Quantum Measurement Theory in FRG approach

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Outlines

1. Measurement in Quantum Mechanics
2. Measurement from field theory point of view
3. SSB as prototype of quantum measurement
4. Interpretation of experiments
5. Conclusions

The basic setup of Quantum Mechanics

- states are elements of a Hilbert space $\in H$
- physical states are normalized $|| \, |\psi\rangle \, ||^2 = 1$
- physical transformations are Hilbert-space homomorphisms: $H_{ph} \to H_{ph} \quad \Rightarrow \quad$ (anti) unitary linear transformations
- trf. of states and operators: $|\psi'\rangle = U \, |\psi\rangle \, , \; A' = U^\dagger A U$
- continuous unitary groups (Lie-groups): $U = e^{-i\omega_a T_a}$
  $\Rightarrow$ generators T_a hermitian

Special 1-parameter (or commutative) Lie-groups

- *time translation*, its generator (def.) Hamiltonian

$$e^{-i\hat{H}t} \, |\psi\rangle = |\psi, t\rangle \quad \Rightarrow \quad i\partial_t \, |\psi\rangle = \hat{H} \, |\psi\rangle$$

- *space translation*, its generator (def.) momentum

$$\delta\hat{q} = i\delta a[\hat{p}, \hat{q}] = \delta a \quad \Rightarrow \quad [\hat{q}, \hat{p}] = i \qquad .$$

Measurement

Perform a transformation which influences the system in the least way (infinitesimal trf.), and detect the change of the state:
$i\delta\,|\psi\rangle = \varepsilon T\,|\psi\rangle \quad \Rightarrow$ generator represents a measurement.

- If $i\delta\,|\psi\rangle = \lambda\varepsilon\,|\psi\rangle$ (eigenstate) then the transformation changes only the phase of the system
  $\Rightarrow$ result of measurement can be represented by a number
  $\Rightarrow$ value of the measurement: λ

But what happens if $i\delta\,|\psi\rangle \not\sim |\psi\rangle$? In a real experiment we still measure a number! How can we obtain it?

Measurement postulate

Measurement postulate:

- the possible measurement values are the eigenvalues of the infinitesimal generator $T\,|n\rangle = \lambda_n\,|n\rangle \quad \Rightarrow \quad$ usually quantized
- the quadratic norm of the eigenvectors $|\,\langle\psi|n\rangle\,|^2$ provides the *probability* to measure λ_n.
- If we measured λ_n, then the system continues time evolution from $|n\rangle$ (*wave function reduction*).

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Challenge

Measurement is non-deterministic, non-causal! How can one build a consistent theory?

Copenhagen interpretation

Copenhagen interpretation

- measurement (observation) is not causal, inherently random.
- throw away deterministic time evolution!
- wave function reduction is instant, and it happens at once in the whole space
- what is a measurement device?
  Neumann-Wigner interpretation: consciousness causes measurement.

Other interpretations

(cf. A.J. Leggett, J. Phys.: Condens. Matter 14 (2002), 415)

- statistical interpretations ⇒ improved versions of the Copenhagen interpretations
- many-worlds interpretation: many worlds, in each of them wave function reduction, but in a collection of them all possibility occurs

  (H. Everett H, Rev. Mod. Phys. 29 (1957) 454)
- objective wave function reduction: nonlinear time evolution, eg. due to gravity effects (Diosi-Penrose-interpretation)

  (L. Diosi, J.Phys.Conf.Ser. 701 (2016) 012019, [arXiv:1602.03772])
- decoherence phenomenon: physics in micro and macro world are not the same; its nature is not clear

Paradoxes and experiments

A QM interpretation should give an account to the questions like:

- causal vs. probabilistic: could it be possible to predict the result of a QM measurement?
- classicality vs. quantum: how local/macroscopic realism appears in a measurement (cf. EPR paradox, Bell-inequalities, Leggett-Garg inequalities, hidden parameters)

  (A. Leggett and A. Garg, PRL 54 (1985), M. Giustina et. al., PRL 115, 250401 (2015))
- what is a measurement device? Schrödinger's cat, conscious observer, detectors, or even spont. symmetry breaking (SSB)?
- time scale of wave function reduction?
- QM measurements: spin (Stern-Gerlach experiment), position, decay of unstable nuclei, etc.

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Field theory point of view

- inherently many-particle approach ⇒ no scale limit for the formalism (from quark to stars is applicable)
- QM (1-particle wave functions) is not fundamental, only a certain approximation of QFT
- In general no separated 1-particle states, interaction mixes $|p\rangle\,, |p_1, p_2\rangle\,, \ldots |p_1, \ldots, p_n\rangle\,, \ldots$ n-particle states $n \to \infty$.
- wave function? ⇒ corresponding notion is propagator
  - n-particle wf. ⇒ fully entangled (indistinguishability)
  - 1-particle propagator: nonlinear evolution equation (DS-eq.)

⇒ linearity in the whole and nonlinearity in a subsystem are not mutually exclusive phenomena

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Educated guess

Exact solution of QFT for the measurement device would provide the phenomenon "wave function reduction"

Why measurement theory is much harder than QCD?

- both require the exact solution of a field theory
- both are complicated many-body problems that can only treated numerically
- prediction the proton mass is possible, because we know microscopically what a proton is
- a measurement device shows properties that is completely irrelevant from the microscopic point of view (what is the difference between a metal tube and a Geiger-Müller counter?)

Strategy

We should find out the relevant quantities of the macroscopic measurement device and relate it to the microscopic (quantum) theory.

Functional Renormalization Group (FRG)

- Exact evolution equation
  for the scale dependence of the effective action (Wetterich-eq.)
  $$\partial_k \Gamma_k = \frac{i}{2} \hat{\partial}_k \,\mathrm{Tr} \ln(\Gamma_k^{(1,1)} + R_k)$$
  Γ_k effective action, k scale parameter, R_k regularization $\hat{\partial}_k = R_k' \frac{\partial}{\partial R_k}$
- fixed points: $\partial_k \Gamma_k = 0$
- around fixed points the effective action can be represented by the relevant operators only
  $\Rightarrow$ FRG Ansatz/effective theory
- scale evolution connects the fixed point regimes

Most important message

The physics should be represented by the relevant operators of the actual fixed point describing the phenomena under investigation.

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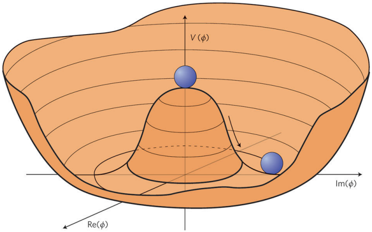
Spontaneous Symmetry Breaking (SSB)

- SSB: the microscopic theory possesses a symmetry which is not manifested in the IR observables
- usual interpretation: the ground state does not respect the symmetry $\Rightarrow$ minima of $\Gamma[\Phi]$



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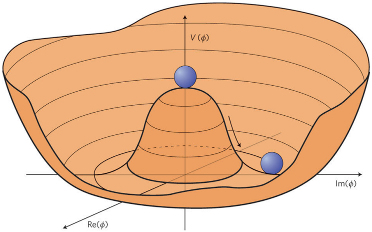


- consistency question: ground state in QM is unique (L. Gross, J. of Func.Anal. 10 (1972) 52) ; why do not we see the lowest energy state which is a linear combination of the states corresponding to classical minima?

Examples of the quantum and classical ground state

Example 1 2-state system with a double-well potential. Classical minima are $|+\rangle$ and $|-\rangle$. Ground state is $|0\rangle = \frac{|+\rangle + |-\rangle}{\sqrt{2}}$

Example 2 2D QM with U(1) symmetric potential (mexican hat). Classical minima correspond to the wave function $\langle x|\varphi\rangle = \delta(x - Re^{i\varphi})$ where R is the distance of the minimum from the origin. Ground state and 1st excited state

$$|0\rangle = \int_0^{2\pi} \frac{d\varphi}{2\pi} |\varphi\rangle , \qquad |1\rangle = \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{\pm i\varphi} |\varphi\rangle$$

Symmetric ground state, no zero mode (discrete spectrum)!

Quantum ground state respects symmetry! – observations??

Consequence

SSB is a classical phenomenon with quantum origin

$\Rightarrow$ it is the simplest example of decoherence!

Description of SSB in FRG

- Usual approach: first determine $\Gamma[\Phi]$, later the ground state
- FRG approach: write up the action around the ground state
  value of the *n*th derivative is the (1PI) *n*-point correlation function
  $\Rightarrow$ symmetry breaking explicitly appears in the action!
  (c.f. talk of András Patkós!)
- Remnant of the symmetry: Ward identities.

In Φ^4 theory

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi)^2 - \frac{M^2}{2}\Phi^2 - \frac{g}{6}\Phi^3 - \frac{\lambda}{24}\Phi^4,$$

and the Ward identity requires

$$g^2 = 3\lambda M^2 \quad \Rightarrow \quad R^2 = \frac{g^2}{3\lambda M^2} = 1.$$

Evolution equations of the couplings

- Use Wetterich equation in LPA

$$\partial_k U = \frac{1}{2}\hat{\partial}_k \int \frac{d^d p}{(2\pi)^d}\ \ln(p_k^2 + \partial_\Phi^2 U), \qquad p_k = \max(|p|, k)$$

where U effective potential

- Expand left and right hand side using the Ansatz
- Match the coefficients; take into account Ward identity

Result $\omega^2 = k^2 + M^2$

$$\partial_k M^2 = \frac{k^{d+1}}{\omega^4}\left(-\lambda + \frac{g^2}{M^2}(1 + \frac{M^2}{\omega^2})\right)$$

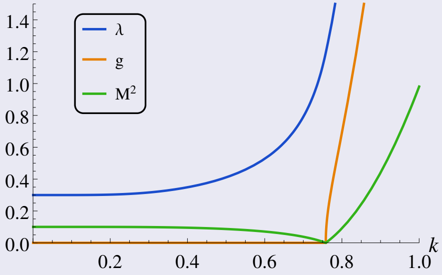
$$\partial_k \lambda = \frac{6k^{d+1}\lambda^2}{\omega^6}$$

$$\partial_k g = \frac{gk^{d+1}}{\omega^6}\left[\frac{9\lambda}{2} + \frac{g^2\omega^2}{3M^4}\right]$$

Results of the scalar model

Renormalized parameters: $\lambda_0 = 0.3,\ \frac{M_0^2}{\Lambda^2} = 0.1,\ g_0 = \pm 0.001$

Mass and couplings



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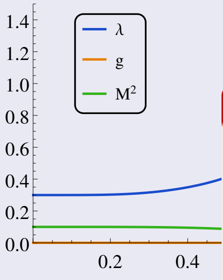
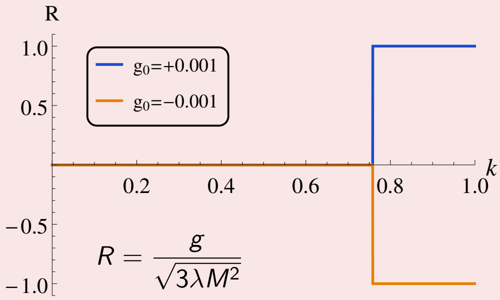


The Ward-identity ratio



$$R = \frac{g}{\sqrt{3\lambda M^2}}$$

Lessons to be generalized

- phase transition at a certain scale (at $k_{ph} = 0.7581430242$)
- described SSB through couplings, without any reference to (classical) fields
- "order parameter" is also a coupling: g, or R
- instead of inequivalent vacua → multiple fixed points
  sign of g_0 decides which is chosen ⇒ deterministic
- changing between fixed points is very fast
  $(R'(k_{ph}) = 1.1 \cdot 10^8!)$

Generalization proposition

- Each classically distinguishable state corresponds to a separate fixed point of the general effective action

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- which fixed point is chosen depends on operators that are very small (unmeasureable) initially
  irrelevant until we approach the measurement device
- fully deterministic, but practically unpredictable

Classical analogy

analogies: pencil placed on its tip, coin flipping, chaos/bifurcation

pencil tumbles deterministically, but still unpredictably

$\Rightarrow$ this happens in FRG in the coupling constant space

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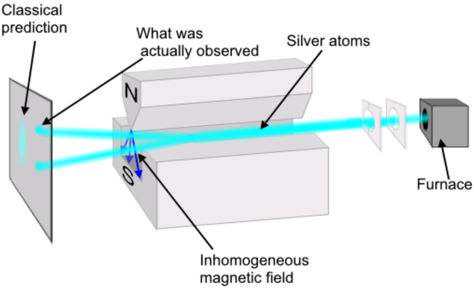
The Stern-Gerlach experiment

Experiment: e^- in x-polarized spin state, eg. $|\psi\rangle = \dfrac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}$,

z-inhomogeneous magnetic field separates the $|\uparrow\rangle$ and $|\downarrow\rangle$ components, detect the incoming particles.

Result: only one of 2 detectors will detect particle, the chance to detect is 50%.



Interpretation of the experiment

Interpretation: time evolution is slow $\Rightarrow$ adiabatic approach

- Creation of e^-: e.g. by photoeffect.
- Flying single e^-: only one fixed point, where the 1-e^- propagation is a good appr. $\Rightarrow$ $\exists\, e^-$ wave function
  state of environment is irrelevant for the e^-.
- e^- near/in the device: complicated system with
  – one unstable fixed point of the incoming e^- (UV)
  – two stable fixed of the measured e^- (IR1, IR2)
  1-e^- propagation (QM) is bad appr. $\Rightarrow$ $\nexists$ wave function
- RG trajectory: starts from UV fp., fast approaches one of the IR fp.s, depending on the state of the complete system
  system-wide "hidden variables" $\Rightarrow$ no macroscopic realism!
- if e^- goes on: the RG flow continues from just one of the fixed points, with definite spin.

Schrödinger's cat

Proposition: take a cat, put it into a box with a bomb coupled to unstable U-atoms; if the U-atom decays, the bomb explodes, the cat dies

Challenge: the U-atom is in a mixture of stable and decayed states

⇒ is the cat also in a mixture of living and dead state? What does the cat perceive?

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Interpretation of Schrödinger's cat thought experiment

Interpretation: there are two fixed points in the system:

- living cat with U-atom and intact bomb (UV)
  has one relevant direction! the initial condition decide how long we stay here
- dead cat with decay products and exploded bomb (IR)
  IR stable fixed point
- the crossover is explosively fast

Consequences

- we are always around one fixed point
- no cat wave function (bad approximation of QFT), no living dead quantum state

Outlines

Conclusions

- Quantum measurement theory can be described by exact QFT tools, like Functional Renormalization Group technique
- there are special fixed points, around which QM is a good approximation, but in general it is not

  $\Rightarrow$ wave function is not relevant in general
- as a consequence $\nexists$ wave function reduction!
- measurement device: several IR stable fixed points with separatrices, all can be the endpoint of the RG evolution, but only one of these!

  instead "many-world" $\Rightarrow$ many fixed points
- role of randomness: which if the IR fixed point is chosen is determined by a force that is small (irrelevant, not measurable) around the UV fp.

  for all practical purposes it is random
- measurement is completely deterministic!

  with system-wide "hidden variables" $\Rightarrow$ no macroscopic realism!