Out of ChLim :} \quad {M_{\eta'}}^2 + {M_\eta}^2 - 2\,{M_K}^2 = \frac{2N_f}{f_\pi^2}\,\chi_{\mathrm{YM}} \quad \left(+O(\frac{1}{N_c})\right)$$

Anomalous part of η_0 mass $\Delta M^2_{\eta_0} = \chi_{\rm YM}\, \frac{2N_f}{f_\pi^2} + O(\frac{1}{N_c})$

QCD chiral behavior (reproduced by (e.g.) DS approach) of the non-anomalous parts of masses of light $q\bar{q}'$ pseudoscalars (i.e., all parts except ΔM_{η_0}) :
$M^2_{q\bar{q}'} = \mathrm{const}\,(m_q + m_{q'}), \ \ (q, q' = u, d, s)$.

$\Rightarrow$ non-anomalous parts of the masses in WVR cancel:
$M_{\eta'}{}^2 + M_\eta{}^2 - 2\,M_K{}^2 \approx \Delta M_{\eta_0}{}^2$, approx. as in ChLim WVR

$$\chi = \int d^4x\, \langle 0|Q(x)Q(0)|0\rangle\ , \qquad Q(x) = \frac{g^2}{64\pi^2}\epsilon_{\mu\nu\rho\sigma}F^a_{\mu\nu}(x)F^a_{\rho\sigma}(x)$$

- $Q(x) =$ topological charge density operator
- In WV rel., χ is the pure-glue, YM one, $\chi_{\rm YM} \leftrightarrow \chi_{quench}$, reproduced reliably by lattice, but for χ of light-flavor QCD, use Di Vecchia-Veneziano

relation: $$\chi = -\frac{\langle\bar{q}q\rangle_0}{\sum\limits_{q=u,d,s}\frac{1}{m_q}} + \mathcal{C}(\text{unknown corrections, higher}\ \mathcal{O}\ \text{in small}\ m_q)$$

.

Results on η and η' (at $T = 0$) with $\Delta M_{\eta_0} = 6\chi_{\rm YM}/f_\pi^2$ from WVR

	$\beta_{\rm fit}$	$\beta_{\rm latt.}$	Exp.
θ	-12.22°	-13.92°	
M_η [MeV]	548.9	543.1	547.75
$M_{\eta'}$ [MeV]	958.5	932.5	957.78
X	0.772	0.772	
3β [GeV^2]	0.845	0.781	

- $X = f_\pi / f_{s\bar{s}}$ as well as the whole $\hat{M}^2_{NA}$ (consisting of M_π and $M_{s\bar{s}}$) are calculated model quantities (in SD approach).
- $\beta_{\rm latt.} = \Delta M_{\eta_0}/(2 + X^2)$ was obtained from $\chi_{\rm YM}(T = 0) = (175.7\ {\rm MeV})^4$
- But is an extension to high T possible, as there is a large mismatch of characteristic temperature scales of the pure-gauge YM ($T_c \sim 270$ MeV) vs. full QCD ($T_c \sim 160$ MeV) with quarks?
- $\Rightarrow$ in WVR, $\chi_{\rm YM}$ is more T-resistant than QCD quantities $M_{\eta,\eta',K}$ and f_π.
- $\Rightarrow$ Conflict with experiment [Horvatić&al,PRD76(2011)] ... Does WVR become unusable as T approaches $T_{\rm Ch}$ of full QCD ?
- But Shore's generalization of WVR does **NOT** have this mismatch of the full QCD and pure-gauge YM temperature scales! Try this?

Shore's generalization of WV valid to all orders in $1/N_c$

$$(f^0_{\eta'})^2 M^2_{\eta'} + (f^0_\eta)^2 M^2_\eta \;=\; \tfrac{1}{3}\big(f^2_\pi M^2_\pi + 2f^2_K M^2_K\big) + 6A \tag{1}$$

$$f^0_{\eta'} f^8_{\eta'} M^2_{\eta'} + f^0_\eta f^8_\eta M^2_\eta \;=\; \tfrac{2\sqrt{2}}{3}\big(f^2_\pi M^2_\pi - f^2_K M^2_K\big) \tag{2}$$

$$(f^8_{\eta'})^2 M^2_{\eta'} + (f^8_\eta)^2 M^2_\eta \;=\; -\tfrac{1}{3}\big(f^2_\pi M^2_\pi - 4f^2_K M^2_K\big) \tag{3}$$

The role of $\chi_{\rm YM}$ taken over by the full QCD topological charge parameter A ,

$$A = \frac{\chi}{1 + \chi\left(\frac{1}{\langle \bar{u}u\rangle\, m_u} + \frac{1}{\langle \bar{d}d\rangle\, m_d} + \frac{1}{\langle \bar{s}s\rangle\, m_s}\right)} \tag{4}$$

- A should behave with T as a full QCD quantity
- ... **but**, at $T = 0$ it is known that $A = \chi_{\rm YM} + \mathcal{O}(\frac{1}{N_c})$

Note (1)+(3) $\Rightarrow (f^0_{\eta'})^2 M^2_{\eta'} + (f^0_\eta)^2 M^2_\eta + (f^8_\eta)^2 M^2_\eta + (f^8_{\eta'})^2 M^2_{\eta'} - 2f^2_K M^2_K = 6A$

- Then, large N_c limit and 'off-diagonal' $f^0_\eta, f^8_{\eta'} \to 0$, as well as $f^0_{\eta'}, f^8_\eta, f_K \to f_\pi$, recovers the standard WV.

Approximate all 3 light condensates by $\langle\bar{q}q\rangle_0$, the chiral-limit one!

This reduces the full QCD topological charge A, Eq. (4), to the remarkable Leutwyler-Smilga relation (LS), which is still valid for both large and small values of m_q:

$$\chi_{\rm YM} = \frac{\chi}{1 + \frac{\chi}{\langle\bar{q}q\rangle_0} \sum\limits_{q=u,d,s} \frac{1}{m_q}} \equiv \widetilde{\chi} \to \widetilde{\chi}(T) \approx A(T)$$

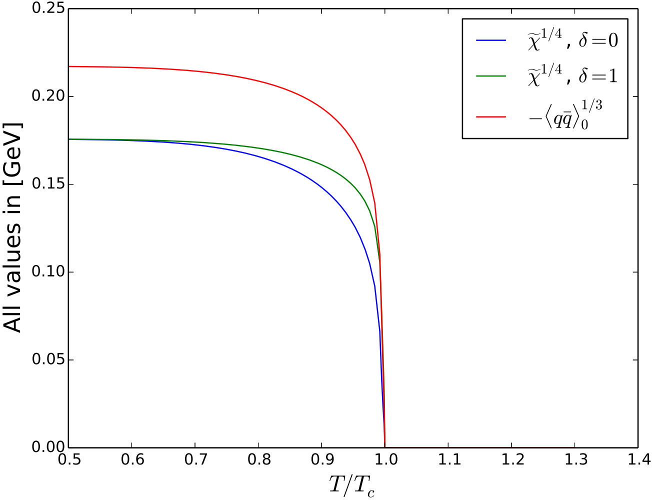
where for the light quarks $$\chi = - \frac{1}{\sum\limits_{q=u,d,s} \frac{1}{m_q \, \langle\bar{q}q\rangle_0}} + \mathcal{C}(m)$$

.

- $\mathcal{C}(m)$ = small corrections of higher orders in small m_q, ... but $\mathcal{C}(m)$ should not be neglected, since $\mathcal{C}(m) = 0$ would imply that $\chi_{\rm YM} = \infty$.
- LS relation fixes the value of the correction at $T = 0$:

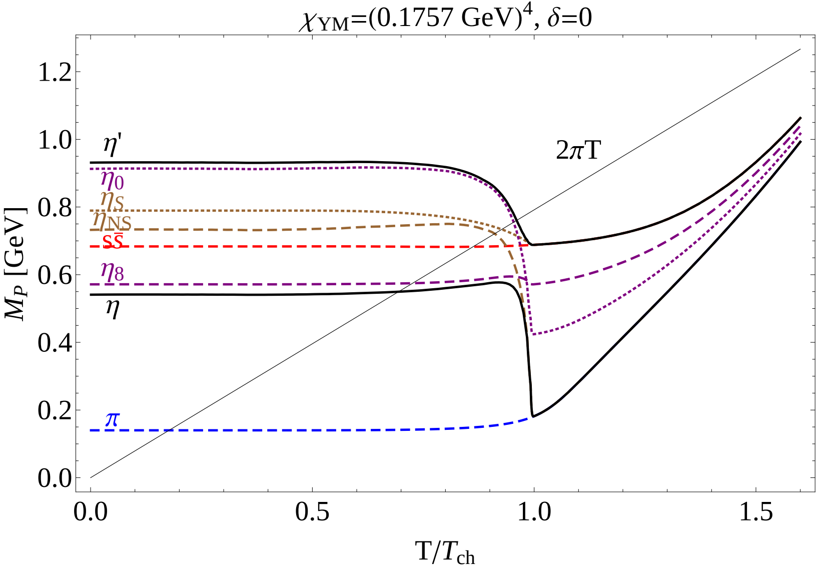
$$\frac{1}{\mathcal{C}(m)} = \sum_{q=u,d,s} \frac{1}{m_q \langle\bar{q}q\rangle_0} - \chi_{\rm YM}(0) \left(\sum_{q=u,d,s} \frac{1}{m_q \langle\bar{q}q\rangle_0} \right)^2 .$$

Chiral condensate $\langle q\bar{q}\rangle_0(T)$ and resulting $\widetilde{\chi}(T)$

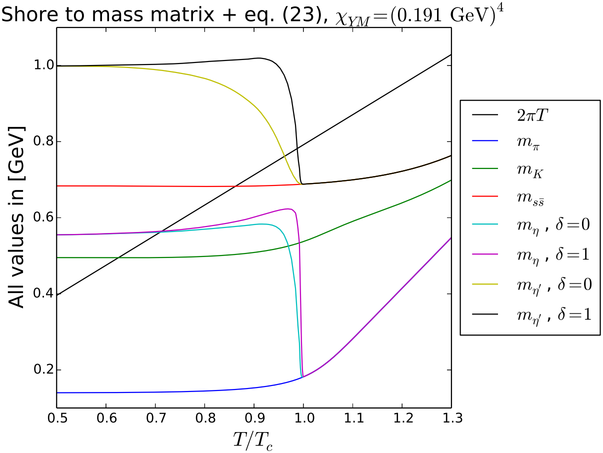


Prediction good for η', but for η not supported by any experiment

[Benić, Horvatić, Kekez and Klabučar, Phys. Rev. D **84** (2011) 016006.]

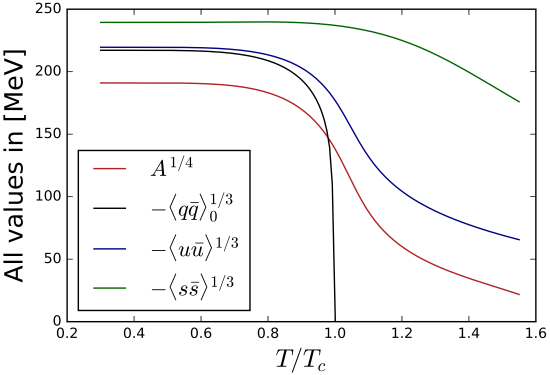


Variations of model, or input or model parameters, do not change much ...



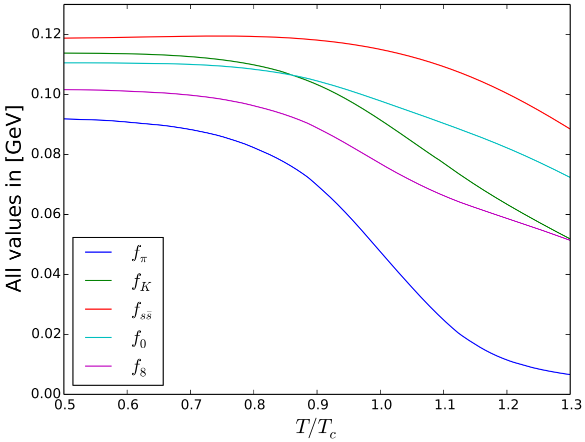
... mass drop prediction still good for η' (where Csörgö and collaborators had found this in RHIC data), but again an even larger mass drop for η, which is not supported by any experiment.

A solution: $U_A(1)$ breaking from realistic condensates



Instead of the fast-falling chiral-limit condensate $\langle\bar{q}q\rangle_0$, try $\langle\bar{q}q\rangle$ condensates with realistic explicit chiral symmetry breaking: replace $m_q\langle\bar{q}q\rangle_0 \to m_q\langle\bar{q}q\rangle$, $(q = u, d, s)$ in χ, like in the original A.

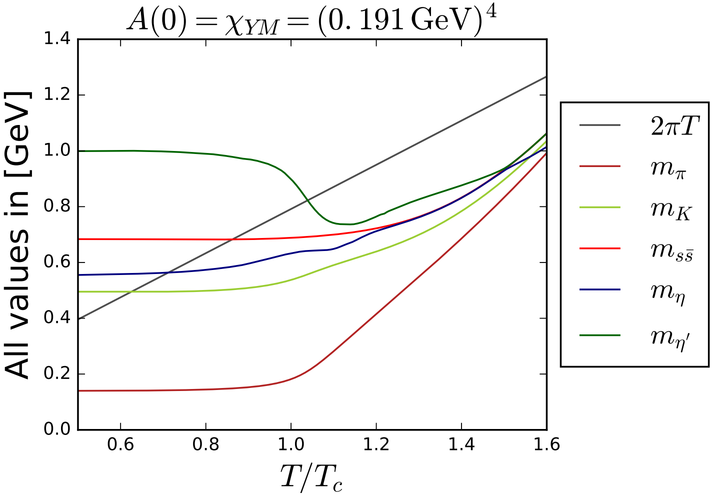
T-dependence of pseudoscalar decay constants



How they influence the elements of the η-η' mass matrix:

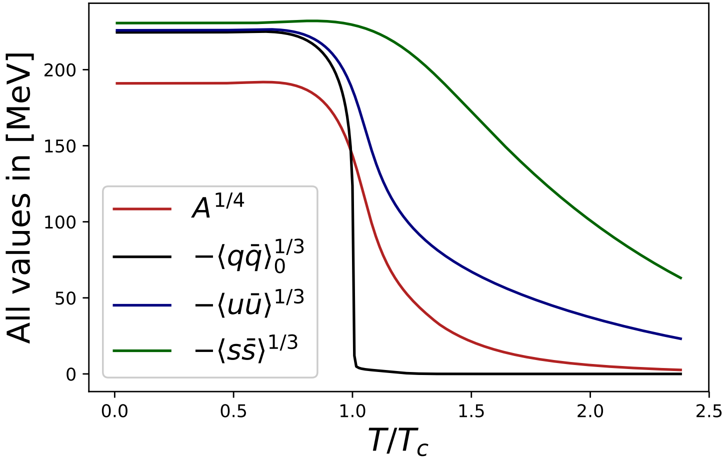
$$M^2_{\rm NS} = M^2_\pi + \frac{4A}{f^2_\pi}, \qquad M^2_{\rm NSS} = \frac{2\sqrt{2}A}{f_\pi f_{s\bar{s}}}, \qquad M^2_{\rm S} = M^2_{s\bar{s}} + \frac{2A}{f^2_{s\bar{s}}}$$

$\Rightarrow$ Acceptable T dependence of light pseudoscalars including η and η'



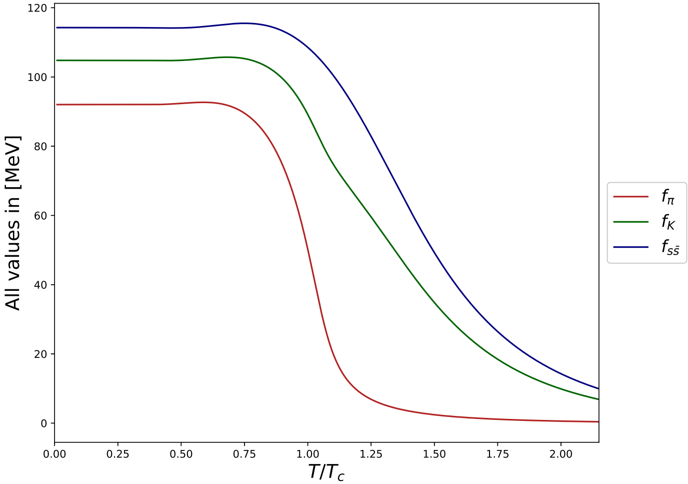
But problematic to find solutions beyond $T \sim 1.6\,T_c$, where the analytic structure of rank-2 models starts causing problems. $\Rightarrow$ Try rank-1 !

T-dependence of the realistic condensates & A in a rank-1 model



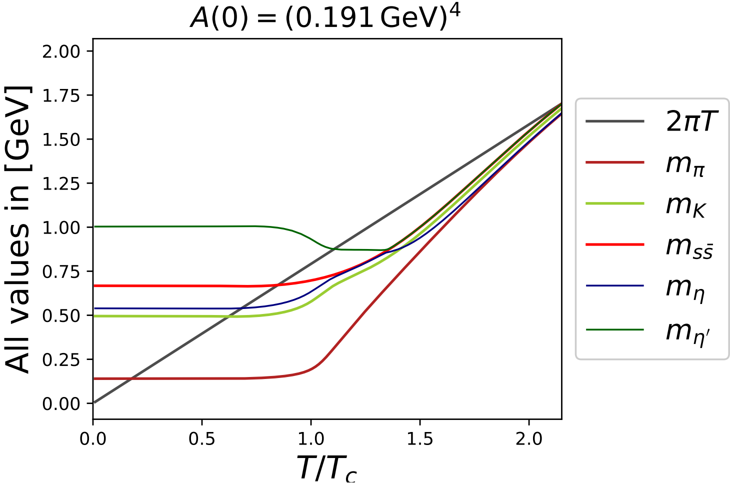
Everything similar as in rank-2, except that higher T's are reached.

T-dependence of pseudoscalar decay constants in a rank-1 model



Also this similar as in rank-2, except that higher T's are reached.

T-dependence of the light pseudoscalars' masses in a rank-1 model



This reaches temperatures of dissolution into very feebly interacting q's & $\bar{q}$'s.

Summary

- Defficiency of separable models regarding reproducing the values and mutual relationships of the temperatures of deconfinement and chiral symmetry restoration in QCD, can be cured by coupling to Polyakov loop.
- Features of the meson description so obtained, give us confidence that even when a separable model is not coupled to a Polyakov loop, one can use temperature rescaling and the notion of relative temperature.
- Our approach ties the $U_A(1)$ SB to the DChSB so closely, that the restoration of the chiral symmetry must lead to the restoration of the $U_A(1)$ symmetry at least partially, on the level of the η' & η masses.
- Condensates with realistic **explicit** ChSB fall with T much more slowly and smoothly than $\langle q\bar{q}\rangle_0$. $\Rightarrow$ similar T-behavior of the topological charge parameter $A(T)$. Then, η does not exhibit any mass drop at all. The behavior of η' is not changed much: $M_{\eta'}(T)$ falls again around T_{Ch} by 300 to 200 MeV, but much slower than in the old calculation with $\langle q\bar{q}\rangle_0$. After the anticrossing (with η), η' becomes a pure $s\bar{s}$ without anomalous contributions, signaling the partial restoration of $U_A(1)$ symmetry.
- This pseudoscalar nonet description had been obtained through a rank-2 separable model. It has now been fully supported by the rank-1 separable models, which can reach much higher temperatures.