Basics of Electroweak Baryogenesis

Zsolt Szép
MTA-ELTE Theoretical Physics Research Group

2021/02/02
ELFT School 2021

- Motivation for EWBG and some conclusions

- On the meaning of Sakharov's conditions
- Review of the EWBG in the minimal SM
  - Fermion number anomaly in SM
  - Mechanism for EWBG
  - Calculation of the effective potential and order of the EWPT

ELKH | Eötvös Loránd Research Network

Motivation 1/2

Observation: baryon asymmetry of the Universe (BAU), that is the Universe seems to contain relatively few antibaryons (for evidence see András Patkós' talk)

- antimatter is not observed in our solar system
- high energy cosmic rays contains only a small amount of antimatter as secondary products
  $\Longrightarrow$ no evidence of BAU on the galactic scale
- absence of hard γ rays from nearby clusters of galaxies, which are expected to be emitted in nucleon-antinucleon annihilation $\Longrightarrow$ matter/antimatter separation on a scale $> 10^{12} M_\odot$
- abundance of light elements (^{1,2,3}H, ^{3,4}He, ^{7}Be, ^{7}Li) explained in primordial nucleosynthesis (BBN) using the baryon-to-photon ratio: P. A. Zyla *et al.* (PDG) PTEP 2020, 083C01 (2020)

$$\eta_{\rm BBN} = \frac{n_B}{n_\gamma} = \frac{n_b - n_{\bar{b}}}{n_\gamma} = (5.8 - 6.5) \times 10^{-10} (95\%\ {\rm CL})$$

CMB gives a narrower range: $\eta_{\rm CMB} = (6.105 \pm 0.055) \times 10^{-10}$

known from cosmology that with 3 light neutrinos $s = 7.04 n_\gamma$ (at present time)

$\Longrightarrow$ using $T_0^{\rm CMB} \simeq 2.725$ K in $n_\gamma = \dfrac{2\zeta(3)}{\pi^2} T^3$ gives $\eta = \dfrac{n_B}{s} = (8.2 - 9.2) \times 10^{-11}$

densities: s: entropy, n_b: baryon number, $n_{\bar{b}}$: antibaryon number, n_γ: photon

Two possibilities/attitudes:

1. the net baryon number of the observed Universe may simply be set by initial conditions
2. a more appealing endeavor is to suppose that the net baryon number is calculable in terms of microscopic physics, like the abundances of light elements in BBN

$\longrightarrow$ baryogenesis (BG) $\equiv$ dynamical generation of the observed value of η

Motivation 2/2

Sakharov's conditions required to obtain BAU starting from a symmetric Universe:
assuming CPT symmetry! JETP Lett. 5, 24 (1967)

1. baryon number violation (nonconservation) $\equiv \not{B}$
2. violation of discrete C & CP symmetries ($\equiv \not{C}$ & $\not{CP}$) C: charge conjugation P: parity
3. deviation from thermal equilibrium

At first BG was proposed in grand unified theories (GUT). Problem with the classical scenario: low reheating temperature after inflation $T_{\rm rh} \sim 10^{11}$ GeV $\ll M_{\rm GUT}$.   Way out: preheating.

Then 't Hooft realized (PRL37 (1976) 8) that in the EW theory **fermion numbers are anomalous**

$\Longrightarrow$ B, L, $B+L$ are anomalous and not conserved

$\Longrightarrow$ w/o Majorana ν mass $B-L$ is accidentally conserved in SM

Due to the anomaly, topology and the vacuum structure of the EW theory plays a role in EWBG.

Anomalous baryon number violation proceeds btw. vacuua with different quantum numbers:
– through tunneling (instantons) $\longleftarrow$ in the vacuum $T=0$ and at low temperature ($T \ll T_{EW}$)

rate: $\Gamma \propto e^{-4\pi/\alpha_W} = e^{-4\pi \times 137 \sin^2\theta_W} \approx e^{-400} \approx 0$ $\sin\theta_W^2 \approx 0.23$

– through sphaleron transitions $\longleftarrow$ rapid for $T > 100$ GeV ($\Gamma_{\rm sph} \gg H$, H: Univ. expansion rate)
$\Longrightarrow$ in a strong phase transition (PT) these transitions could in principle equilibrate to zero (reduce, if there are conserved charges) any baryon asymmetry produced by GUT $\perp B-L$

Since the minimal SM provides all the necessary ingredients for baryogenesis, it was suggested that anomalous baryon violation in the weak interactions can produce baryon excess during a strong 1$^{\rm st}$ order EWPT   Kuzmin, Rubakov & Shaposhnikov, PLB155, 36 (1985)

Some conclusions and further motivations 1/2

EWBG in SM fails for two reasons:

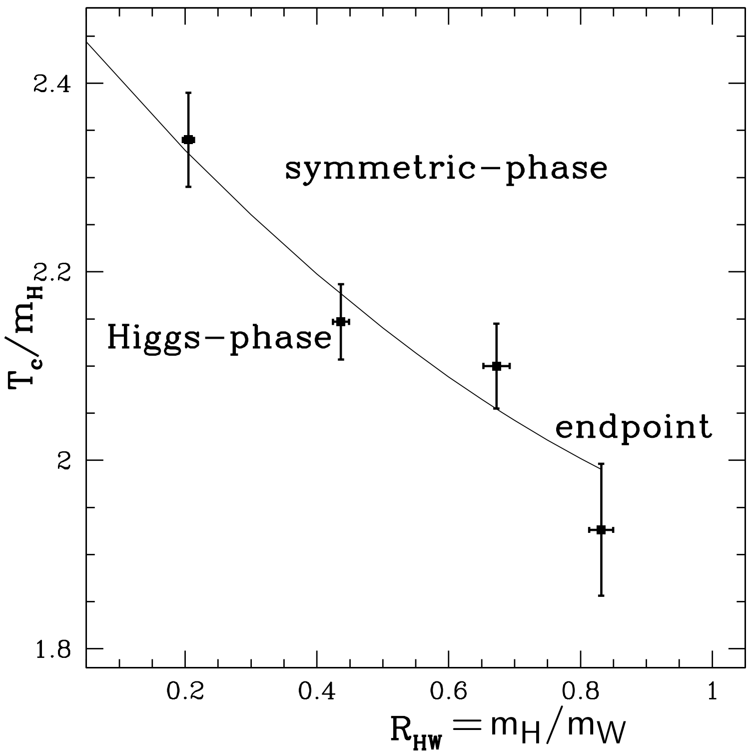
– estimated amount of CP violation is too small: $d_{\mathbf{CP}}/(100\mathrm{GeV})^{12} \sim 10^{-20}$

measure of $\not{CP}$: $d_{\mathbf{CP}} = J \times (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)$

$J = s_{12} s_{23} s_{31} c_{12} c_{23} c_{31}^2 \sin \delta_{\mathrm{CP}}$ Jarlskog invariant   no $\not{CP}$ if $d_{\mathbf{CP}}$ vanishes

– the EWPT is of 1$^{\mathrm{st}}$ order only for Higgs mass $m_H \lessapprox 73$ GeV
Settled btw. '95-'98 as a result of a collective effort: lattice (mainly) + analytic calculations

Csikor et al., PRL82 (1999) 21



– continuum extrapolated result in 4D SU(2)-Higgs model

– done w/o fermions and the $U(1)_Y$ gauge boson

– PT order from the behavior of Lee-Yang zeros for $V \to \infty$

– figure shows: PT is of 1$^{\mathrm{st}}$ order for $m_H < 66.5 \pm 1.4$ GeV

– compared with results of the 3D version of the lattice model (Gürtler et al., PRD56, 3888) showed the reliability of dimensional reduction (pert. integration of the heavy modes)

– perturbative inclusion of fermions (top) gave for SM:

$$m_H < 72.4 \pm 1.7 \text{ GeV}$$

– already **at that time** $m_H > 89.8$ GeV $\Longrightarrow$ SM BG ruled out

Some conclusions and further motivations 2/2

EWPT was re-analyzed after the discovery of the Higgs in the 3d SU(2)-Higgs lattice model including recent algorithmic and action improvements D'Onofrio et al., PRL113 (2014) 141602

EWPT parameters, reflecting intrinsic SM properties, relevant also in low-scale leptogenesis:

– sharp crossover & $T_c = (159.5 \pm 1.5)$ GeV $\longleftarrow$ determined from max. of $\frac{dv}{dT}$

– sphaleron rate: symmetric phase ($T > T_c$): $\Gamma_{\rm sp}^{(s)} = (18 \pm 3)\alpha_w^5 T^4$

broken ph. ($130\text{GeV} < T < T_c$): $\ln \frac{\Gamma_{\rm sp}^{(b)}}{T^4} = (0.83 \pm 0.01) T[\text{GeV}] - (147.7 \pm 1.9)$

– freeze-out temperature: $T_{\rm sp,\, d} = (131.7 \pm 2.3)$ GeV $\longleftarrow$ B-violating (sphaleron) transitions decouple in the early Universe

Why considering EWBG in extensions of SM?

- EWBG is driven by the Higgs field, and a new particle introduced to solve the PT problem of SM EWBG couples strongly to the Higgs.
- EWBG motivated simple extensions of SM also provide a **testable dark matter candidate**. Also, in a 1$^{\rm st}$ order PT bubble collisions, sound waves and MHD turbulence are detectable sources of **gravitational waves**.
- EWBG requires new physics close to the EW scale $\Longrightarrow$ **EWBG is predictive and falsifiable** simple models of EWBG have already been excluded
  - by lack of direct discovery of the new light particles
  - by limits from electric dipole moment searches, which are extremely sensitive to CP violation
- Many well-motivated possibilities remain and there are some new ideas (see talk by A. Patkós).

Bibliography

– K. Funakubo, *CP Violation and Baryogenesis at the Electroweak Phase Transition*, Prog.Theor.Phys. 96, 445 (1996)

– Lectures at the Summer School in High Energy Physics and Cosmology, Trieste, 1998
A. Riotto, *Theories of Baryogenesis*, arXiv:hep-ph/9807454
M. Quiros, Finite temperature field theory and phase transitions, arXiv:hep-ph/9901312

– N. Petropoulos, *Baryogenesis at the electroweak phase transition*, hep-ph/0304275

– J. M. Cline, *Baryogenesis*, arXiv:hep-ph/0609145, Lectures at Les Houches Summer School, Particle Physics and Cosmology: The Fabric of Spacetime, 2006

– V. A. Rubakov, *Cosmology and the Large Hadron Collider*
lecture dedicated to the memory of V. L. Ginzburg at Lebedev Physical Institute, 2010

– J. M. Cline, *TASI Lectures on Early Universe Cosmology: Inflation, Baryogenesis and Dark Matter, 2018*, arXiv:1807.08749

– D. Bödeker and W. Buchmüller. *Baryogenesis from the weak scale to the GUT scale*
arXiv:2009.07294

Word of caution: hereafter everything is illustrative: in some cases I have not checked signs, factors of i, numerical factors and, sometimes, the meaning of the formulas!

Meaning of Sakharov's conditions

1) If the initial net baryon number in the Universe was zero and the baryon number were conserved, the Universe today would still be symmetric.

2) $C\psi C^{-1}=i\gamma^2\gamma^0\bar\psi^{\mathsf{T}}$, $P\psi(x)P^{-1}=\gamma^0\bar\psi^{\mathsf{T}}(x_{\mathsf{p}})$, $T\psi(x)T^{-1}=\gamma^1\gamma^3\bar\psi^{\mathsf{T}}(x_{\mathsf{T}})$ $\qquad x=(t,\mathbf{x})$ $x_{\mathsf{p}}=(t,-\mathbf{x})$

$CJ^\mu(x)C^{-1}=-J^\mu(x)$, $\quad PJ^\mu(x)P^{-1}=J_\mu(x_{\mathsf{p}})$, $\quad TJ^\mu(x)T^{-1}=J_\mu(x_{\mathsf{T}})$, $\qquad x_{\mathsf{T}}=(-t,\mathbf{x})$

current: $J^\mu=\bar\psi(x)\gamma^\mu\psi(x)$ $\qquad$ charge: $Q(t)=\int d^3x\, J^0(x_0,\mathbf{x}) \overset{\mathbf{x}\to-\mathbf{x}}{=} \int d^3x\, J_0(x_{\mathsf{p}})$

for $\dot Q=0:\quad CQC^{-1}=(CP)Q(CP)^{-1}=(CPT)Q(CPT)^{-1}=-Q$ and $PQP^{-1}=Q$

Remark: both $\not{C}$ and $\not{CP}$ are needed. We can see this in two ways.

A) Hamiltonian time evolution of the Universe from an baryon-symmetric initial state (I)

State of the Universe described by the density operator $\rho(t)=\sum_n p_n\,|\psi_n(t)\rangle\langle\psi_n(t)|$, $\sum_n p_n=1$

$|\psi_n(t)\rangle$ state vector in Schrodinger picture

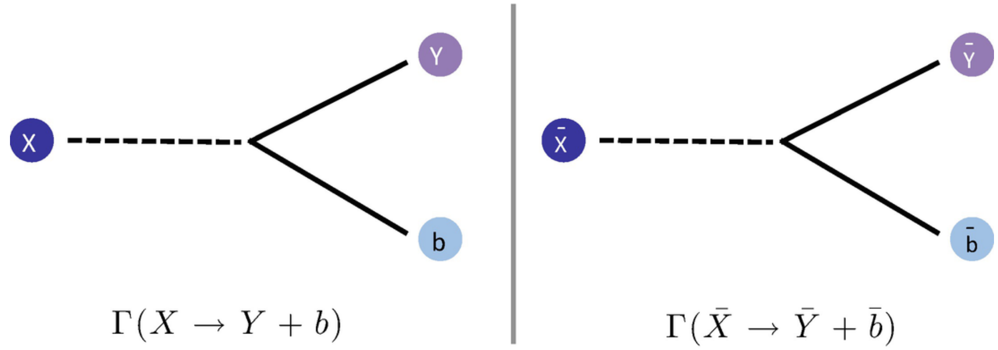
Quantum Liouville equation: $\quad i\dfrac{\partial\rho(t)}{\partial t}=L\rho(t)$ with $\quad L\rho(t)=[H,\rho(t)]$

$\Longrightarrow$ formal solution $\rho(t)\equiv\rho_I e^{-iLt}$ with $\rho_I\equiv\rho(t{=}t_I)$ such that $\langle B\rangle(t{=}t_I)\equiv\mathrm{Tr}[\rho_I B]=0$

If $\mathcal{S}\in\{C,CP\}$ is a symmetry $\quad\Longrightarrow\quad [H,\mathcal{S}]=0 \quad\Longrightarrow\quad [\rho(t),\mathcal{S}]=0\quad\Longrightarrow$

$$\langle B\rangle(t)=\mathrm{Tr}[\rho(t)B]=\mathrm{Tr}\Big[\underbrace{\mathcal{S}^{-1}\mathcal{S}}_{\mathbb{1}}\,\rho(t)B\Big]\underset{[\rho,\mathcal{S}]=0}{\overset{\text{cycl.}}{=}}\mathrm{Tr}\Big[\rho(t)\,\underbrace{\mathcal{S}B\mathcal{S}^{-1}}_{-B}\Big]=-\,\mathrm{Tr}[\rho(t)B]\Longrightarrow\langle B\rangle(t)=0$$

B) Decay process when CP is conserved



baryon numbers: $B_X = B_Y = 0$ and $B_b = 1$

For a left-handed particle $Y_{\rm L}$ $\quad C : Y_{\rm L} \to \bar{Y}_{\rm L} \quad P : Y_{\rm L} \to Y_{\rm R} \quad \Longrightarrow \quad CP : Y_{\rm L} \to \bar{Y}_{\rm R}$

$$\left.\begin{array}{l}\Gamma(X\to Y_{\rm L}b_{\rm L}) \overset{CP}{=} \Gamma(\bar{X}\to\bar{Y}_{\rm R}\bar{b}_{\rm R}) \\ \Gamma(X\to Y_{\rm R}b_{\rm R}) \overset{CP}{=} \Gamma(\bar{X}\to\bar{Y}_{\rm L}\bar{b}_{\rm L})\end{array}\right\} \overset{+}{\Longrightarrow} \underbrace{\Gamma(X\to Y_{\rm L}b_{\rm L})+\Gamma(X\to Y_{\rm R}b_{\rm R})}_{\Gamma(X\to Y+b)} \overset{CP}{=} \underbrace{\Gamma(\bar{X}\to\bar{Y}_{\rm R}\bar{b}_{\rm R})+\Gamma(\bar{X}\to\bar{Y}_{\rm L}\bar{b}_{\rm L})}_{\Gamma(\bar{X}\to\bar{Y}+\bar{b})}$$

$$\Delta B = 1 \qquad\qquad \Delta B = -1$$

so, decays of particles and antiparticles create and destroy baryon number at the same rate
$\Longrightarrow$ net baryon number created in these processes vanish when CP is conserved,
even when C is violated, i.e. $\Gamma(X \to Y_{\rm L} + b_{\rm L}) \neq \Gamma(\bar{X} \to \bar{Y}_{\rm L} + \bar{b}_{\rm L})$

3) In thermal equilibrium:
forward and reverse reactions occur at the same rate, for both matter and antimatter processes:

$$\Gamma(X \to Y + b) = \Gamma(Y + b \to X) \text{ and } \Gamma(\bar{X} \to \bar{Y} + \bar{b}) = \Gamma(\bar{Y} + \bar{b} \to \bar{X})$$

$\Longrightarrow$ even if C and CP are simultaneously violated, i.e. $\Gamma(X \to Y + b) \neq \Gamma(\bar{X} \to \bar{Y} + \bar{b})$ the baryon number produced in B violating forward and reverse reaction rates of each of the matter and antimatter processes cancel out

Another way to see this:

$$\langle B \rangle_{\text{eq}} := \text{Tr}\left[e^{-\beta H} B\right] = \text{Tr}\left[\underbrace{(CPT)^{-1}(CPT)}_{\mathbb{1}} e^{-\beta H} B\right] \underset{\text{cycl.}}{=} \text{Tr}\left[(CPT) e^{-\beta H} B (CPT)^{-1}\right]$$

$$\underset{[CPT,H]=0}{=} \text{Tr}\left[e^{-\beta H} \underbrace{(CPT) B \big(CPT\big)^{-1}}_{-B}\right] = -\,\text{Tr}\left[e^{-\beta H} B\right] = -\langle B \rangle_{\text{eq}} \quad \Longrightarrow \quad \langle B \rangle_{\text{eq}} = 0$$

Simultaneous C and CP violation means $\Gamma(X \to Y + b) \neq \Gamma(\bar{X} \to \bar{Y} + \bar{b})$.
Only when the reaction on one side is favored in a period of non-equilibrium evolution of the Universe, net baryon number is produced.

Interactions of known particles are in thermal equilibrium for $T \in (\sim 100, 10^{12})$ GeV, *i.e.* $T > T_{\text{EW}}$.

Departure from thermal equilibrium can be attained:
1. through the out-of-equilibrium decay of a heavy particle (decay rate: Γ_X)
2. during a phase transition leading to the spontaneous breaking of a global symmetry

Scenario 1. (*e.g.*: GUT BG, BG through leptogenesis)
$T \gg M_X$: all particles are in thermal equilibrium $\Rightarrow n_X \simeq n_{\bar{X}} \simeq n_\gamma$ (up to factors related to Bose and Fermi species, internal d.o.f.)

If $\Gamma_X > H$: equilibrium abundance $\frac{n_X^{\rm eq}}{n_\gamma} = \frac{n_{\bar{X}}^{\rm eq}}{n_\gamma} \simeq \left(\frac{M_X}{T}\right)^{3/2} e^{-M_X/T}$ can be maintained,
as X and $\bar{X}$ can decay $\Longrightarrow$ no departure from equilibrium abundances
If $\Gamma_X \lesssim H$: X and $\bar{X}$ cannot decay on the expansion time scale $\tau_U \sim H^{-1}$ of the Universe

$T_{d,X} > M_X$: X and $\bar{X}$ are weakly interacting $\Rightarrow$ cannot catch up with the Univ.'s expansion
$\Longrightarrow$ decouple from the thermal bath while still being ultrarelativistic: $n_{X,\bar{X}} \simeq n_\gamma \simeq T_{d,X}^3$
$T < T_{d,X}$ **overabundance** of $X, \bar{X}$: $n_{X,\bar{X}} \gg n_{X,\bar{X}}^{\rm eq} \Longrightarrow$ departure from thermal equilibrium

Scenario 2. The assumed 1st order PT needed for EWBG is driven by:
– thermal fluctuations $\longleftarrow$ importance of loop corrections to classical potential — old approach
– classical potential $\longleftarrow$ loop corrections are assumed to be small — new approach

Still: baryon asymmetry can develop while B-violating interactions are still in thermal equilibrium
departure from thermal equilibrium needed to preserve the created baryon asymmetry
requirement: CPT breaking interaction for some time — Cohen & Kaplan, PLB192 (1987) 251
scenario: thermalon ϕ coupled to J_B^μ through a dim. 5 op. $\delta\mathcal{L} = \Lambda^{-1} J_B^\mu \partial_\mu \phi$ (resulted at $\Lambda \gtrsim T$)
acts as an eff. chemical potential when it is spatially homogeneous and slowly varying:

$$\partial_0\phi/\Lambda \equiv \mu_{\rm eff} \quad \Longrightarrow \quad \delta\mathcal{L} = \mu_{\rm eff} n_B$$

Fermion number anomaly in EWSM

Compared to a vector-like theory (QED, QCD), the fermionic part of the EW Lagrangian:

$$\mathcal{L}_f = \sum_{j=1}^{N_G=3} \left[\bar{\Psi}^j_L i\not{D}(\vec{W}, B)\Psi^j_L + \bar{\Psi}^j_R i\not{D}(B)\Psi^j_R\right], \qquad \Psi^j_L = \{L^j_L, q^j_L\}, \quad \Psi^j_R = \{L^j_R, q^j_R\}$$

contains left- and right-handed fields in different representations (chiral theory) $\mathrm{P}_{R/L} = \frac{1}{2}(1 \pm \gamma_5)$

$$SU(2) \text{ doublet fields}: \quad L^i_L = \mathrm{P}_L \left\{ \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu, \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \right\} \quad q^i_L = \mathrm{P}_L \left\{ \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix} \right\}$$

$$SU(2) \text{ singlet fields}: \quad L^i_R = \mathrm{P}_R \left\{ \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu, \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \right\} \quad q^i_R = \mathrm{P}_R \left\{ \begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix} \right\}$$

☛ only the left-handed field couple to the SU(2) gauge bosons $\vec{W}_\mu$ (chiral theory)

$$D_\mu(B) = \mathbb{1}_{2\times2}\partial_\mu - ig'B_\mu Y, \quad D_\mu(\vec{W}, B) = \mathbb{1}_{2\times2}\partial_\mu - ig\vec{T}\cdot\vec{W}_\mu - ig'B_\mu Y \qquad T_a = \frac{\sigma_a}{2}$$

☛ all fields, except for $\nu^e_R, \nu^\mu_R, \nu^\tau_R$, couple to the $U(1)_Y$ gauge boson B_μ:

$$\text{quark with:} \quad Y^q_L = \begin{pmatrix} 1/6 & 0 \\ 0 & 1/6 \end{pmatrix} \qquad Y^q_R = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix}$$

$$\text{leptons with:} \quad Y^l_L = \begin{pmatrix} -1/2 & 0 \\ 0 & -1/2 \end{pmatrix} \qquad Y^l_R = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$$

convention for Y: electric charge is $Q = T_3 + Y = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + Y$

There are many **accidental global symmetries**. In particular, there are vector-like symmetries:

$$\begin{aligned}\psi(x) &\to e^{i\theta}\psi(x)\\ \bar\psi(x) &\to e^{-i\theta}\bar\psi(x)\end{aligned} \quad\Longleftrightarrow\quad \begin{aligned}\psi_L(x) &\to e^{i\theta}\psi_L(x), &\psi_R(x) &\to e^{i\theta}\psi_R(x)\\ \bar\psi_L(x) &\to e^{-i\theta}\bar\psi_L(x), &\bar\psi_R(x) &\to e^{-i\theta}\bar\psi_R(x)\end{aligned}$$

leading at classical level to conserved Noether charges (when applied to quarks and leptons):

baryon number $$B = \frac{1}{3}\sum_{\psi_q^j}\int d^3x\, \bar\psi_q^j(x)\mathbb{1}_{2\times 2}\otimes\gamma^0\psi_q^j(x) \qquad \psi_q^j \in \{q_L^j, q_R^j\}$$

lepton number $$L = \sum_{\psi_l^j}\int d^3x\, \bar\psi_l^j(x)\mathbb{1}_{2\times 2}\otimes\gamma^0\psi_l^j(x) \qquad \psi_l^j \in \{l_L^j, l_R^j\}$$

But, at quantum level the fermion number of the left-handed (LH) and right-handed (RH) fermions is anomalous (triangle anomaly).

see Ch. 6.4.1 in K. Fujikawa and H. Suzuki, Path Integrals and Quantum Anomalies (2004)

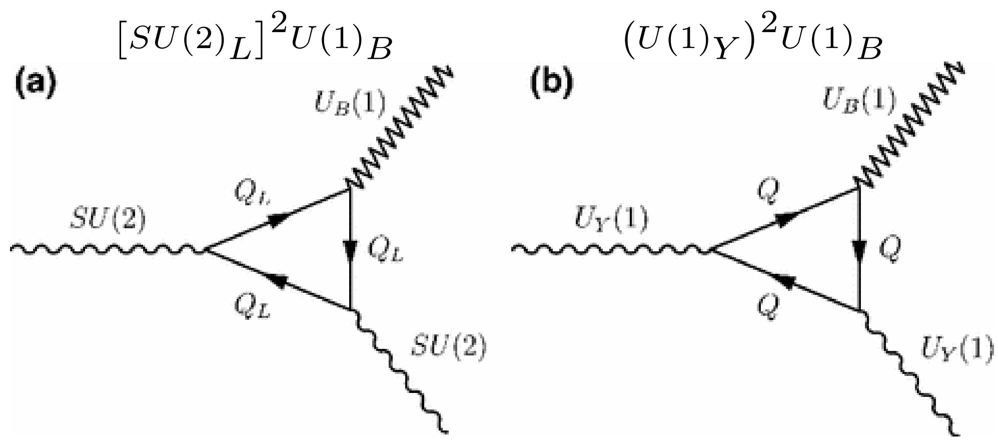
In the SM LH and RH fermions couple with different strength to the gauge bosons
$\Longrightarrow$ both baryon and lepton numbers are anomalous

$\Longrightarrow$ relevant symmetry group of the EWSM:

$$\left\{SU(2)_L\times U(1)_Y\right\}_{\text{gauge}} \times \left\{U(1)_{B+L}\right\}_{\substack{\text{global \&}\\ \text{anomalous}}} \bigtimes_{i=1}^{N_G=3} \left\{U(1)_{\frac{B}{3}-L_i}\right\}_{\text{global}}$$

Anomaly factor for baryon (J_B^μ) and lepton (J_l^μ) currents

Triangle graphs giving nonvanishing contribution to the anomaly of the baryon current:



figures from: N. D. Barrie, *Cosmological Implications of Quantum Anomalies*, Springer theses, 2018

One can use the formula derived for chiral anomaly: (Eqs. (30.78)-(30.80) of Schwartz, QFT & SM (2014))

$$\partial_\mu J_a^\mu = \frac{\epsilon^{\mu\nu\alpha\beta}}{64\pi^2}\left(\sum_{\text{LH particles}} A_r^{abc} g_L^2 F_{\mu\nu}^b F_{\alpha\beta}^c - \sum_{\text{RH particles}} A_{r'}^{abc} g_R^2 F_{\mu\nu}'^{\,b} F_{\alpha\beta}'^{\,c}\right)$$

$$A_r^{abc} = \mathrm{tr}\left[T_r^a\{T_r^b, T_r^c\}\right]$$

anomaly factor
T_r^a generator in representation r

- no index a for baryon and lepton currents ($U(1)_B$ and $U(1)_l$ sym.) $\Rightarrow T_r^a = 1$
- no index of b and c for $U(1)_Y$ current ☛ doublet nature of LH particles has to be included
- for $[SU(2)_L]^2 U(1)_{B/l}$: $T_r^{b/c} = \frac{\sigma^{b/c}}{2} \Longrightarrow A^{bc} = \mathrm{tr}\left[1\left\{\frac{\sigma^b}{2}, \frac{\sigma^c}{2}\right\}\right] = \frac{\delta^{bc}}{2}\mathrm{tr}\,\mathbb{1}_{2\times 2} = \delta^{bc}$
- for $[U(1)_Y]^2 U(1)_{B/l}$: $T_r^a = T_r^b = T_r^c = 1 \Longrightarrow A = 2$

Anomaly coeff. for baryon current

1) contribution of $\left[SU(2)_L\right]^2 U(1)_B$ ☛ only LH quarks couple to $\vec{W}_\mu$: coupling $g_L \propto g$ that LH quarks are in a doublet was already taken care when computing the trace

$$\sum_{q_L} A^{bc} g_L^2 = g^2 \delta^{bc} N_G \times \underset{\text{color}}{3} \times \underset{\text{B number}}{\frac{1}{3}} = g^2 \delta^{bc} N_G$$

2) contribution of $\left[U(1)_Y\right]^2 U(1)_B$ ☛ both LH & RH quarks couple to B_μ: coupling $g_{L/R} \propto g'$

$$\sum_{q_L} A g_L^2 - \sum_{q_R} A g_R^2 = 2 \times N_G \times 3 \times \frac{1}{3} \times \left\{ \underset{\substack{\text{two quarks} \\ \text{in a doublet}}}{2} \left(\frac{1}{6}\right)^2 - \left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right] \right\}$$

$$= 2N_G \left[\frac{1}{18} - \frac{5}{9}\right] = -N_G$$

Anomaly coeff. for lepton current

contribution of $\left[SU(2)_L\right]^2 U(1)_B$

$$\sum_{l_L} A^{bc} g_L^2 = g^2 \delta^{bc} N_G$$

contribution of $\left[U(1)_Y\right]^2 U(1)_B$

$$\sum_{l_L} A g_L^2 - \sum_{l_R} A g_R^2 = 2 \times N_G \left[2\left(-\frac{1}{2}\right)^2 - (-1)^2\right] = -N_G$$

$\Longrightarrow$ identical contribution:

$$\boxed{\partial_\mu J_B^\mu = \partial_\mu J_l^\mu = \frac{N_G}{64\pi^2} \epsilon^{\mu\nu\alpha\beta} \left[g^2 F^b_{\mu\nu}(W) F^b_{\alpha\beta}(W) - g'^2 f_{\mu\nu}(B) f_{\alpha\beta}(B)\right]}$$

Topology and the Chern-Simons number

To see the consequence of $\partial_\mu J_B^\mu = \partial_\mu J_l^\mu = \frac{N_G}{64\pi^2}\epsilon^{\mu\nu\alpha\beta}\left[g^2 F^b_{\mu\nu}F^b_{\alpha\beta} - g'^2 f_{\mu\nu}f_{\alpha\beta}\right]$ write:

$$\frac{g^2}{64\pi^2}\epsilon^{\mu\nu\alpha\beta}F^b_{\mu\nu}F^b_{\alpha\beta} = \partial_\mu K^\mu \qquad K^\mu = \frac{g^2}{32\pi^2}\epsilon^{\mu\nu\alpha\beta}\left(F^a_{\nu\alpha}W^a_\beta - g\epsilon_{abc}W^a_\nu W^b_\alpha W^c_\beta\right)$$

$$\frac{g'^2}{64\pi^2}\epsilon^{\mu\nu\alpha\beta}f_{\mu\nu}f_{\alpha\beta} = \partial_\mu k^\mu \qquad k^\mu = \frac{g'^2}{32\pi^2}\epsilon^{\mu\nu\alpha\beta}f_{\nu\alpha}B_\beta$$

Neglect integrals like $\int d^3x \nabla\cdot\vec{K}$ and introduce $N_{\mathsf{CS}} = \int d^3x K^0, n_{\mathsf{CS}} = \int d^3x k^0$

$$B(t) - B(0) = L(t) - L(0) = N_G[N_{\mathsf{CS}}(t) - N_{\mathsf{CS}}(0) - (n_{\mathsf{CS}}(t) - n_{\mathsf{CS}}(0))]$$

For an abelian gauge field $f_{\mu\nu} \neq 0$ is required for $k_\nu \neq 0$, but for a non-abelian field, one can have $K^\mu \neq 0$ even with $F^a_{\mu\nu} = 0$ due to the term $\propto g$ in K^μ (i.e. in the pure gauge case)
$\Longrightarrow$ for vacuum configurations $\Delta n_{CS} = 0$

Consider in $W^a_0 = 0$ gauge a field that it pure gauge: $\boldsymbol{W}_{\mathsf{vac}} = \frac{i}{g}U^{-1}\nabla U$ ($W_i = W^a_i\sigma^a/2$) with $U \in SU(2)$ such that $U \to \mathbb{1}$ when $|\vec{x}| \to \infty$. Such a configuration is a ground state, and since $SU(2) \simeq S^3 \Rightarrow U(x) : S^3 \mapsto S^3$. According to the homotopy theory, such mappings fall into equivalence classes characterized by winding (Chern-Simons) number N_{CS}: two mappings to the same class if U_1 can be continuously transformed to U_2.

N_{CS} invariant under ‘small’ gauge transformations, i.e., continuously connected to the identity.
$N_{\mathsf{CS}} \neq 0$ for ’large’ gauge transformation (not continuously connected to the identity), of the form:

$$\boldsymbol{U^{(1)}(x) = \frac{x_0 + i\vec{x}\cdot\vec{\sigma}}{r}}, \quad r = (x_0^2 + |\vec{x}|^2)^{1/2} \quad \ldots \quad U^{(n)}(x) = \left[U^{(1)}\right]^n$$

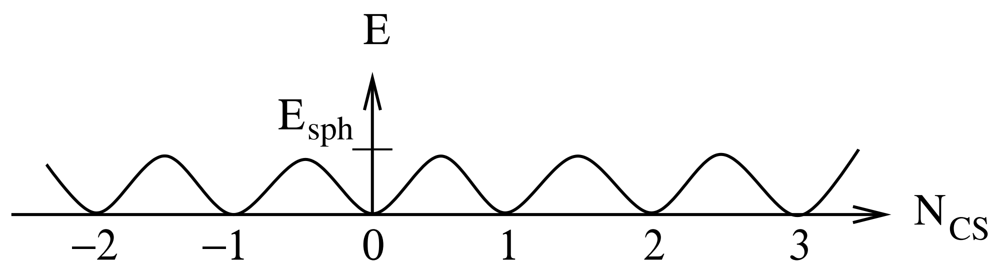
SM for $\theta_w = 0$ is the SU(2) Higgs theory: $SU(2)_L$ gauge boson coupled to the Higgs field H.
In order to explore the vacuum structure we choose a constant Higgs, the gauge $W_0^a = 0$

From the ground state $\mathcal{G}^{(0)}_{\rm vac} = \left\{ W_i^{(0)} = 0,\, H^{(0)} = (0, v),\, N_{\rm CS} = 0 \right\}$

one can reach **infinitely many ground states**

$$\mathcal{G}^{(n)}_{\rm vac} = \left\{ W_i^{(n)} = \frac{i}{g_2} \left(U^{(n)}\right)^{-1} \nabla U^{(n)},\, H^{(n)} = U^{(n)} H^{(0)},\, N_{\rm CS} = n \right\}$$

separated by an energy barrier:



$\Delta N_{\rm CS} = 1$ in a transition from $\mathcal{G}^{(n)}_{\rm vac}$ to $\mathcal{G}^{(n\pm1)}_{\rm vac} \Longrightarrow \Delta B = \Delta L = \pm N_G$

Each transition creates 9 LH quarks ($N_c \times N_G$) and 3 LH leptons (one per generation)

$$3 \sum_{i=1}^{3} q_L^i + \sum_{i=1}^{3} \ell_L^i \longleftrightarrow 0$$

$SU(2)_L$ Sphaleron

Klinkhammer and Manton's type **ansatz at $T \neq 0$ for the sphaleron** $\phi_0^{\text{sp}} = \{W_i^a(\mathbf{x}), H(\mathbf{x})\}$

$$W_i^a(\mathbf{x})\sigma^a dx^i = -\frac{2i}{g} f(r) U_\infty dU_\infty^{-1} \qquad H(\mathbf{x}) = \frac{v(T)}{\sqrt{2}} h(r) U_\infty \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$U_\infty = U^{(1)}(x_0 = 0, x_1, x_2, x_3) \qquad r = gv(T)|\mathbf{x}| \text{ : dimensionless radial coordinate}$$

The gauge and Higgs field **radial profile functions** $f(r)$ and $h(r)$:
– are determined numerically by imposing the stationary conditions on the energy functional:

$$r^2 \frac{d^2 f}{dr^2} = 2f(1-f)(1-2f) - \frac{1}{4} r^2 h^2 (1-f) \qquad \frac{d}{dr}\left(r^2 \frac{dh}{dr}\right) = 2h(1-f)^2 - \frac{\lambda}{g^2} r^2 h (1-h^2)$$

– satisfy the boundary conditions: $f(0) = h(0) = 0 \quad \text{and} \quad f(\infty) = h(\infty) = 1$

– yields the energy functional: $S_3[\phi_0^{\text{sp}}] \equiv \frac{E_{\text{sp}}}{T} = \frac{4\pi v(T)}{gT} B\left(\frac{\lambda}{g^2}\right)$

S_3 is the action of the SU(2) Higgs model (at $T = 0$ it is the classical action)

$$B\left(\frac{\lambda}{g^2}\right) = \int_0^\infty dr \left[4\left(\frac{df}{dr}\right)^2 + \frac{8}{r^2} f^2 (1-f)^2 + \frac{r^2}{2}\left(\frac{dh}{dr}\right)^2 + h^2(1-f)^2 + \frac{1}{4}\left(\frac{\lambda}{g^2}\right) r^2 (1-h^2)^2\right]$$

$B(\lambda/g^2)$ is weakly varying and of $\mathcal{O}(1)$ and for $T = 0$: $\quad 8\text{GeV} \lesssim E_{\text{sp}} \lesssim 14\text{GeV}$
Chern-Simons number (w/o the factor of N_G), that is baryonic and leptonic charge of a sphaleron
$N_{\text{CS}}(\phi_0^{\text{sp}}) = 1/2$

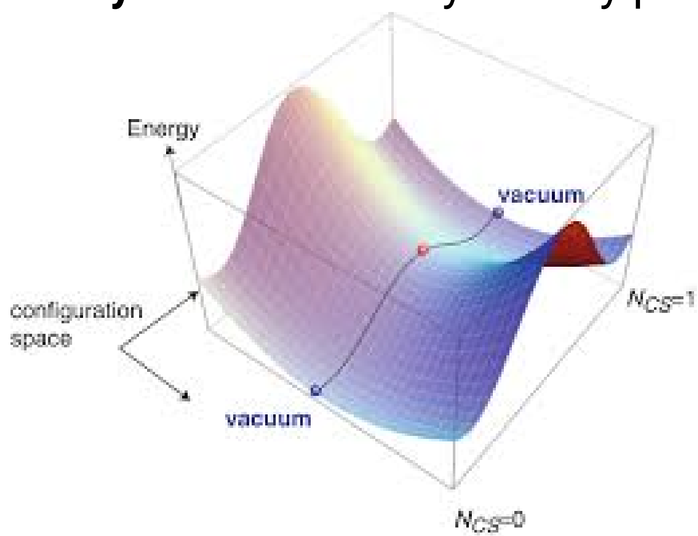
$SU(2)_L$ Sphaleron

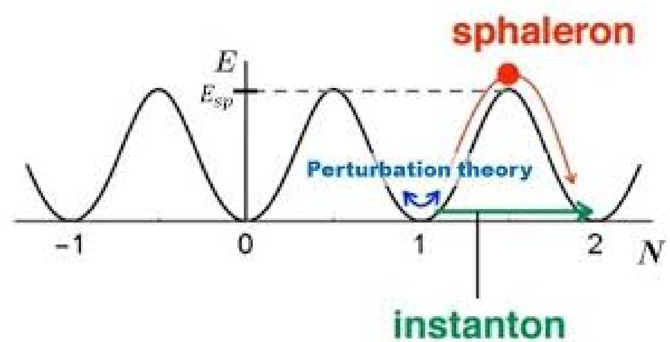
Distinction to be made: sphaleron solution vs. sphaleron (baryon number violating) process

Terminology concerning the solution of a classical field equations:
– instanton: localized, finite-action solution for imaginary time t ($t^2 \leq 0$, Euclidean)
– soliton: static, stable, finite-energy solution for real time t ($t^2 \geq 0$)
– sphaleron: static, unstable, finite-energy solution for real time t

(see F. Klinkhamer, Sphalerons in the SM)

sphaleron sol. – exists **only** in the broken symmetry phase (Higgs vev $v \neq 0$)

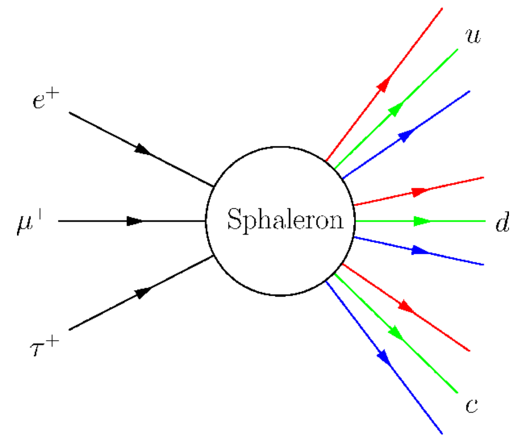




– role in the **washout factor** and **baryon number preservation criterion (BNPC)**

depletion rate: $\frac{\partial n_B}{\partial t} = -k(T) n_B, \quad k(T) = -N_G \frac{13}{2} \frac{\Gamma_{\rm sp}(T)}{VT^3} \sim A(T) e^{-E_{\rm sp}/T}$

$SU(2)_L$ Sphaleron



sphaleron process
– means conversion of quarks into leptons and vice-versa

e.g. $3\bar{l}_L \longrightarrow 9q_L$

$\Delta B = 9 \times \frac{1}{3} - 0 = 3 \quad \Delta L = 0 - 3 \times (-1) = 3 \quad \Delta Q = Q_f - Q_i$

– L and B change by 3 units, $B - L$ conserved, $B + L$ violated

– occurs both in the symmetric and the broken phase

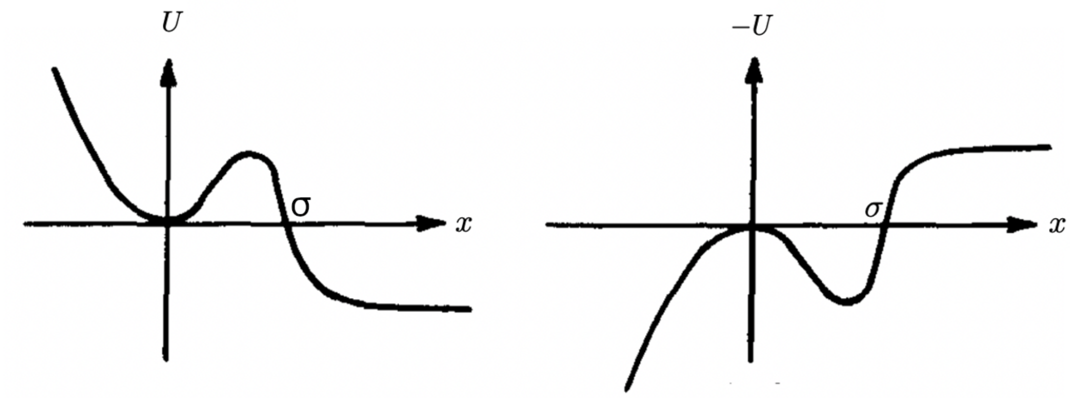
– effective in the symmetric phase, but cannot be calculated perturbatively

$\Gamma_{\rm sp}^{(s)} \propto (\alpha_w^4 T^4) \alpha_w \ln \alpha_w^{-1}$ from dimensional analysis   form: Bödeker NPB559 (1999) 502

only relevant scale: $\mathcal{O}(g^2 T)$ mass scale of the transverse gauge bosons
at this scale pert. expansion breaks down: contribution of same order from an ∞ nr. of diagrams
$\Longrightarrow$ proportionality factor κ determined in lattice simulations D'Onofrio et al., PRL113 (2014) 141602 and G D. Moore, PRD62 (2000) 085011

– suppressed in the broken phase, where sphaleron solution exists

Bounce solution and false vacuum decay



QM tunneling (barrier penetration): in WKB approximation the probability of finding the particle at escape point σ with zero kinetic energy is $P_{\mathrm{WKB}} \propto e^{-2\int_0^\sigma \sqrt{2U}}$

$m=1,\ E=0$

Coleman (PRD15 (1977) 2929) reformulated the problem: probability given by action related to the **bounce solution** to the Euclidean EoM of a particle in inverted potential ($\tau = it$)

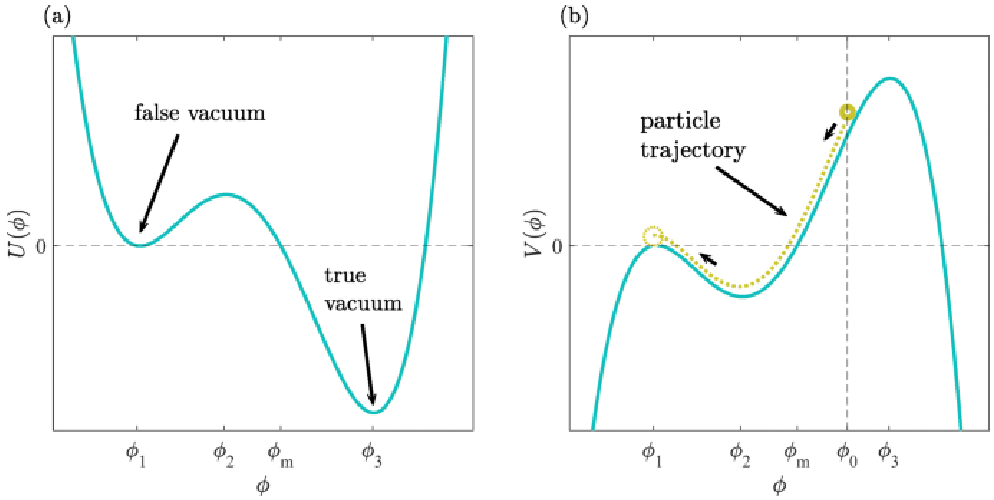
$$P \propto e^{-S_E(x_b)}, \quad S_E(x) = \int_{-\infty}^{\infty} d\tau L_E, \quad L_E = \frac{1}{2}\left(\frac{dx}{d\tau}\right)^2 + U(x)$$

bounce solution $\boldsymbol{x_b(\tau)}$: $x(\tau=-\infty)=0 \longrightarrow x(0)=\sigma \longrightarrow x(\tau=\infty)=0$

N.B.: particles bouncing back at $x=\sigma \to$ the factor of 2 in exponent of P_{WKB} is taken care of

WKB approximation corresponds to a classical trajectory, so in a path integral formulation of the transition amplitude corrections comes from fluctuations around the classical path. At quadratic order, Gaussian integral $\to$ fluctuation operator $\partial_\tau^2 + \omega^2$ in the determinant is not positive definite!

bounce sol. is a saddle point $\Longrightarrow$ negative eigenvalue of fluct. operator $\Longrightarrow$ imaginary part in the ground state energy $\Longrightarrow$ decay Callan & Coleman PRD16 (1977) 1762



Field theoretical description of nucleation: system minimizes its free energy by transition from metastable state (ϕ_1) to the true vacuum (ϕ_3) through classically forbidden configurations. The **bounce solution** ϕ_B is a $O(4)$ symmetric solution ($O(3)$ for $T \neq 0$) of the Euclidean field equation (EoM). Fluctuation around the bounce solution leads to imaginary free energy.

Need to calculate:
a) the classical action of a bubble solution (bounce solution)
b) the quadratic fluctuations around the classical (bounce) solution
c) the sum over infinitely many bounce solutions.

At finite temperature the decay rate of the false vacuum: A. D. Linde, NPB 216 (1983) 421

$$\frac{\Gamma(T)}{V} = \frac{|\omega_-|}{\pi}\left(\frac{S_3(\phi_B, T)}{2\pi T}\right)^{3/2}\left(\frac{\det'[-\Delta + V''(\phi_B, T)]}{\det[-\Delta + V''(\phi_1, T)]}\right)^{-1/2} e^{-S_3(\phi_B, T)/T}$$

bounce $\phi_B(\boldsymbol{x})$ is an $O(3)$ symmetric (static) solution to $\nabla^2\phi = U'(\phi)$ b.c. $\lim_{|\boldsymbol{x}|\to\infty} \phi(\boldsymbol{x}) = \phi_1$

– bounce solution is a saddle point $\Rightarrow$ fluctuation operator around it has an unstable mode $\Rightarrow$ ω_- frequency is imaginary
– prefactor is the zero eigenvalue (mode) contribution present due to the invariance of the bounce solution under translation of its center: 3 of them, each giving $S_3(\phi_B, T)/(2\pi T)$

On the evaluation of the sphaleron rate in the broken phase

To get the decay rate we need to consider the free energy of a dilute sphaleron gas:

$$\Gamma_{\text{sp}} = \frac{|\omega_-|}{\pi T}\text{Im }F_{\text{s.g.}} = \frac{|\omega_-|}{\pi}\text{Im }\frac{Z_{\text{sp}}}{Z_0} = \frac{|\omega_-|}{\pi}\text{ Im }e^{-\left(\Gamma_{\text{eff}}[\phi^{\text{sp}}]-\Gamma_{\text{eff}}[\phi^{\text{EW}}]\right)}$$

The effective action $\Gamma_{\text{eff}}[\phi^{\text{sp}};T] = S[\phi_0^{\text{sp}}] + \Gamma_{1-\text{loop}}[\phi_0^{\text{sp}}]$ is computed in the 3d SU(2) Higgs model in temporal-axial gauge ($W_0^a = 0$) from:

Carson et al. PRD42 (1990) 2127

$$Z = \int \mathcal{D}H\mathcal{D}W e^{-S_3[H,W;T]} \qquad S_3 = \frac{1}{T}\int d^3x\left[|D_iH|^2 + \frac{1}{4}W^a_{ij}W^a_{ij} + V(H,T)\right]$$

giving: $$\Gamma_{\text{eff}}[\phi^{\text{sp}}] = S_3[\phi_0^{\text{sp}}] + \frac{1}{2}\ln\left(\frac{\det\mathcal{O}_{\text{bos}}(\phi_0^{\text{sp}})}{\det\mathcal{O}_{\text{bos}}(\phi^{\text{EW}})}\right) - \ln\left(\frac{\det\mathcal{O}_{\text{FP}}(\phi_0^{\text{sp}})}{\det\mathcal{O}_{\text{FP}}(\phi^{\text{EW}})}\right)$$

Evaluation of the fluctuation determinants gives:

Arnold & McLerran PRD36 (1987) 581

$$\Gamma_{\text{sp}} = \frac{|\omega_-|}{2\pi}[\mathcal{N}\mathcal{V}]_{\text{tr}}[\mathcal{N}\mathcal{V}]_{\text{rot}}\left[\frac{v(T)}{gT}\right]^3\kappa e^{-\frac{E_{\text{sp}}}{T}} \Longrightarrow \frac{\Gamma_{\text{sp}}}{V} = \frac{|\omega_-|}{2\pi}\mathcal{N}_{\text{tr}}(\mathcal{N}\mathcal{V})_{\text{rot}}[gv(T)]^3\left[\frac{v(T)}{gT}\right]^3\kappa e^{-E_{\text{sp}}/T}$$

$$\mathcal{V}_{\text{tr}} = V(gv(T))^3$$

Can be written in terms of the sphaleron energy using $E_{\text{sp}}(T) = \frac{4\pi v(T)}{g}B\left(\frac{\lambda}{g^2}\right) = \frac{2m_W(T)}{\alpha_W}B\left(\frac{\lambda}{g^2}\right)$

– $|\omega_-|$ contribution of the negative eigenvalue (unstable mode)
– $\mathcal{N}\mathcal{V}$ obtained in terms of profile functions $f(r)$ and $h(r)$ from integrating the spatial zero modes of the sphaleron using the method of collective coordinates

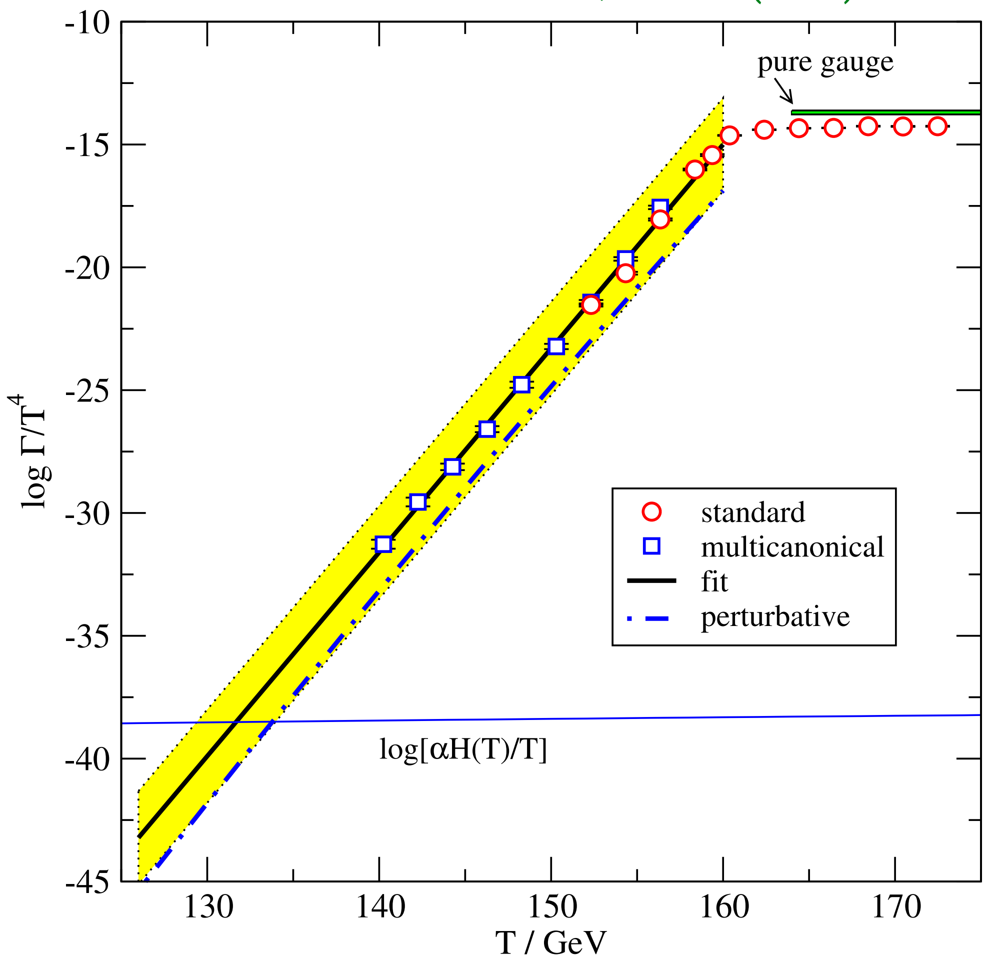
$\mathcal{V}_{\text{rot}} = 8\pi^2$ is the volume of the $SO(3)$ group

Measured on the lattice close to equilibrium is the Chern-Simons diffusion rate

$$\Gamma_{\text{diff}}(T) = \lim_{V,t\to\infty} \frac{\langle Q^2(t)\rangle_T}{Vt}, \qquad Q(t) = N_{\text{CS}}(t) - N_{\text{CS}}(0),$$

which is twice the sphaleron rate.

D'Onofrio et al., PRL113 (2014) 141602



After adjusting a constant, the perturbative result of Burnier et al,, J. Cosmol. Astropart. Phys. 02 (2006) 007, in which $E_{\text{sp}}(T)$ was evaluated with a 2-loop potential, agrees with the lattice result and can be use to extend the latter to values of T where there is no lattice data.

Implication of sphaleron transitions in presence of conserved charges

correction to the simple prediction: $B_f = -L_f = \frac{1}{2}(B-L)|_i$ f: final i: initial $B_i = 0$
obtained using that $B = \frac{B+L}{2} + \frac{B-L}{2}$ and that sphaleron process erases $B+L$ but preserves $B-L \Rightarrow (B-L)|_i = (B-L)|_f$

– rapid EW interactions in the early Universe between Higgs fields in $H = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix}$, $W^\pm$, quarks and leptons enforces equilibrium relations btw. chemical potentials

– processes only involve the left-handed fields

– the charge neutrality (and weak isospin I_3 for $T > T_c$ – not for $T < T_c$, as there $SU(2)_L$ is broken) of the Universe must be preserved

$$T \gtrsim T_c : \quad B = \frac{28}{79}(B-L) \qquad L = -\frac{51}{79}(B-L)$$

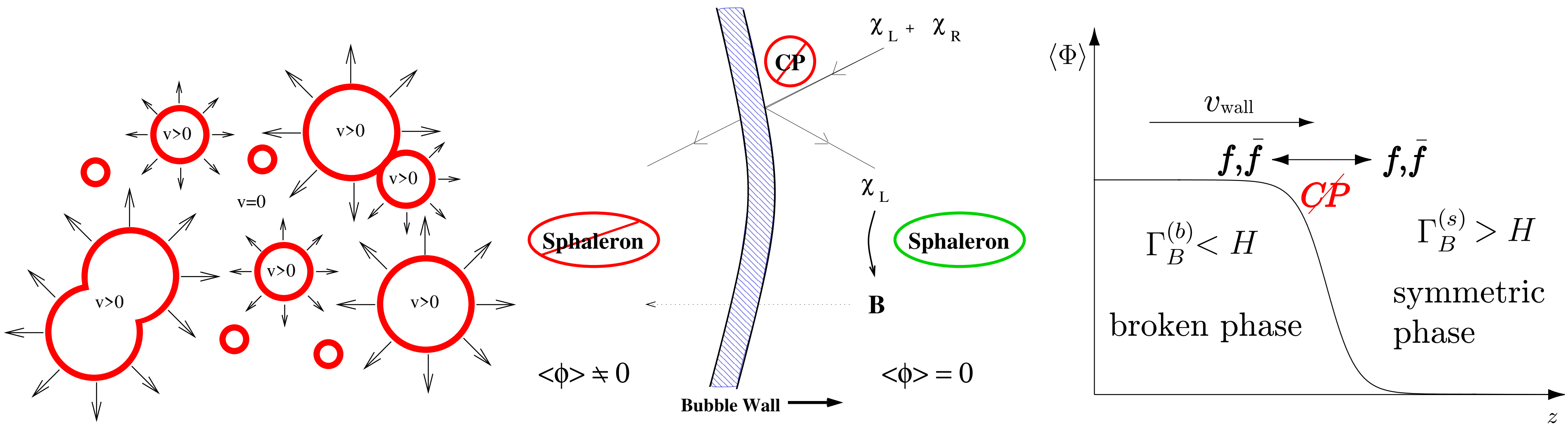
$$T < T_c : \quad B = \frac{12}{37}(B-L) \qquad L = -\frac{25}{37}(B-L)$$

Harvey & Turner, PRD42 (1990) 3344

$SU(3)_c$ **sphaleronlike** transitions can modify these relations Mohapatra & Zhang, PRD45 (1992) 2699
In QCD there is no fermion number violation, only chiral charge can be generated, due to chiral anomaly, that is net RH quark number can be converted in LH quark number, which can bias the baryon asymmetry generation by $SU(2)$ sphalerons (which only affects LH fields)

EWBG mechanism in SM

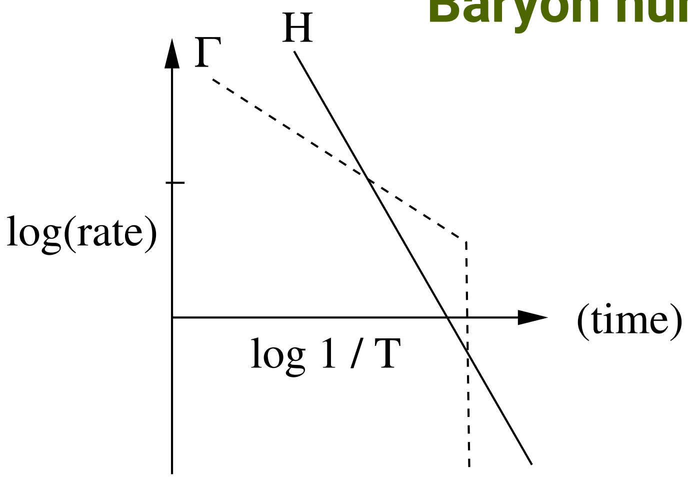
Sakharov's conditions satisfied: **1.** $\not{B}$ realized by the finite-T anomalous sphaleron processes, **2.** $\not{CP}$ comes from the Cabbibo-Kobayashi-Maskawa matrix, **3.** Out of equilibrium realized via $1^{\rm st}$ order PT with bubble nucleation and expansion in a supercooled Universe.
As the Universe expands and cools, at $T_N < T_{\text{c,EW}}$ bubbles of true vacuum (broken phase $v \equiv \langle\phi\rangle \neq 0$) are created in the environment of false vacuum (symmetric phase, $v = 0$).



Steps: 1) C and CP violating interaction with the bubble wall creates chiral asymmetry in particle densities in front of the wall (difference of transmission to the bubble btw. the part. & antipart.). **2)** Particles diffuse in the symmetric phase (bubble thickness, shape and velocity plays a role, scattering and diffusion described by quantum transport equations) **3)** Sym. phase EW sphaleron transitions process LH particles producing a net baryon charge. **4)** Rapidly expanding bubble engulf the created baryons (before equilibration, as inverse sphaleron processes reduce B), and broken phase sphaleron process rate abruptly drops in order to preserve the net baryon number.

$\mathcal{O}(70)$ alternative BG scenarios listed in a talk by M. Shaposhnikov at COSMO12

Baryon number preservation criterion (BNPC)



for $T \in (\sim 100, 10^{13})$ GeV $\Gamma_{\rm sp} > H$

we need abrupt decrease below T_c

For an approximate expression of the washout criterion: Patel & Ramsey-Musolf, JHEP07 (2011) 29

– integrate the depletion rate $\frac{dn_B}{dt} = -\frac{13N_G}{2}\frac{\Gamma_{\rm sp}}{VT^3}n_B$ from nucleation time $t=0$ where $T(t=0)=T_N \lesssim T_c$ to $\Delta t_{\rm EW}$ $\Longrightarrow \frac{n_B(\Delta t_{\rm EW})}{n_B(0)} = \exp\left[-\frac{13N_G}{2}\int_0^{\Delta t_{\rm EW}} dt \frac{\Gamma_{\rm sp}(T(t))}{VT^3(t)}\right]$

– set acceptable dilution factor: $\frac{n_B(\Delta t_{\rm EW})}{n_B(0)} > e^{-X} \quad (*)$

– 1) assume constant integrand over t, 2) use the expression of the sphaleron rate,
3) take the double logarithm of both sides of $(*)$

$\Longrightarrow$ bound on $\frac{v(T_c)}{T_c}$: $\frac{4\pi B}{g}\frac{v(T_C)}{T_C} - 6\ln\frac{v(T_C)}{T_C} > -\ln X - \ln\left(\frac{\Delta t_{\rm EW}}{t_H}\right) + \ln \mathcal{Z} + \ln\kappa$

result: $\boldsymbol{v(T_c)/T_c = \mathcal{O}(1)}$

t_H : Hubble time $\mathcal{Z} = \left(\frac{13n_f}{2}\right)\mathcal{N}_{\rm tr}(\mathcal{NV})_{\rm rot}\left(\frac{|\omega_-|t_H}{\pi}\right)$

Evaluation of $V_{\rm eff}$ – illustration of the ring resummation

Consider the partition function Z of the **Yukawa model** (scalar + fermion):

$$\mathcal{L}=\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi-V_{\rm cl}(\varphi)+\bar\psi\left(i\partial\!\!\!/-g\varphi\right)\psi,\quad V_{\rm cl}(\varphi)=\frac{m^2}{2}\varphi^2+\frac{\lambda}{4}\varphi^4,\qquad Z=\int\mathcal{D}\varphi\mathcal{D}\bar\psi\mathcal{D}\psi\, e^{iS}$$

To obtain the **ring resummation** in the effective potential: **1)** integrate out the fermions

$$\int\mathcal{D}\bar\psi\mathcal{D}\psi\,\exp\left\{\int_x\bar\psi[i\partial\!\!\!/-g\varphi(x)]\psi\right\}=\mathrm{Det}[i\mathcal{S}^{-1}(\varphi)],\qquad i\mathcal{S}^{-1}(x,y)=\left[i\partial\!\!\!/_x-g\varphi(x)\right]\delta(x-y)$$

$$\Longrightarrow Z=\int\mathcal{D}\varphi e^{iS_{\rm eff}[\varphi]}\quad\text{with}\quad S_{\rm eff}[\varphi]=\underbrace{\int_x\left[\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi-V_{\rm cl}(\varphi)\right]}_{S_{\rm b}[\varphi]}+\underbrace{-i\,\mathrm{Tr}\log[i\mathcal{S}^{-1}(\varphi)]}_{S_{\rm f}[\varphi]}$$

2) do the shift $\varphi(x)\to v+\varphi(x)$ and expand $S_{\rm eff}[\varphi]$ around the homogeneous background v

$$S_{\rm b}[\varphi(x)+v]=S_{\rm b}(v)+\int_x\varphi(x)\frac{\delta S_{\rm b}}{\delta\varphi}\Big|_{\varphi=v}+\frac{1}{2}\int_x\int_y\varphi(x)i\mathscr{D}^{-1}(x,y)\varphi(y)+\int_x\underbrace{\mathcal{L}_I(v,\varphi(x))}_{\lambda(\varphi^4+4v\varphi^3)/4}$$

$$S_{\rm f}[\varphi(x)+v]=-i\,\mathrm{Tr}\log[i\mathcal{S}_0^{-1}]+g\,\mathrm{tr}\int_x\mathcal{S}_0(x,x)\varphi(x)-\frac{1}{2}\int_x\int_y\varphi(x)\underbrace{ig^2\,\mathrm{tr}\left[\mathcal{S}_0(x,y)\mathcal{S}_0(y,x)\right]}_{\Pi(x,y)}\varphi(y)+\cdots$$

$$i\mathscr{D}^{-1}(x,y)\equiv\frac{\delta^2 S_{\rm m}[\varphi]}{\delta\varphi(x)\delta\varphi(y)}\Big|_{\varphi=v}=\left[-\partial_x^2-m^2(v)\right]\delta(x-y)\qquad m^2(v)=\frac{d^2V_{\rm cl}}{dv^2}=m^2+3\lambda v^2\qquad i\mathcal{S}_0^{-1}=i\partial\!\!\!/-gv$$

$\cdots$ represents infinitely many one-loop n-point functions, beyond the 3- and 4- point ones correcting the tree-level vertices in $\mathcal{L}_I(v,\varphi(x))$

3) Do the remaining functional integral in **Gaussian approximation**, i.e. keeping in the expansion of $S_{\text{eff}}[\varphi]$ only terms quadratic in $\varphi(x)$

$$\int \mathcal{D}\varphi \exp\left\{\frac{1}{2}\int_x\int_y \varphi \cdot iG^{-1} \cdot \varphi\right\} = \mathrm{Det}\left[iG^{-1}\right]^{-\frac{1}{2}} \qquad iG^{-1}(x,y) \equiv \left.\frac{\delta^2 S_{\text{eff}}[\varphi]}{\delta\varphi(x)\delta\varphi(y)}\right|_{\varphi=v} = i\mathscr{D}^{-1}(x,y) - \Pi(x,y)$$

The effective potential in the Gaussian approximation in momentum space: ($K^\mu = (k_0, \vec{k})$)

$$V_{\text{eff}}^{\mathsf{GA}}(v) = V_{\text{cl}}(v) - \frac{i}{2}\int_K \log\left(iG^{-1}(K)\right) + i\,\mathrm{tr}_{\mathsf{D}}\int_K \log\left(i\mathcal{S}_0^{-1}(K)\right)$$

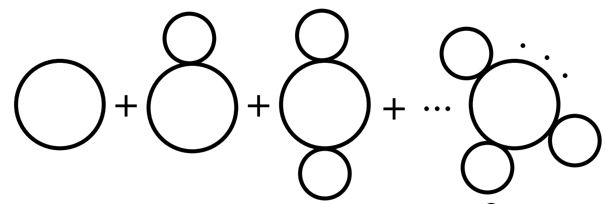
The second term on the r.h.s. represents the ring resummation:

$$\begin{aligned} \log\left(iG^{-1}\right) &= \log\left(i\mathscr{D}^{-1} - \Pi\right) = \log\left(i\mathscr{D}^{-1}\right) + \log\left(1 + i\mathscr{D}\Pi\right) \\ &= \log\left(i\mathscr{D}^{-1}\right) - \sum_{n=1}^{\infty}\frac{1}{n}\left(-\,i\mathscr{D}\Pi\right)^n \end{aligned}$$

Ring resummation is needed in a **scalar φ^4 theory** in order to deal with the **IR divergences** produced at high T (massless limit) by the static mode (zero Matsubara frequency).

The self-energy Π in the scalar poropagator G contains at least the scalar tadpole $\propto T^2$ needed because otherwise the tree-level curvature mass of the scalar is negative for small v

Several approximations are possible:

– localization $\Pi(K) \to \Pi(K=0) \quad \Longrightarrow \ln G^{-1} =$ 

– resummation of the zero mode only in a φ^4 model   curvature mass: $M^2 = m^2 + \Pi(K=0)$

$$\sum\!\!\!\!\!\!\!\int_K \ln\left(K^2+M^2\right) = \sum\!\!\!\!\!\!\!\int_K{}' \ln\left(K^2+ \underbrace{M^2}_{\to m^2}\right) + T\int \frac{d^3k}{(2\pi)^3}\ln\left(K^2+M^2\right) \qquad {}' : \text{zero mode left out}$$

$$\longrightarrow \sum\!\!\!\!\!\!\!\int_K \ln\left(K^2+m^2\right) + \underbrace{\mu^\epsilon T\int\frac{d^{d-\epsilon}k}{(2\pi)^{d-\epsilon}}\Big[\ln\left(K^2+M^2\right)-\ln\left(K^2+m^2\right)\Big]}_{-\frac{T}{6\pi}\left[(M^2)^{3/2}-(m^2)^{3/2}\right]}$$

– dimensional reduction: integrating out only the non-static scalar modes ($n \neq 0$ in $\omega_n = 2\pi n T$)
– high temperature expansion: reproduces the leading order result of the dimensional reduction

Improvement: include loop corrections of different topology than the daisy-type of the ring resummation

At finite temperature: $k_0 \to i\nu_n$, $\nu_n = (2n+1)\pi T$ for fermions & $k_0 \to i\omega_n$, $\omega_n = 2\pi n T$ for bosons

$$\int_K \to iT\sum_n \int \frac{\mathrm{d}^3 k}{(2\pi)^3}$$ summation over Matsubara frequencies

W/o ring resummation, that is $G \to \mathcal{D}$, the scalar and fermion contributions to the **1-loop effective potential** contains each a divergent vacuum $V^{(0)}$ & a finite thermal $V^{(1)}$ parts:

$$\mathbf{V}_{1-\text{loop}} = V^{(0)}_{1-\text{loop}} + V^{(1)}_{1-\text{loop}}$$

$$V^{(0)}_{1-\text{loop}} = g_s(-1)^{2s}\left[-\frac{i}{2}\int \frac{d^4K}{(2\pi)^4}\ln\left[-K^2+m^2(v)-i\varepsilon\right]\right] \qquad s:\text{spin}$$

$$= (-1)^{2s}\frac{g_s}{64\pi^2}\left[m^4(v)\left(\ln\frac{m^2(v)}{\Lambda^2}-\frac{1}{2}\right)+2\Lambda^2 m^2(v)\right] + \underset{\text{v}-\text{indep}}{\mathcal{O}(\Lambda^4)} \qquad \Lambda:\text{cut-off}$$

$$V^{(1)}_{1-\text{loop}} = g_s(-1)^{2s}T\int \frac{d^3k}{(2\pi)^3}\ln\left[1 \pm e^{-\sqrt{k^2+m^2(v)}/T}\right] \qquad \pm:\text{fermion/boson}$$

$g_s = 1$ for $s = 0$ (scalar), $g_s = 4$ for $s = \frac{1}{2}$ (fermion) and $g_s = 1$ for $s = 3$ (massive vector boson)

Calculation of $v(T_c)/T_c$ in the SM

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{GF+ghost}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{Yukawa}}$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu H)^\dagger D^\mu H + \mu^2 (H^\dagger H) - \lambda (H^\dagger H)^2, \qquad D_\mu = \partial_\mu + ig\frac{\sigma^a}{2}W^a + ig'\frac{1}{2}B_\mu$$

$$\mathcal{L}_{\text{GF}} = \frac{1}{2\xi}\left[\partial^\mu W^a_\mu - \frac{1}{2}\xi g v G^a\right]^2 - \frac{1}{2\xi}\left[\partial^\mu B_\mu - \frac{1}{2}\xi g' v G_1\right]^2 \qquad H = \frac{1}{\sqrt{2}}\begin{pmatrix} G_2 + iG_3 \\ v + h + iG_1 \end{pmatrix}$$

S-V cross terms from $\mathcal{L}_{\text{GF}}$ and $(D_\mu H)^\dagger D^\mu H$ combine to produce total divergences $\Rightarrow$ give 0

Approx. 1: Only top quark is included (heaviest fermion) ☚ largest contribution to the eff. pot.
In early '90s, before experimental discovery/confirmation in '95, top mass of ≈ 130 GeV was used in calculations. Precise value: m_t=(172.76 ± 0.3)GeV

Tree-level masses: $m_t^2(v) = \frac{y_t^2}{2}v^2$, $\quad m_h^2(v) = -\mu^2 + 3\lambda v^2$, $\quad m_{G_i}(v) = -\mu^2 + \lambda v^2$, i$=1, 2, 3$

$m_W^2(v) = \frac{g^2}{4}v^2$, $\quad m_Z^2(v) = (g^2 + g'^2)\frac{v^2}{4}$, $\quad m_\gamma^2(v) = 0$, $\quad m^2_{\text{ghost}} = 0$ for $\xi = 0$ (Landau gauge)

☛ photon and ghosts are massless $\Longrightarrow$ no v-dependent contribution to the eff. pot.

Approx. 2: Assuming $m_h < m_W$ contribution of Higgs sector (h, $G_{1,2,3}$) not included in the eff. pot.

$$V_{\text{eff}}(v) = V_{\text{cl}}(v) + V_{\text{CT}}(v) + V^{(0)}_{1-\text{loop}}(v) + V^{(1)}_{1-\text{loop}}(v), \quad V_{\text{cl}}(v) = -\frac{\mu^2}{2}v^2 + \frac{\lambda}{4}v^4$$

Renormalization done using the conditions $\frac{d^n}{dv^n}\left[V_{\text{CT}}(v) + V^{(0)}_{1-\text{loop}}(v)\right]$=$0$, n=$1, 2$, which preserve the $T=0$ tree-level VEV and Higgs mass values, v_0 and $m_h(v_0)$

$$V^{(0)}_{1-\text{loop, R}}(v) = \frac{1}{64\pi^2} \sum_{i=\text{t,W,Z}} n_i (-1)^{2s_i} \left[m_i^4(v) \left(\ln \frac{m_i^2(v)}{m_i^2(v_0)} - \frac{3}{2} \right) + 2 m_i^2(v) m_i^2(v_0) \right]$$

$$V^{(1)}_{1-\text{loop}}(v) = \frac{T^4}{2\pi^2} \left[\sum_{i=\text{Z,W}} n_i J_b\left(\frac{m_i^2(v)}{T^2} \right) - n_t J_f\left(\frac{m_t^2(v)}{T^2} \right) \right]$$

dof: n_Z=3(polarization), n_W=2($W^{\pm}$)×3=6, n_h=1, n_t=2(spin)×2(part. + $\overline{\text{part}}$)×3(color)=12

Approx. 3: high-T expansion (HTE) in the thermal part of $V_{\text{eff}}(v)$ good even for the top

$$J_b(a^2) = \int_0^\infty dx x^2 \ln\left[1 - e^{\sqrt{x^2+a^2}}\right] = -\frac{1}{3} \int_0^\infty dx \frac{x^4}{\sqrt{x^2+a^2}} \frac{1}{e^{\sqrt{x^2+a^2}} - 1}$$

$$= -\frac{\pi^4}{45} + \frac{\pi^2}{12} a^2 - \frac{\pi}{6} (a^2)^{3/2} - \frac{a^4}{32} \ln \frac{a^2}{c_b} + \mathcal{O}(a^6)$$

$$J_f(a^2) = \int_0^\infty dx x^2 \ln\left[1 + e^{\sqrt{x^2+a^2}}\right] = \frac{1}{3} \int_0^\infty dx \frac{x^4}{\sqrt{x^2+a^2}} \frac{1}{e^{\sqrt{x^2+a^2}} + 1}$$

$$= \frac{7\pi^4}{360} - \frac{\pi^2}{24} a^2 - \frac{1}{32} a^4 \ln \frac{a^2}{c_f} + \mathcal{O}(a^6)$$

$$c_b = 16 c_f = 16\pi^2 e^{-2\gamma_E + 3/2}$$

Using $m_i^2(v) = \boldsymbol{m_i^2(v_0)} v^2/v_0^2 \implies v$ and T dep. separate and **vacuum mass** appears $\implies$

Parametrization: $V^{\text{HTE}}_{\text{eff, 1-loop}}(v,T) = D(T^2 - T_0^2)v^2 - ETv^3 + \lambda(T) v^4/4 \equiv V_T(v)$

$$D = \frac{2m_W^2 + m_Z^2 + 2m_t^2}{8v_0^2}, \quad E = \frac{2m_W^3 + m_Z^3}{4\pi v_0^3}, \quad T_0^2 = \frac{m_h^2 - 8Bv_0^2}{4D}, \quad B = \frac{3(2m_W^4 + m_Z^4 - 4m_t^4)}{64\pi^2 v_0^4}$$

$$\lambda(T) = \lambda - \frac{3}{16\pi^2 v_0^4} \left(2m_W^4 \ln \frac{m_W^2}{C_b T^2} + m_Z^4 \ln \frac{m_Z^2}{C_b T^2} - 4m_t^4 \ln \frac{m_t^2}{C_f T^2} \right)$$

$$\boldsymbol{m_i^2 \equiv m_i^2(v_0)}$$

see, e.g., Quiros, arXiv:hep-ph/9901312

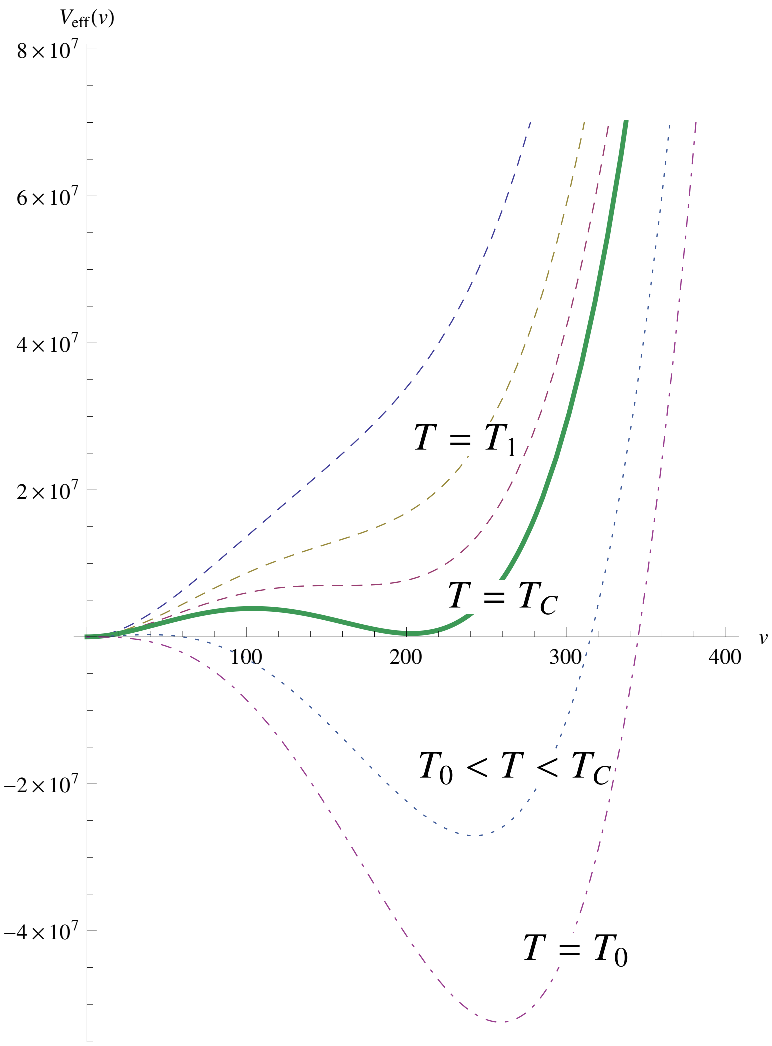
$$C_{b/f} = c_{b/f} e^{-3/2}$$

Behavior of $V_T(v) = D(T^2 - T_0^2)v^2 - ETv^3 + \lambda(T)v^4/4$

For $E = 0$ PT is 2$^{\text{nd}}$ order with $T_c = T_0$ and OP $v(T) = T_0\sqrt{\frac{2D}{\lambda(T)}\left[1 - \frac{T^2}{T_0^2}\right]}$ for $T < T_0$

For $E \neq 0$ PT is 1$^{\text{nd}}$ order and T_0 is the lower spinodal temperature: $V''_{T_0}(0) = 0$

plot from arXiv:1511.00579v3



–as T decreases from high values, a nontrivial minimum appears at the spinodal temperature T_1:

$$V'_{T_1}(v_1) = V''_{T_1}(v_1) = 0 \Longrightarrow \begin{cases} T_1^2 = T_0^2\left[1 - \dfrac{9E^2}{8D\lambda(T_1)}\right]^{-1} \\ \dfrac{v_1}{T_1} = \dfrac{3E}{2\lambda(T_1)} \end{cases}$$

– minima become degenerate at T_c:

$$\left.\begin{array}{l} \overbrace{\underbrace{0 = V_{T_c}(0)}_{\text{trivial}} = V_{T_c}(v_c)} \\ \overbrace{0 = V'_{T_c}(0)} = V'_{T_c}(v_c) \end{array}\right\} \Longrightarrow \begin{cases} T_c^2 = T_0^2\left[1 - \dfrac{E^2}{\lambda(T_c)D}\right]^{-1} \\ \dfrac{v_c}{T_c} = \dfrac{2E}{\lambda(T_c)} \end{cases}$$

– $v = 0$: – is metastable minimum for $T < T_c$
– survives as a minimum down to $T = T_0$ where it turns into a maximum

Result and some remarks on the calculation of $V_{\rm eff}$

Using v_0=246.2 GeV, m_h=125 GeV, m_W=80.4 GeV, m_Z=91.2 GeV and m_t=172.9 GeV gives

$$B \simeq -4.4\,10^{-3},\ D \simeq 0.17,\ E \simeq 9.6\,10^{-3},\ \lambda(T_c) \approx \frac{m_h^2}{2v_0^2} \approx 0.13$$

E. Senaha, Symmetry 12 (2020) 733

$\Longrightarrow T_c \simeq 163.4$GeV and $v_c \simeq 24.3$GeV $\Longrightarrow \frac{v_c}{T_c} \simeq 0.15 \ll \mathcal{O}(1)$ needed by BNPC

$\Longrightarrow$ BNPC satisfied only for $m_h \lesssim 48$ GeV

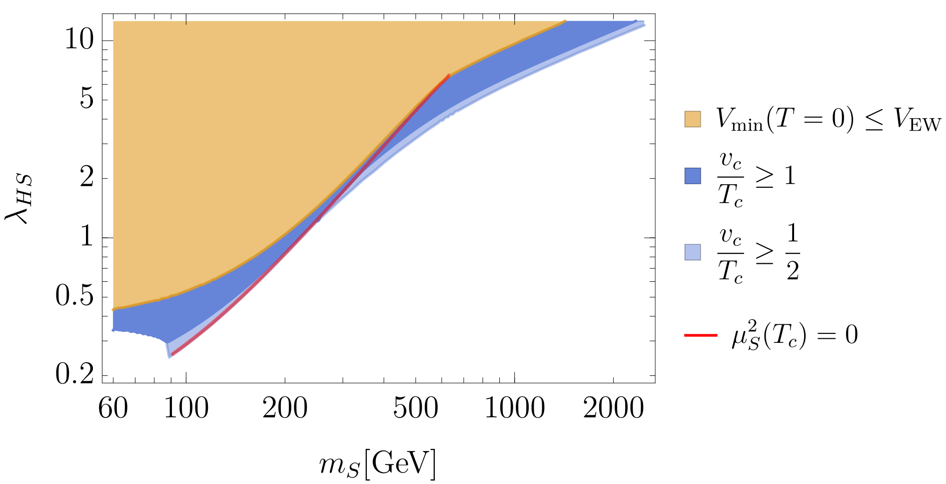
– ring resummation decreases v_c/T_c because the Debye mass reduces E

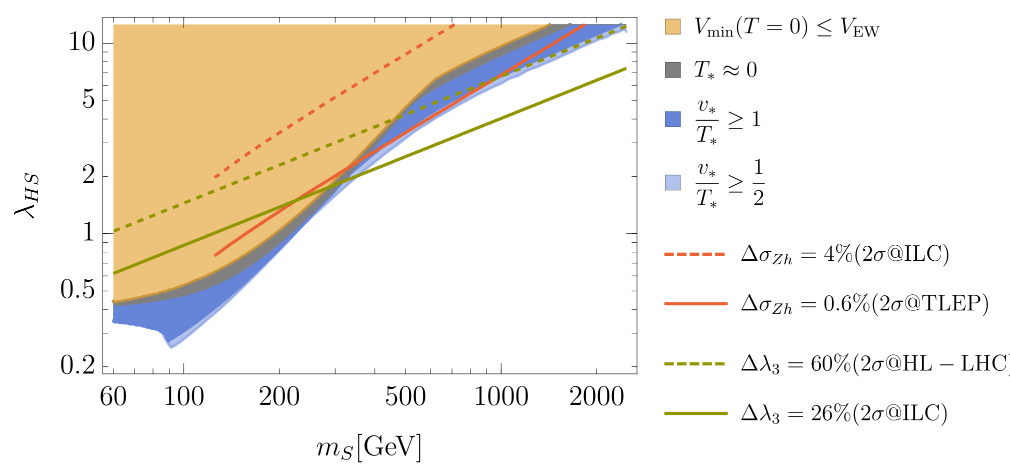
M. Carrington, PRD45 (1992) 2933 M. Dine et al., PRD46 (1992) 550

– need to increase E end/or decrease $\lambda(T_c)$ $\longleftarrow$ possible with extra scalars, as suggested in

Anderson & Hall, PRD45 (1991) 2685

Scalar singlet S coupled to SM Higgs: $V_{\rm cl}(H,S) = -\mu^2|H|^2 + \lambda|H|^4 + \lambda_{HS}|H|^2S^2 + \frac{\mu_S^2}{2}S^2 + \frac{\lambda_S}{4}S^4$





yellow & grey: excluded $T_\star$ nucleation temp. Beniwal et al, JHEP08 (2017) 108

Problems:

1. Gauge dependence: with this approach $\frac{v_c}{T_c}$ is not physical, as is gauge-parameter dependent

$$\frac{v_c}{T_c} = \frac{2E}{\lambda(T_c)} = \frac{3 - \xi^{3/2}}{48\pi\lambda}\left[2g^3 + \left(g^2 + g'^2\right)^{3/2}\right] + \ldots \qquad \Longrightarrow \text{gauge-dependent BNPC}$$

Gauge-independent T_c in the ring resummation scheme was obtained using Nielsen identities

Patel & Ramsey-Musolf, JHEP07 (2011) 29

Gauge-dependence at 2-loop order is not known, it was investigated only in the DR theory:
– minimum of gauge dependent 2-loop eff. pot. is gauge independent M. Laine, PRD51 (1995) 4525
– **mass and vertex resummation** in an optimized 1-loop calculation give gauge-independent self-energies Buchmüller & Philipsen, NPB443 (1995) 47

2. Reliability of the perturbative expansion

Based on 2-loop calculations, common consensus on perturbation theory seems to be:
– perturbative evaluation of the effective potential is problematic in the SM due the nonperturbative magnetic mass scale and for $m_h \gtrsim 70$GeV (also in SM extensions for large scalar coupling)
– the order of the phase transition cannot be reliable studied, although greater accuracy can be achieved in the DR theory Fodor & Hebecker, NPB432 (1994) 127, Buchmüller et al., Annals Phys. 234 (1994) 260, K. Farakos et al., Nucl.Phys. B425 (1994) 67, Buchmüller & Philipsen, NPB443 (1995) 47, Kainulainen et al., JHEP06 (2019) 075
– in fluctuation-driven phase transitions perturbation theory can at most serve as a guidance in exploring the parameter space, and in the context of EWBG, in pointing to those regions were baryon number preservation is more likely.