Renormalisation flow in a Yukawa model with quadratic symmetry breaking

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Outline:

- The model, its symmetries and spectra
  (Reminder of the Cakovec-2016 talk)
- Spectra in Local Potential Approximation (LPA)
  interpretation through RG-flow structure
- Effect of the wavefunction renormalisation (LPA'):
  adiabatic deformation of the LPA-flow
- Conclusions

The model

Complex scalar and single Dirac-fermion with chiral Yukawa-interaction

$$\Phi = \frac{1}{\sqrt{2}}(\Phi_1(x) + i\Phi_2(x)), \qquad \psi_{R/L} = \frac{1}{2}(1 \pm \gamma_5)\psi.$$

The symmetric part of the scale dependent action $\mathbf{k < \Lambda}$:

$$\Gamma^{(k)}_{SYM} = \int d^4x \left[Z_\psi \bar{\psi} \partial\!\!\!/ \psi + Z_\phi \partial_m \Phi^* \partial_m \Phi + U_k(\Phi^*\Phi) + h_k(\bar{\psi}_R \psi_L \Phi^* + \bar{\psi}_L \psi_R \Phi) \right].$$

Symmetries:

$$\psi \to e^{i\alpha}\psi, \qquad \bar{\psi} \to e^{-i\alpha}\bar{\psi} \qquad \Phi \to \Phi, \qquad \mathrm{U}(1),$$

$$\psi \to e^{i\gamma_5\Theta}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{i\gamma_5\Theta}, \qquad \Phi \to e^{-2i\Theta}\Phi, \quad U_A(1).$$

$U_A(1)$: fermion mass-term can be generated only via non-zero Φ-condensate.

Quadratic explicit breaking of $U_A(1)$

$$\Gamma^{(k)}_{ESB} = \Pi_k \int_p [\Phi(-p)\Phi(p) + \Phi^*(-p)\Phi^*(p)].$$

Symmetry: $U(1) \times U_A(1) \rightarrow U(1) \times Z(2)$

Π_k : $\langle \Phi\Phi + \Phi^*\Phi^* \rangle_k \neq 0$ Superposition of oppositely charged condensates

Spontaneous breaking:

$$U^{(k)}_{INV} = M_k^2 \Phi^*\Phi + \frac{\lambda_k}{6}(\Phi^*\Phi)^2 \rightarrow U^{(k)}_{SSB} = \frac{\lambda_k}{6}\left(\Phi^*\Phi - \frac{v_k^2}{2}\right)^2$$

Field expectation: $\Phi_0 = \Phi_0^* = \frac{u_k}{\sqrt{2}} \rightarrow$ equations of the condensate:

$$\left(M_k^2 - 2|\Pi_k| + \frac{\lambda_k}{6}u_k^2\right)\frac{u_k}{\sqrt{2}} = 0, \quad \leftarrow \mathrm{INV}$$

$$\left(\frac{\lambda_k}{6}(u_k^2 - v_k^2) - 2|\Pi_k|\right)\frac{u_k}{\sqrt{2}} = 0, \quad \leftarrow \mathrm{SB}$$

Infrared spectrum of the scalar sector ($k = 0$)

Denominator of the bosonic propagators:

INV ($u_0 = 0, \;\; \psi = 0$)
$\Delta(\, \Gamma_B^{(2)} \,) = (q^2 + M_0^2)^2 - 4|\Pi_0|^2 \quad \to \quad m_1^2 = M_0^2 - 2|\Pi_0|, \quad m_2^2 = M_0^2 + 2|\Pi_0|$

SB ($u_0 \neq 0, \quad \psi = 0$)
$\Delta(\, \Gamma_B^{(2)} \,) = \left(q^2 + \frac{\lambda_0}{3} v_0^2 + 4|\Pi_0| \right) (q^2 + 4|\Pi_0|)$

$$\to m_1^2 = \tfrac{\lambda_0}{3} v_0^2 + 4|\Pi_0| = \tfrac{\lambda_0}{3} u_0^2, \qquad m_2^2 = 4|\Pi_0|.$$

Separation of the symmetric and broken symmetry regions (critical surface):

$$M_0^2 = 2|\Pi_0| \quad \to m_1^2 = 0, \;\; m_2^2 = 4|\Pi_0|$$

The masses m_1^2 and m_2^2 go continuously through the critical surface.

The second mass corresponds in the SB-phase to the pseudo-Goldstone field from the $U_A(1)$ breakdown.

Fermion mass:

$$m_\psi = h_{k=0} \tfrac{u_0}{\sqrt{2}}$$

Strategy for solving RGE

Toy model of top-Higgs system with enlarged scalar sector:

Is it possible to identify the PGB with the Higgs ($m_G = m_2 = m_{Higgs}$)?

How large can be made m_{hb}/m_G?

How does influence Π_Λ the m_{hb}/m_G ratio?

Strategy:

Tune h_Λ, M_Λ^2 with fixed Π_Λ/M_Λ^2 and λ_Λ

to get

$$h_0 = \sqrt{2}\frac{m_{top}}{u_{SM}} = \frac{173}{246}, \qquad \frac{8\Pi_0}{h_0^2 u_{SM}^2} = \frac{m_{Higgs}^2}{m_{top}^2} = \left(\frac{125}{173}\right)^2.$$

Read out

$$\frac{m_G^2}{m_{hb}^2} = \frac{3h_0^2}{2\lambda_0}\frac{m_{Higgs}^2}{m_{top}^2}$$

Wetterich RGE's for the effective action I. *symmetric phase*

$$\partial_t M^2 = -2h^2 \int_q \hat{\partial}_t \frac{1}{Z_\psi^2 q_R^2} + \frac{2\lambda}{3} \int_q \hat{\partial}_t \frac{Z_\phi q_R^2 + M^2}{\Delta(q_R^2)},$$

$$\partial_t \lambda = 6h^4 \int_q \hat{\partial}_t \frac{1}{Z_\psi^4 q_R^4} - \frac{\lambda^2}{3} \int_q \hat{\partial}_t \frac{5(Z_\phi q_R^2 + M^2)^2 + 16|\Pi|^2}{\Delta^2(q_R^2)},$$

$$\partial_t h = h^3 \Pi \int_q \hat{\partial}_t \frac{1}{Z_\psi^2 q_R^2 \Delta(q_R^2)},$$

$$\partial_t \Pi = -\frac{\lambda\Pi}{3} \int_q \hat{\partial}_t \frac{1}{\Delta(q_R^2)}.$$

Note: Evolution of the Yukawa coupling is very slow with diminishing $\Pi_\Lambda / M_{r\Lambda}^2$

Wetterich RGE's for the effective action II. *broken symmetry phase*

$$\frac{3}{4}\partial_t m_G^2 = -2h^2\hat{\partial}_t \int_q \frac{1}{Z_\psi^2 q_R^2 + m_\psi^2} + \frac{\lambda}{6}\hat{\partial}_t \int_q \frac{4Z_\phi q_R^2 + m_{hb}^2 + m_G^2}{\Delta(q_R^2)},$$

$$\partial_t m_{hb}^2 = 2h^2\hat{\partial}_t \int_q \frac{-Z_\psi^2 q_R^2 + m_\psi^2}{(Z_\psi^2 q_R^2 + m_\psi^2)^2}$$
$$+\frac{\lambda}{6}\hat{\partial}_t \int_q \frac{1}{\Delta(q_R^2)}\left[4Z_\phi q_R^2 + 7m_{hb}^2 + 3m_G^2 - \frac{m_{hb}^2}{\Delta(q_R^2)}\left(4Z_\phi q_R^2 + m_{hb}^2 + m_G^2\right)^2\right],$$

$$\partial_t h = -\frac{h^3}{2}(m_{hb}^2 - m_G^2)\hat{\partial}_t \int_q \frac{-Z_\psi^2 q_R^2 + m_\psi^2}{(Z_\psi^2 q_R^2 + m_\psi^2)^2 \Delta(q_R^2)}$$
$$+ \frac{h^3 m_{hb}^2}{4}\hat{\partial}_t \int_q \frac{1}{Z_\psi^2 q_R^2 + m_\psi^2}\left[\frac{3}{(Z_\phi q_R^2 + m_{hb}^2)^2} - \frac{1}{(Z_\phi q_R^2 + m_G^2)^2}\right].$$

Wetterich RGE's for the effective action III. *broken symmetry phase*

$$\partial_t \lambda = \partial_t^F \lambda + \partial_t^B \lambda, \qquad \Delta_{nm} = \frac{\delta^{n+m}\Delta}{\delta\Phi^n \delta\Phi^{*m}}$$

$$\frac{2}{3}\partial_t^F \lambda = 4h^4 \hat{\partial}_t \int_p G_\psi^2 [1 - 4m_\psi^2 G_\psi + m_\psi^4 G_\psi^2],$$

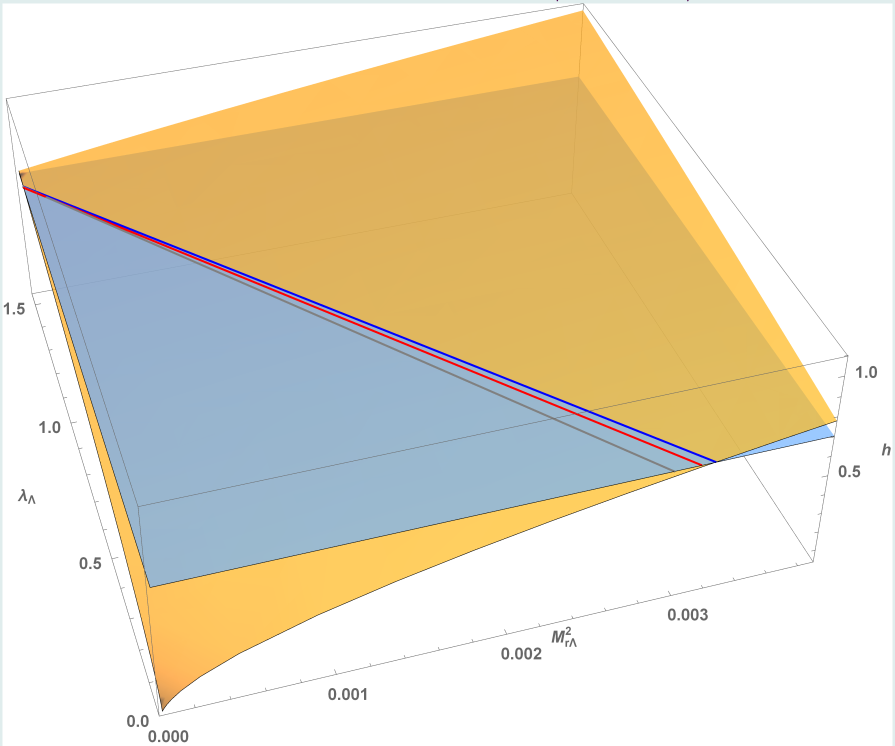
$$\frac{2}{3}\partial_t^B \lambda = \frac{1}{2}\hat{\partial}_t \int_p \left\{ \frac{\Delta_{22}}{\Delta} - \frac{1}{\Delta^2}\left[2\Delta_{12}\Delta_{10} + 2\Delta_{21}\Delta_{01} + 2\Delta_{11}^2 + \Delta_{20}\Delta_{02}\right]\right.$$

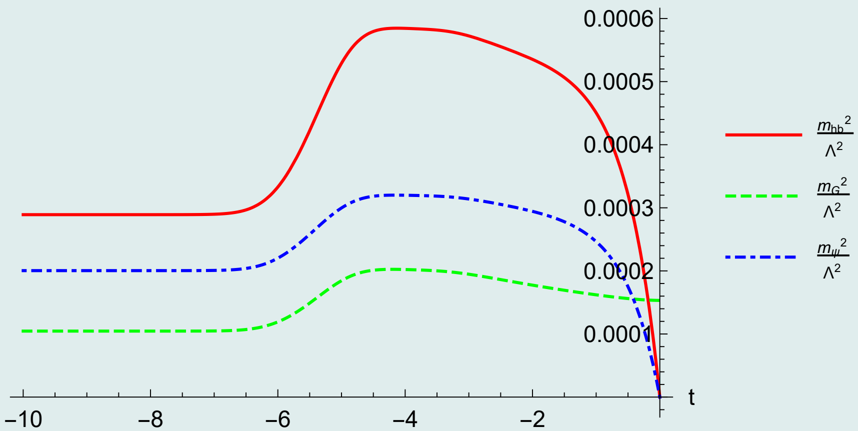
$$\left. + \frac{2}{\Delta^3}\left[\Delta_{20}\Delta_{01}^2 + \Delta_{02}\Delta_{10}^2 + 4\Delta_{11}\Delta_{10}\Delta_{01}\right] - \frac{6\Delta_{10}^2\Delta_{01}^2}{\Delta^4}\right\}.$$

$$G_G(q_R^2) = \frac{1}{Z_\Phi q_R^2 + m_G^2}, \quad G_{hb}(q_R^2) = \frac{1}{Z_\Phi q_R^2 + m_{hb}^2}, \quad G_\psi = \frac{1}{Z_\psi^2 q_R^2 + m_\psi^2},$$

$$\Delta = G_G G_{hb}$$

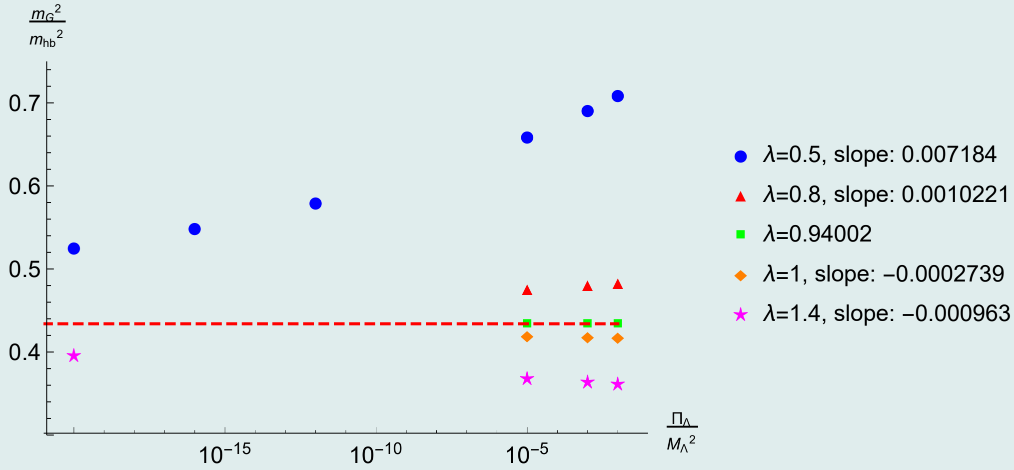
Phase structure and spectra in $LPA, \quad (\eta_\psi = \eta_\phi = 0)$

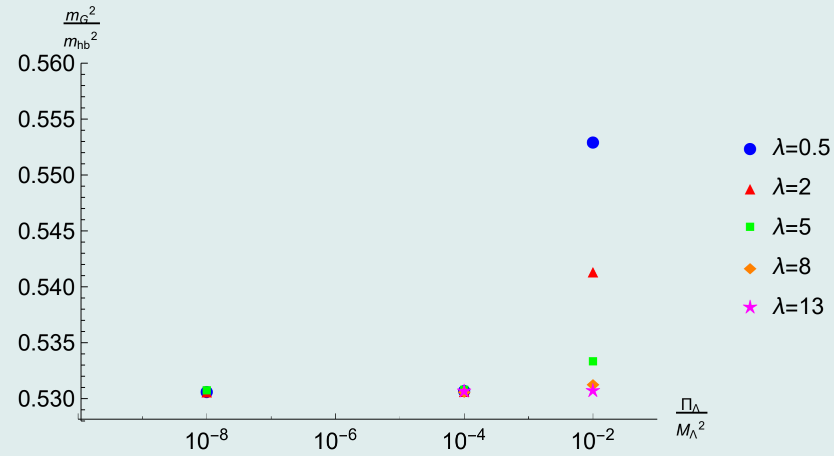




$$|\Pi_\Lambda|/M^2_\Lambda = 0.01, \quad h \equiv h_\Lambda$$

Scalar mass ratio vs. $\Pi_\Lambda / M_\Lambda^2$





$h_\Lambda \approx 0.7$ $\qquad$ $h_\Lambda \approx 3$

Important observation:

Scalar mass ratio apparently approaches a value insensitive to $\Pi_\Lambda / M_\Lambda^2$

Conjecture:

$\lambda(k=0)$ for $|\Pi_\Lambda|/M_\Lambda^2 \to 0$ is a unique function of h_Λ

What is behind this systematics observed from the numerical solution of the RGE's?

The RG-flow in the symmetric phase

RGE's of dimensionless couplings

$$\partial_t \Pi_r + 2\Pi_r = \frac{4\lambda_r \Pi_r v_d}{3} \frac{1 + M_r^2}{((1 + M_r^2)^2 - 4\Pi_r^2)^2},$$

$$\partial_t M_r^2 + 2M_r^2 = 4h_r^2 v_d - \frac{4\lambda_r v_d}{3} \frac{(1 + M_r^2)^2 + 4\Pi_r^2}{((1 + M_r^2)^2 - 4\Pi_r^2)^2},$$

$$\partial_t \lambda_r + (4 - d)\lambda_r = -24h_r^4 v_d + \frac{4\lambda_r^2 v_d (1 + M_r^2)}{3((1 + M_r^2)^2 - 4\Pi_r^2)^3} \left(5(1 + M_r^2)^2 + 52\Pi_r^2\right),$$

$$\partial_t h_r + \frac{1}{2}(4 - d)h_r = -\frac{2\Pi_r h_r^3 v_d}{(1 + M_r^2)^2 - 4\Pi_r^2} \left[\frac{2(1 + M_r^2)}{(1 + M_r^2)^2 - 4\Pi_r^2} + 1\right].$$

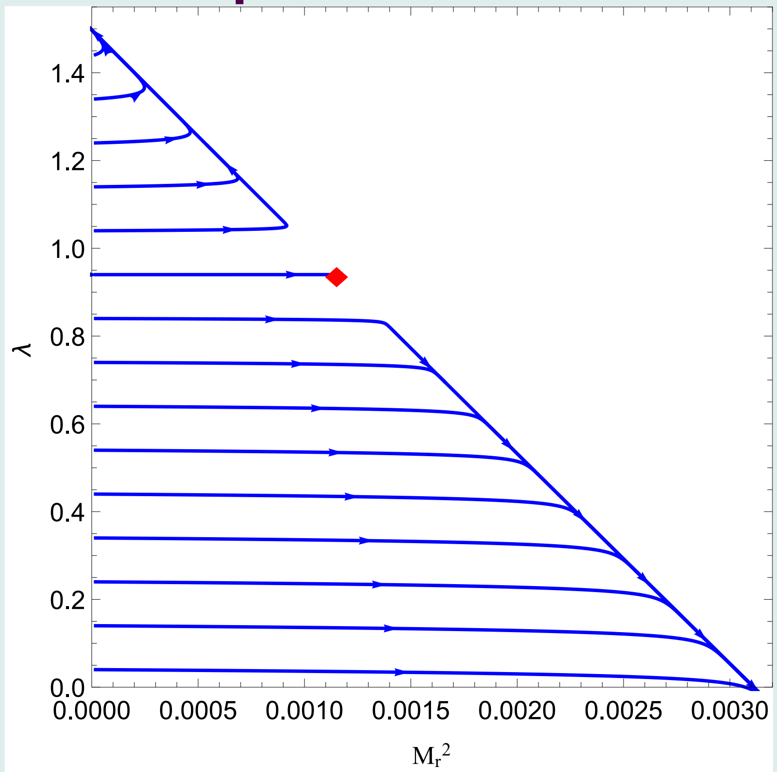
Fixed point solution in $d = 4$:

$$\frac{\lambda_{UV}^{*2}}{h_{UV}^4} \approx \frac{18}{5}, \quad \frac{M_{r,UV}^{*2}}{h_{UV}^2} \approx 2v_4 \left(1 - \sqrt{\frac{2}{5}}\right), \quad \Pi_{r,UV}^* = 0, \quad v_4 = (32\pi^2)^{-1}.$$

With $h_\Lambda = 173/246$ it suggests $\lambda_{UV}^* \approx 0.94$.

Compare with λ_Λ belonging to the limiting m_G^2/m_{hb}^2 ratio!

The RG-flow in the symmetric phase



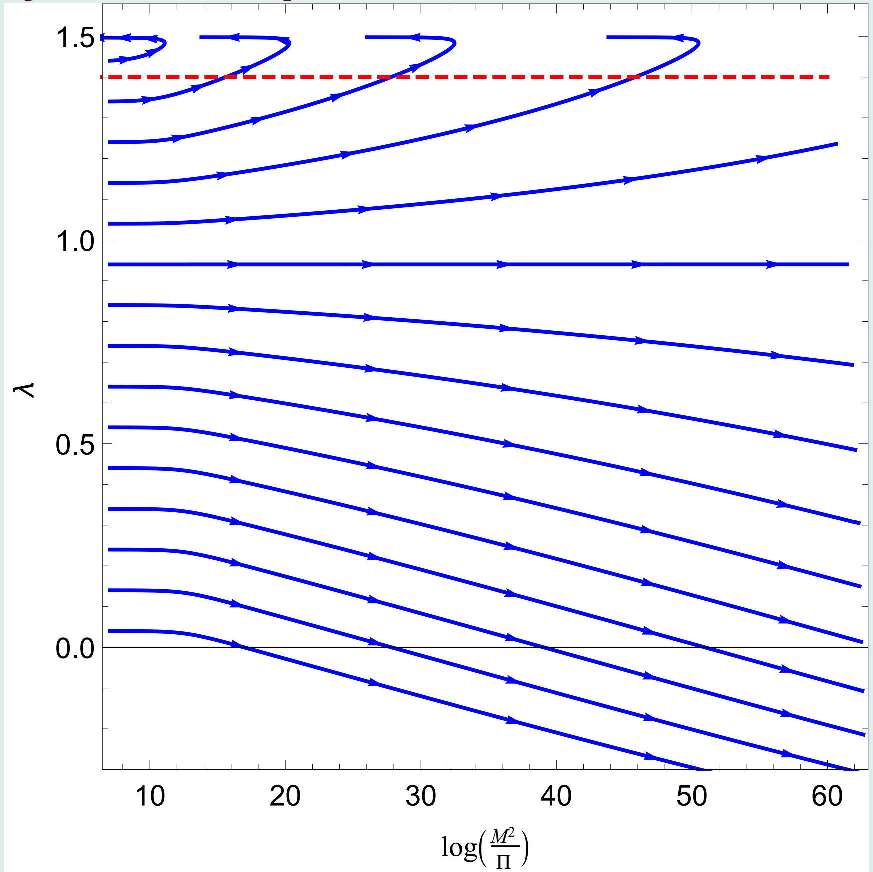
Two classes of trajectories:

i) running into Landau-singularity, ii) running into instability separated by the renormalized trajectory ending in the fixed point.

Δt needed for the evolution in the broken phase is independent of Π_Λ.

The smaller is Π_Λ the longer is the evolution along a flow-line in the symmetric phase.

The RG-flow in the symmetric phase



Fixed λ_Λ, $M_\Lambda^2/\Pi_\Lambda \to \infty$ selects the renormalized trajectory

Predicts(!) $m_G^2/m_{hb}^2 \approx 1.5\sqrt{5/18} \times m_G^2/m_\psi^2$ ($= 0.42$ compare to figure!!)

Is this picture robust enough when one improves solutions of the RGE's?

Effect of wavefunction renormalisation (LPA')

$$\partial_t \Pi_r + (2-\eta_\phi)\Pi_r = \frac{4\lambda_r \Pi_r v_4}{3} \frac{1+M_r^2}{((1+M_r^2)^2 - 4\Pi_r^2)^2}\left(1-\frac{\eta_\phi}{6}\right),$$

$$\partial_t M_r^2 + (2-\eta_\phi)M_r^2 = 4h_r^2 v_4 \left(1-\frac{\eta_\psi}{5}\right) - \frac{4\lambda_r v_4}{3}\frac{(1+M_r^2)^2 + 4\Pi_r^2}{((1+M_r^2)^2 - 4\Pi_r^2)^2}\left(1-\frac{\eta_\phi}{6}\right),$$

$$\partial_t \lambda_r - 2\eta_\phi \lambda_r = -24 h_r^4 v_4 \left(1-\frac{\eta_\psi}{5}\right) + \frac{4\lambda_r^2 v_d (1+M_r^2)}{3((1+M_r^2)^2 - 4\Pi_r^2)^3}\times$$

$$\times \left(5(1+M_r^2)^2 + 52\Pi_r^2\right)\left(1-\frac{\eta_\phi}{6}\right),$$

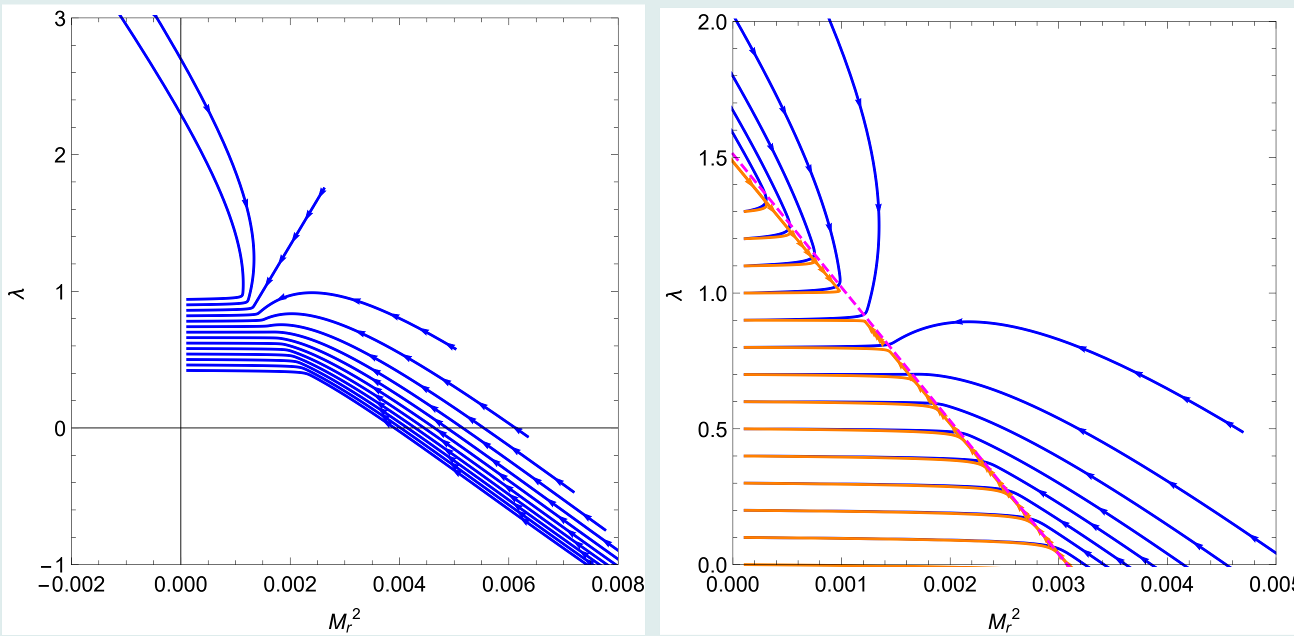
$$\partial_t h_r^2 - (\eta_\phi + 2\eta_\psi) h_r^2 = -\frac{4\Pi_r h_r^4 v_4}{(1+M_r^2)^2 - 4\Pi_r^2}\left[\frac{2(1+M_r^2)}{(1+M_r^2)^2 - 4\Pi_r^2}\left(1-\frac{\eta_\phi}{6}\right) + 1 - \frac{\eta_\psi}{5}\right].$$

Algebraic equations of the anomalous dimensions:

$$\eta_\psi = h_r^2 v_4 \frac{(1+M_r^2)^2 + 4\Pi_r^2}{((1+M_r^2)^2 - 4\Pi_r^2)^2}\left(1-\frac{\eta_\phi}{5}\right), \qquad \eta_\phi = h_r^2 v_4 (4-\eta_\psi).$$

Slow (logarithmic) variation of the Yukawa-coupling h

$$h_r^2(t) = \frac{h_r^2(t_0)}{1 - 2h_r^2(t_0)C(t - t_0)}, \qquad C \approx v_4\left(4 + \frac{2}{1 + M_{rUV}^{*2}}\right) < \frac{3}{16\pi^2}$$



$h_r(t = -\infty) = 0.7$

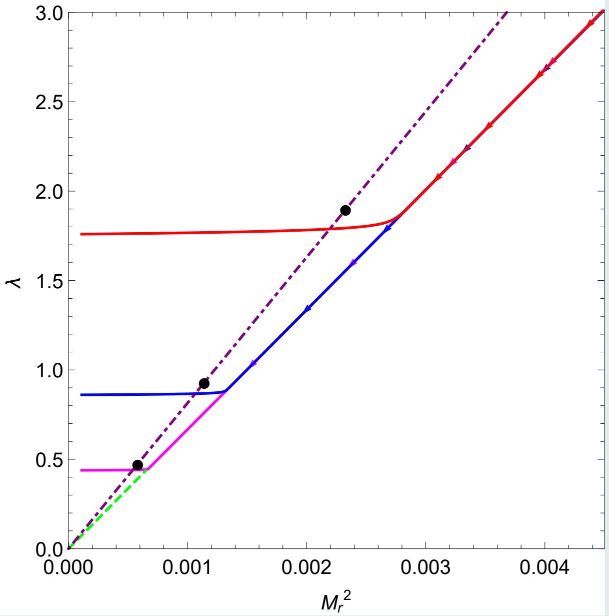
Initial data $(h_{r\Lambda}, \Pi_{r\Lambda})$ labels the points in the plane.

”Neutral” flow-line separates regions of RG-trajectories ending with instability and RG-trajectories ending with Landau-singularity

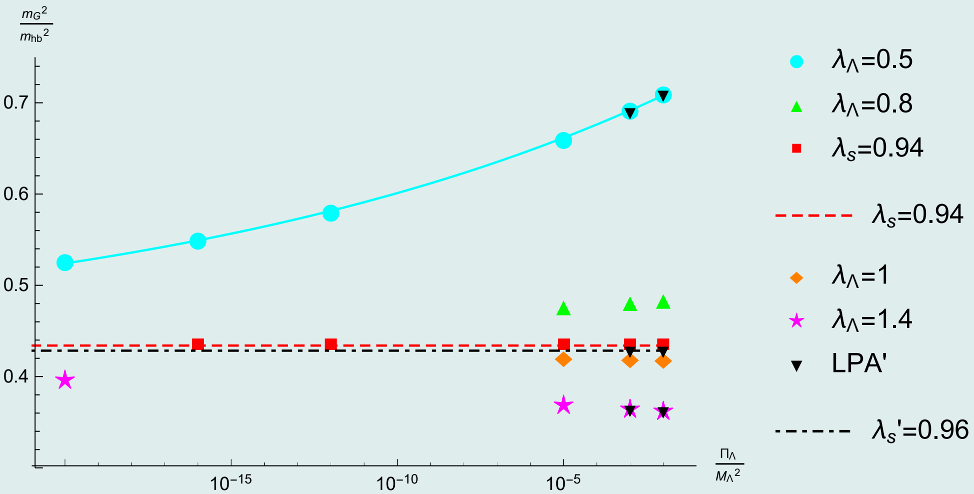
LPA'-flow $\rightarrow$ adiabatic overlay of LPA flow patterns belonging to slowly varying $h_r(t)$

Slow (logarithmic) variation of the Yukawa-coupling h

The ”neutral” flow-line represents adiabatic shift of the fixed-line of LPA.



No substantial change in the prediction for the scalar mass-ratio



Conclusions

Yukawa-models of general global symmetry can be constructed which possess

– an LPA fixed line, parametrized by the Yukawa-coupling (invariant in LPA);

– the improved (exact?) RG-evolution follows adiabatically the LPA fixed point structure, although

no fixed point solution of the exact RGE's exists.

Specific to $U_L(1) \times U_R(1)$ symmetric model:

Quadratic explicit symmetry breaking leads to unique prediction of m_G^2/m_{hb}^2, as a function of m_ψ^2/m_G^2, when $M_\Lambda^2/|\Pi_\Lambda| \to \infty$.