Particle physics model of inflation

ELFT Winter School

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- First half of second lecture: Inflation from renormalization group running
- Second half of second lecture : Higgs portal inflation

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Standard Cosmology

Cosmological inflation is motivated by the shortcomings of the standard model of cosmology. Let us first discuss the latter.

- We model the time evolution of the universe as the solution of the Einstein equations. The solution we are interested in corresponds to a homogeneous and isotropic universe. This is called the **Cosmological Principle**. This is true for scales larger than 250 million light years.

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$$ds^2 = -dt^2 + a(t)^2 dr^2. \tag{1}$$

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- The Einstein equations give two differential equations, called **Friedmann equations** governing the behavior of the scale parameter $a(t)$.

Important definitions (I)

The **Friedmann equations** are

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = \frac{\rho}{3M_{\rm Pl}} - \frac{k}{a^2} \quad \text{and} \quad \frac{\ddot{a}}{a} \equiv \dot{H} + H^2 = -\frac{1}{6M_{\rm Pl}}(\rho + 3p), \qquad (2)$$

where H is called the Hubble parameter and the density ρ and pressure p are obtained from the stress-energy tensor. The parameter k corresponds to the spatial curvature.

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The ratio $w = p/\rho$ is called the **equation of state**. If the universe is filled with a medium comprised of multiple components, then the scale factor is going to behave differently at different stages during the evolution of the universe.

Universe is	w	$a(t)$	ρ
matter dominated	0	$t^{2/3}$	a^{-3}
radiation dominated	1/3	$t^{1/2}$	a^{-4}
dark energy/ cosmological constant (Λ) dominated	-1	$e^{\Lambda t}$	Λ

Table: Summary of the equation of state parameter w, the scale factor a and density ρ in a flat ($k = 0$) FRW universe in **different scenarios.**

Important definitions (II)

It is going to be convenient to introduce a new time variable, the **conformal time** τ. It simplifies the metric so that it looks like a Minkowski one:

$$ds^2 = a^2(-d\tau^2 + dr^2), \quad d\tau = dt/a. \tag{3}$$

▶

Universe is dominated by	matter	radiation	dark energy
$a(\tau)$	τ^2	τ	$-1/\tau$

Table: The scale factor a as a function of the conformal time τ.

Conformal diagram

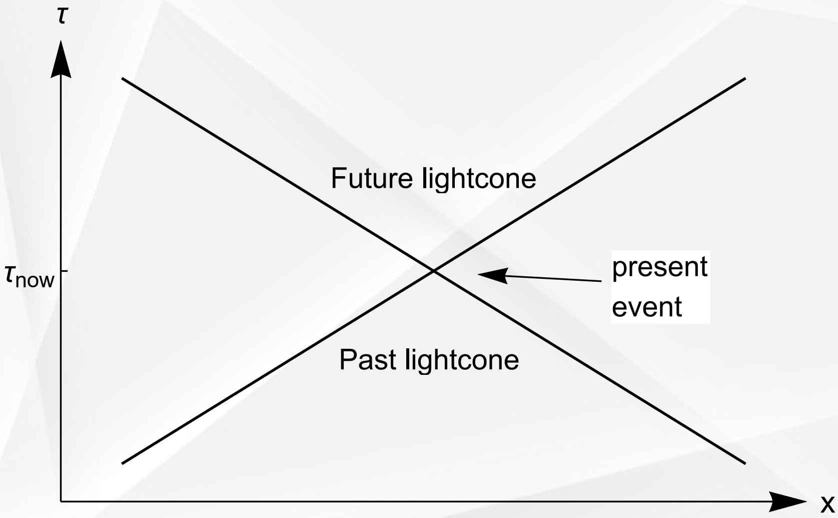

Figure: Conformal diagram in the FLRW metric. It is called conformal because the time axis *t* is replaced by the conformal time τ.

The (comoving) Hubble horizon

The comoving **Hubble horizon** (or the radius of the Hubble sphere) is $(aH)^{-1}$. If we know the speed of an object (*now*) and the Hubble parameter (*now*), then we can calculate its distance *now* from **Hubble's law**.

$$v = Hr \quad \Rightarrow \quad r = \frac{v}{H}. \tag{4}$$

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Obviously, the speed of light is constant, so the maximum distance from where we can **detect light** signals *now* is

$$\frac{c}{H}. \tag{5}$$

Most of the time we work in **comoving coordinates**, and thus define the comoving Hubble radius by factoring out the effect of expansion:

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Universe is dominated by	matter	radiation	dark energy
The comoving Hubble sphere	grows	grows	shrinks

Table: The behavior of the comoving Hubble sphere for different w.

The fine-tuning problem

The first Friedmann equation can be rewritten in the following form

$$H^2 = \frac{\rho}{3M_{Pl}^2} - \frac{k}{a^2} \Longrightarrow 1 - \Omega = -\frac{k}{(aH)^2} \quad (7)$$

Remember: $k = 0$ corresponds to a flat, $k = +1$ to a closed 3-sphere and $k = +1$ to an open 3-hyperboloid shape for the universe. The parameter Ω is called the **density parameter** $\Omega = \rho/(3H^2M_{\mathrm{Pl}}^2)$ and its value is **measured** today

$$\Omega = 1.00 \pm 0.02 \quad (8)$$

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This implies that the **universe is flat** $k = 0$. We know that $(aH)^{-1}$ grows during the regular evolution of the universe and thus $k = 0$ is a **repulsive fixed** point.

The horizon problem

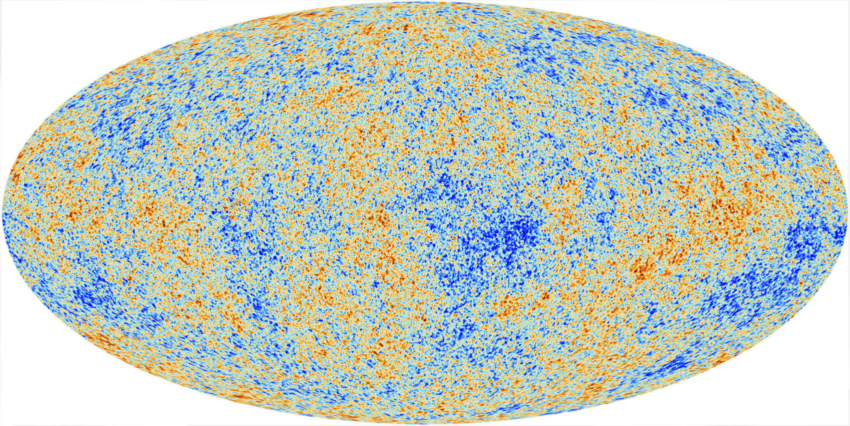

Figure: The CMB background from the Planck experiment. The red spots are the hottest, the blue ones are the coldest sports in the CMB, yet the relative difference between them is 10^{-5}. It is almost perfectly homogeneous!

The horizon problem

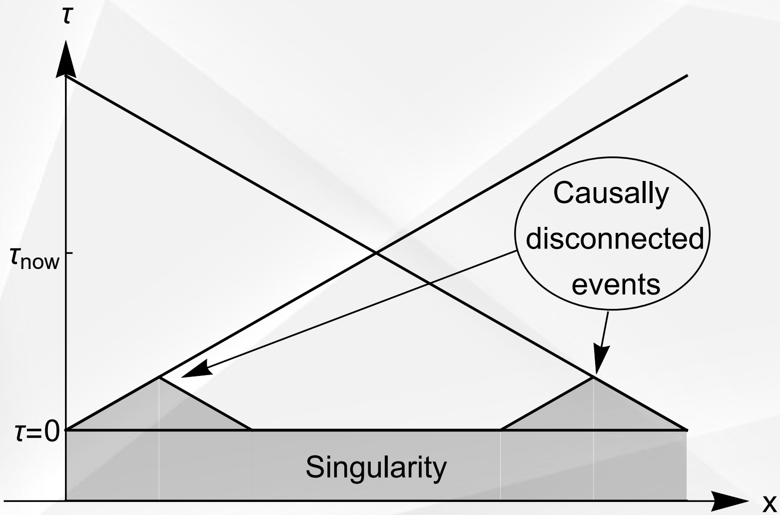

Figure: If you look up at the sky, you will see several causally disconnected patches of the universe! Again: are the causally separated regions fine tuned to be in thermal equilibrium?

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Defining inflation

- Cosmological inflation is an **era** during the evolution of the universe.

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- During this period of time the universe is dominated by something, which mimics dark energy, i.e. the **universe expands exponentially and the comoving Hubble horizon shrinks**.

- The cosmological inflation takes place after the Big Bang and **before the radiation dominated era** of the universe.

The horizon problem revisited

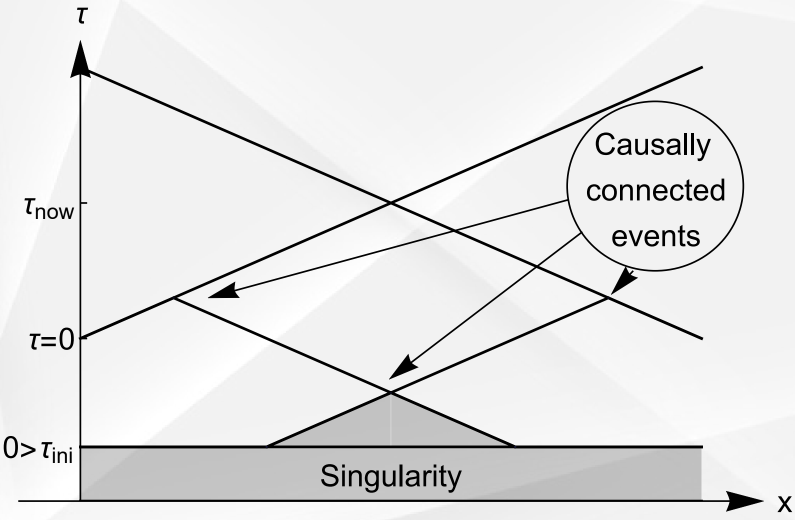

Figure: The inflation provides extra conformal time, during which causal contact is established!

Anisotropy problem resolved

Let us address now the **anisotropy** observed in the CMB. Actually, this is the most robust solution of cosmic inflation. We say, that the **fluctuations** produced during inflation will act **as seeds** for the structure formation during the regular evolution of the universe. But how?

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You only need three things to keep in mind: the **comoving Hubble horizon shrinks during inflation**, and expands during the regular evolution of the universe. Lastly, the comoving wavelength of a fluctuation is constant. (This last one comes from the literal definition of the comoving distance.)

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The **fluctuation** eventually **grows to superhorizon scale** during inflation and won't affect the universe in any way. As the universe expands after inflation these fluctuations eventually fall into the horizon and act as inhomogeneities.

Anisotropy problem resolved

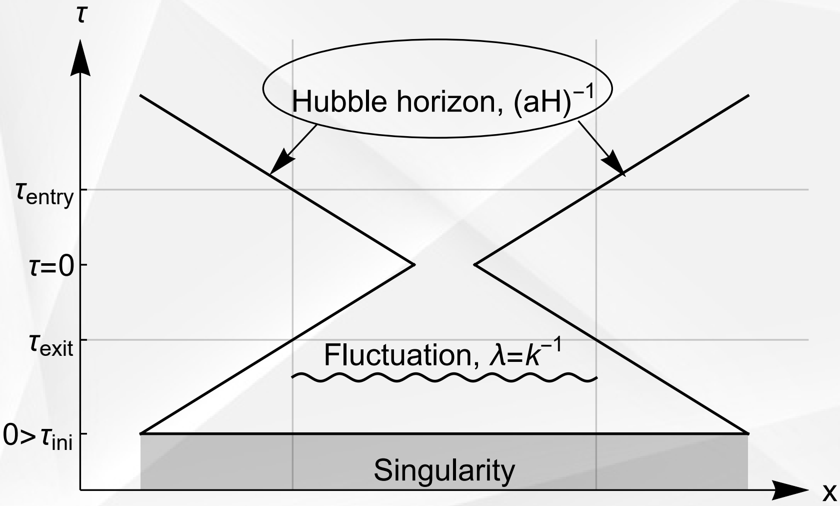

Figure: See the previous slide!

What can be the dynamical source of inflation?

Our first thought may be that it is Einstein's cosmological constant Λ. However, that **cannot be** the case! Matter and radiation are diluted exponentially quickly and $w = -1$ forever, since Λ is also independent of time. There is no exit from this. A positive (negative) **cosmological constant** dominated universe is also called (anti) de Sitter universe.

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The **simplest way** to have a finite inflationary epoch is by assuming that a **scalar field** was coupled to gravity such that

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} R + \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) \right) \tag{9}$$

It is just a singlet scalar field with canonical kinetic term.

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There are a **lot of other options**. We are going to cover two in the second half of the lectures.

Relevant equations and quantities

In order to analyze the dynamics we need several pieces of information. A great simplification comes from the **homogeneity** requirement of the Einstein equations $\phi(\vec{r}, t) = \phi(t)$. The **equation of motion** for ϕ takes the form

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \tag{10}$$

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We can read off **from the stress-energy tensor** the density ρ and pressure p:

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Then, the equation of state is

$$w = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \tag{12}$$

In order to get inflation we need $w \approx -1$, i.e $\dot{\phi}^2 \ll V(\phi)$! The **potential energy has to dominate over the kinetic energy**, hence we call it **slow-roll** inflation.

Stopping inflation

The Friedmann equations take the forms

$$H^2 = \frac{1}{3M_{Pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right), \quad (13)$$

$$H^2 + \dot{H} = H^2(1-\varepsilon) = -\frac{1}{3M_{Pl}^2}(\dot{\phi}^2 - V(\phi)) \quad (14)$$

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This actually gives us the clue that **inflation stops when** $\varepsilon = 1$!! Using the second Friedmann equation we can express

$$\varepsilon = \frac{1}{2}M_{Pl}^{-2}\frac{\dot{\phi}^2}{H^2}. \tag{16}$$

This agrees perfectly with our requirement to have $w \approx -1$: If the kinetic energy of the field grows too large, inflation eventually stops by itself.

Sufficient amount of inflation

We want inflation to resolve the **Horizon problem**, which sets a lower boundary on the **amount of expansion** during the inflationary epoch. This is measured in $N = \ln(a_{end}/a_{start})$, which is called the e-folding number. Our current size of Hubble horizon imply

$$N \geq 50 \tag{17}$$

Quantities observed today correspond to $60 > N > 50$, depending on the specifics of the inflationary model.

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In order to **secure a long enough inflationary period** we prescribe

$$|\dot{\varepsilon}| < 1 \tag{18}$$

and obtain

$$\frac{\dot{\varepsilon}}{2\varepsilon H} - \varepsilon = \frac{\ddot{\phi}}{\dot{\phi} H} \equiv -\eta_H \tag{19}$$

The quantity η_H is our second slow-roll parameter. Note: we have not used any approximations so far. The subscript H means it is expressed via the Hubble parameter, and the negative sign is a convention.

Visualizing inflation

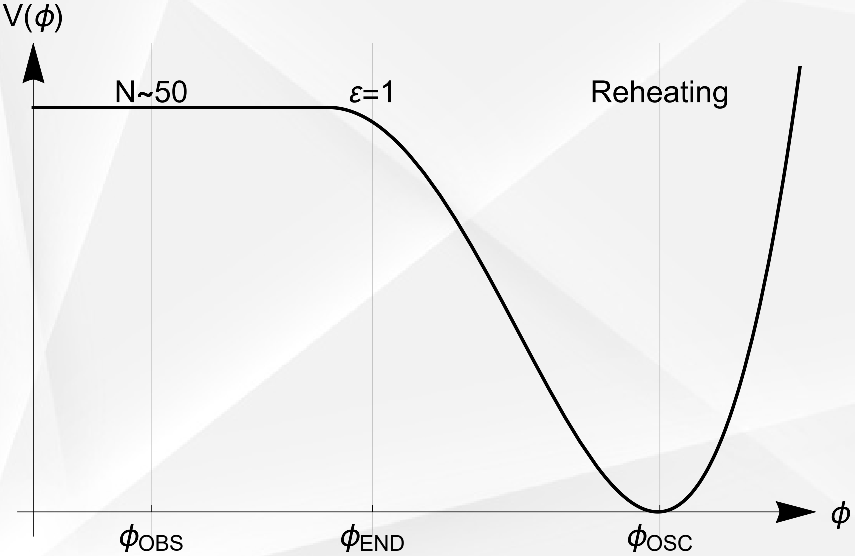

Figure: The inflationary potential $V(\phi)$. We define $\Lambda = \rho^{1/4}$ as the energy scale of inflation.

Relation of inflationary predictions to observables

We can make observations of primordial **density fluctuations**. These can be sorted to scalar fluctuations (directly related to fluctuations in the curvature $\mathcal{R}$) and primordial gravitational waves. We call the latter **tensor fluctuations**.

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The variance of the curvature fluctuations is usually parametrized as

$$\Delta^2_{\mathcal{R}} \simeq A_s \left(\frac{k}{k_*}\right)^{n_s-1}, \tag{20}$$

where k_* is a pivot scale dependent on the parameters of the measurement. The Planck satellite measures

$$A_s = 2.2 \times 10^{-9}, \quad \text{and} \quad n_s = 0.964 \pm 0.005 \tag{21}$$

with the pivot scale $k_* = 0.05\,\mathrm{Mpc}^{-1}$.

Relation of inflationary predictions to observables

The variance of the tensor fluctuations is parametrized in the same way as that of the density fluctuations

$$\Delta_t^2 = A_t \left(\frac{k}{k_*}\right)^{n_t} \tag{22}$$

Tensor fluctuations are not observed yet. We are only able to measure an upper bound on the ratio of the amplitude of tensor and scalar fluctuations

$$r = \frac{A_t}{A_s} < 0.14 \tag{23}$$

Relation of inflationary predictions to observables

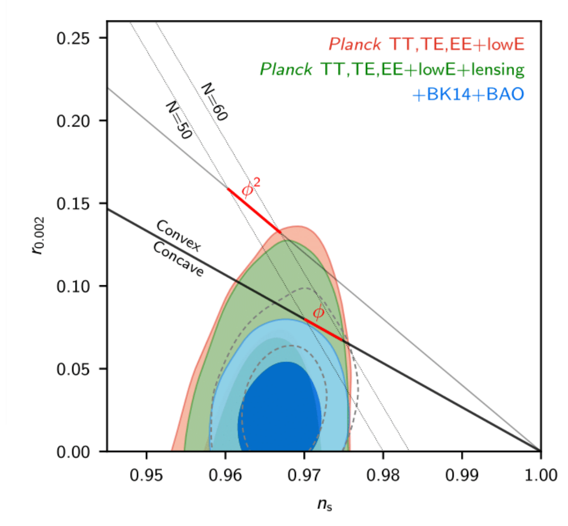

Figure: Observational bounds from the Planck satellite. This is the most recent data, the experiment ended in late 2013. The subscript refers to $k_* = 0.002\ \mathrm{Mpc}^{-1}$, a different pivot scale used for tensor modes.

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We have no time to **derive** the inflationary **predictions** for $\Delta^2_{\mathcal{R}}$ and Δ^2_t. A very good read detailing the procedure is for instance arXiv:0907.5424.

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The **density perturbations** are proportional to the perturbations in the scalar field. This leads to the formula

$$\Delta^2_{\mathcal{R}} \simeq \frac{1}{2M^2_{\text{Pl}}} \frac{1}{\varepsilon} \left(\frac{H}{2\pi} \right)^2_{k=aH}, \tag{24}$$

where $k = aH$ means that for a fluctuation with wavelength k^{-1}, it is evaluated at the point when it is the size of the Hubble horizon. Note that this quantity is **model dependent** due to ε! (It requires a direct assumption for the velocity of the scalar field.)

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The above formula implies the relations

$$A_s = \frac{1}{24\pi^2}\frac{1}{\varepsilon}\frac{V(\phi)}{M^4_{\mathrm{Pl}}}\bigg|_{\phi=\phi_{OBS}} \tag{25}$$

(using $H^2 \simeq V/(3M^2_{\mathrm{Pl}})$ during slow-roll) and

$$n_s - 1 = \frac{d\ln\Delta^2_{\mathcal{R}}}{d\ln k}\bigg|_{k=aH} \simeq 2\eta_H - 4\varepsilon. \tag{26}$$

Relation of inflationary predictions to observables

Inflation predicts that the variance of the **tensor fluctuations** are

$$\Delta_t^2 = \frac{2}{\pi^2} \frac{H^2}{M_{\rm Pl}^2}\bigg|_{k=aH}. \tag{27}$$

Note that this formula only depends on *H*. It does **not depend on the specifics** of the inflationary model.

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They leave a unique imprint on the **polarization pattern** of the CMB: the **B-mode** polarization pattern. In 2014 the BICEP2 experiment reported such an observation. Unfortunately later in 2015, the Planck experiment has shown that galactic foregrounds (dust) are responsible for the detected B-modes.

The computations simplify significantly in the regime $\dot{\phi}^2 \ll V(\phi)$, this is called the **slow-roll approximation**

$$H^2 \simeq \frac{1}{3M_{\rm Pl}^2} V(\phi), \tag{29}$$

$$\varepsilon \simeq \epsilon = \frac{1}{2} M_{\rm Pl}^2 \left(\frac{V'(\phi)}{V(\phi)} \right)^2, \tag{30}$$

$$\eta_H \simeq \eta - \epsilon \qquad \text{with} \quad \eta = M_{\rm Pl}^2 \frac{V''(\phi)}{V(\phi)}, \tag{31}$$

$$N = \int H dt \simeq \frac{1}{M_{\rm Pl}} \int_{\phi_{END}}^{\phi_{OBS}} \frac{d\phi}{\sqrt{2\epsilon}} \tag{32}$$

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- Find the value ϕ_{END} for which inflation stops ($\epsilon = 1$).
- Using $N = 60$, find ϕ_{OBS} corresponding the observed fluctuations.
- We can now simply compute n_s and r by substitution.

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$$V(\phi) = \frac{1}{2} m^2 \phi^2. \tag{33}$$

- Let us compute the observables n_s, r and constrain m^2!
- As a first step, compute ϵ and η.
- Find the value ϕ_{END} for which inflation stops ($\epsilon = 1$).
- Using $N = 60$, find ϕ_{OBS} corresponding the observed fluctuations.
- We can now simply compute n_s and r by substitution.
- Use the measured value A_s to constrain the parameters in the potential.

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- We can **use zero temperature QFT** at the times when the temperature was low.

Renormalization scale as cosmic timescale?

- **At finite temperature**, the thermally corrected effective potential carries the correct information, which can be used for instance to identify phase transitions:

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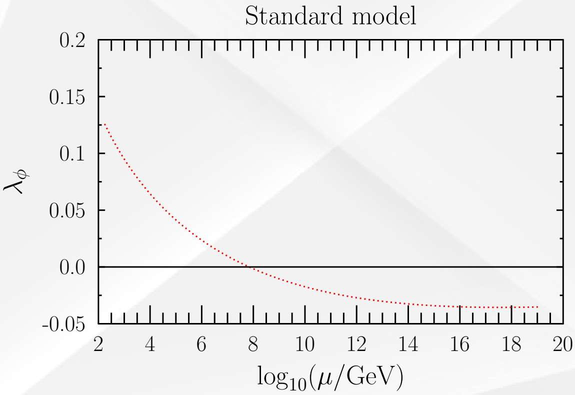
- Fortunately, we do not have to deal with the thermal effects, as we are interested in a **very cold era** of the universe. We use the straightforward choice and during the inflation we set μ to be the energy scale $\rho^{1/4}$ of inflation.

Problem with the SM

The **coupling strength** λ_ϕ of the scalar quartic vertex of the standard model becomes **negative** (at high energies) during its renormalization group flow :



The corresponding one-loop beta function is

$$\beta^{(1)}_{\lambda_\phi} = \frac{1}{(4\pi)^2}\left(24\lambda_\phi^2 - 6c_t^4 + \frac{3}{8}g_Y^4 + \frac{9}{8}g_L^4 + \frac{3}{4}g_Y^2 g_L^2 + \lambda_\phi\left(12c_t^2 - 3g_Y^2 - 9g_L^2\right)\right)$$

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- Even the most precise computations[1] support this. Higher loop calculations push the scale μ_0 higher about (10^{11} GeV), for which $\lambda_\phi(\mu_0) = 0$.

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- This calls for an **extension of the SM**. We elaborate here an inflationary model[2] based on the particular extension[3].

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- The particle content of the model coincides with that in the standard model of particle interactions, supplemented with one **complex scalar field**. We also allow for one (or more) Dirac-, or Majorana-type right-handed neutrinos.

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- The extended **scalar potential** has the form:

$$V(\phi,\chi) = V_0 - \mu_\phi^2|\phi|^2 - \mu_\chi^2|\chi|^2 + \frac{1}{2}(|\phi|^2, |\chi|^2)C\begin{pmatrix}|\phi|^2\\|\chi|^2\end{pmatrix},$$

with C being the quartic coupling matrix $C = \begin{pmatrix}2\lambda_\phi & \lambda\\ \lambda & 2\lambda_\chi\end{pmatrix}$.

Vacuum stability

- The **criteria** for the potential to be perturbative and stable are

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4\pi > \lambda_\phi &> 0, \\
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- After spontaneous symmetry breaking the model has two vacuum expenctation values (v and w) and two scalar particles with physical masses $M_{h/H}$. In order **to be consistent**, we have to set:

$$v(M_Z) = 246 \text{ GeV} \quad \text{and} \quad M_h(M_Z) = 125 \text{ GeV}.$$

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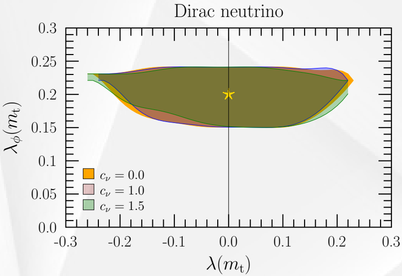
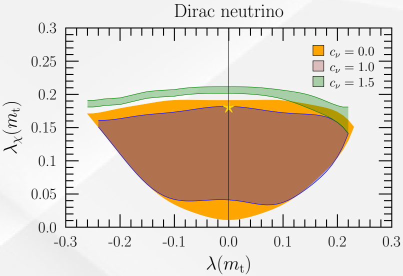
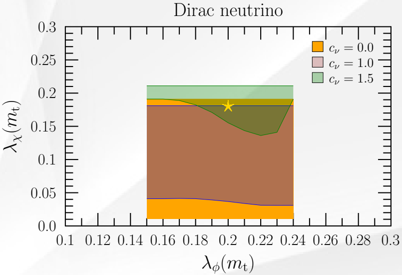
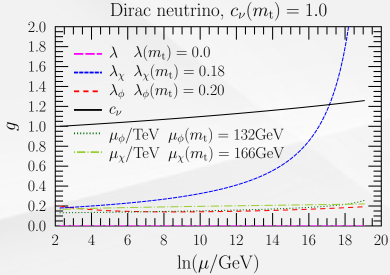
- Finally, we **constrain the parameter space** spanned by the unknown couplings through the stability and consistency conditions.

Vacuum stability









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- The most important difference is that we now have a **multiple field inflationary model**. Actually, the multiple-field models are favored when one attempts to make a connection with particle physics, at least because of the vacuum stability requirement.

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- The most important difference is that we now have a **multiple field inflationary model**. Actually, the multiple-field models are favored when one attempts to make a connection with particle physics, at least because of the vacuum stability requirement.

- We want to explore the **parameter region**, for which the inflationary potential produces predictions consistent with **observations**.

Multiple field inflation

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- In practice, we define an 'adiabatic field' σ and an 'entropy field' s (we have $\dot{s} = 0$) and **rotate the original fields** (ϕ, χ) into (σ, s).

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- The slow-roll parameters and the formula for n_s are also different for multiple fields.

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- The multiple field formula for n_s is very long and its derivation takes a complete paper[4].

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Constraining the inflationary model

- As a first guide, we look for an inflationary potential with **parameters consistent** (at least in the orders of magnitude) **with the RG running**. For instance $\lambda_i, |\lambda| \sim O(10^{-1} - 10^{-2})$ and $\mu_i^2/\text{GeV} \sim O(10^4)$ with $i = \phi, \chi$.

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- We can only get **predictions** consistent with the observations **when the true minimum of the potential is far** $\sim M_{\text{Pl}}$ from the origin and consider **the curvaton scenario**.

Slow-roll trajectory

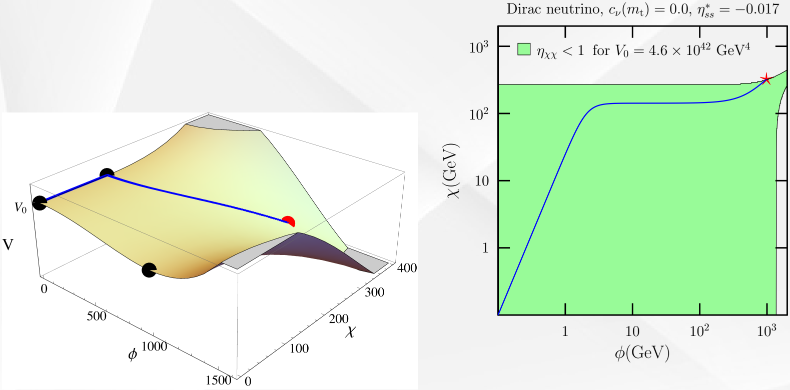

Figure: Left: A possible trajectory of the rolling of the scalar fields. The black dots denote the extrema of the potential. Right: Projection of the trajectory onto the $\phi - \chi$ plane. The red star denotes the end of inflation on the trajectory, marked with a red dot on the three-dimensional picture.

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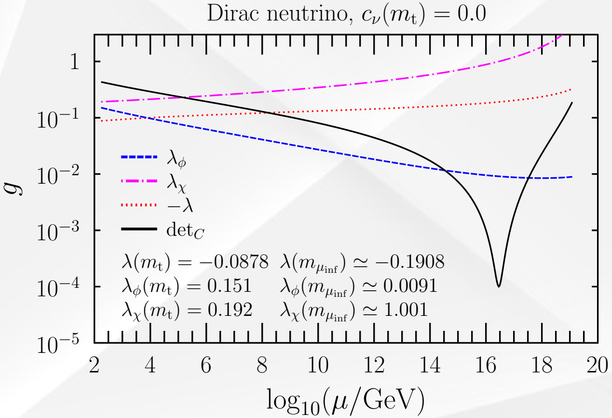
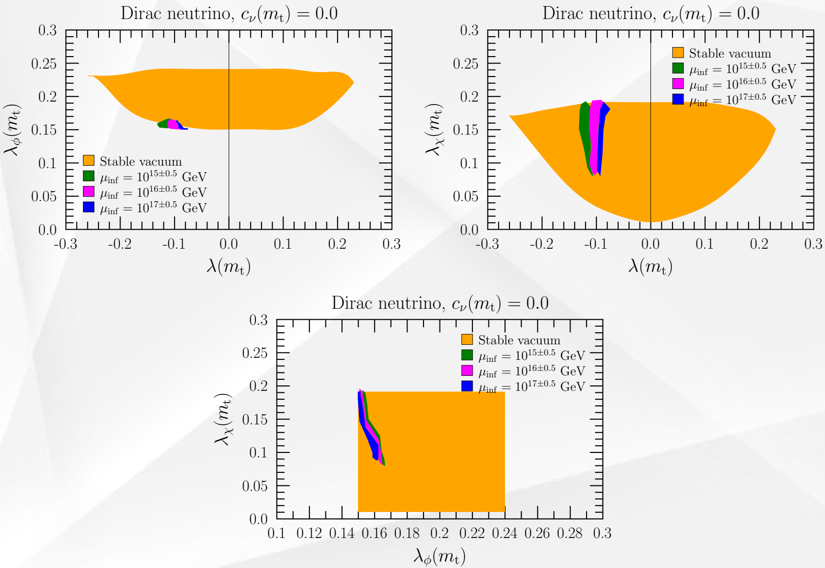
- We do this together with selecting the parameter region where we can obtain large values for the minimum of the potential! The global minimum of the potential is inversely proportional to $\det_C^{1/2}$. **Smaller $\det_C$ pushes the minimum farther away from the origin.**

A typical RG running



The final parameter space



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- Examining the SM Lagrangian, we can see that the operator $H^\dagger H$ is a dimension 2, Lorentz invariant operator.
- The Ricci scalar R in the EH action is also a dimension 2, Lorentz invariant operator.
- The combination $(H^\dagger H)R$ is therefore a **renormalizable operator** and can be generated radiatively. We might as well add it to the complete Lagrangian.

Set-up

- The relevant part of the action in unitary gauge $H^\dagger = (0, h/\sqrt{2})$ is

$$S = \int d^4x \sqrt{-g} \left(-\frac{M_{\mathrm{Pl}}^2}{2} \left(1 + \xi \frac{h^2}{M_{\mathrm{Pl}}^2} \right) R + \frac{1}{2} g^{\mu\nu} (\partial_\mu h)(\partial_\nu h) - U(h) \right) , \quad (42)$$

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- Obviously, we exclude $\xi < 0$ as the action may become unbounded from below. We can also observe, that this action reduces to the SM case if $\sqrt{\xi} h \ll M_{\rm Pl}$. Values $\xi \sim O(M_{\rm Pl}/\text{GeV})$ are completely excluded, so **h has to be large** to see new physics.

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- The factor multiplying $-(M_{\mathrm{Pl}}^2/2)R$ is called the **conformal factor** Ω. If $\Omega \neq 1$, then we are in the **Jordan frame**. There exists a so-called conformal transformation of the metric which transforms the action into the **Einstein frame** where

$$\hat{S} = \int d^4x \sqrt{-\hat{g}} \left(-\frac{M_{\mathrm{Pl}}^2}{2} \hat{R} + \dots \right), \quad (43)$$

where the quantities in the Einstein frame are denoted with an overhat.

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- We need to introduce a **canonical field** χ, for which this action takes the form

$$\hat{S} = \int d^4x \sqrt{-\hat{g}} \left\{ \frac{\hat{g}^{\mu\nu}}{2} (\partial_\mu \chi)(\partial_\nu \chi) - \hat{U}(\chi) \right\} . \tag{45}$$

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$$U(h) \simeq \frac{\lambda_h}{4} h^4 \xrightarrow[\text{infl.}]{\text{during}} \hat{U}(\chi) = \frac{\lambda_h M_{\rm Pl}^4}{4\xi^2} \frac{(\Omega^2 - 1)^2}{\Omega^4}$$

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- Every other part of the SM Lagrangian is **suppressed by** Ω^{-2} (kinetic terms) or Ω^{-4} (non-kinetic terms).

Predictions of the Higgs inflation

- The predictions of this model are in excellent agreement with the observational data

$$n_s = 1 - \frac{2}{N} = 0.967, \quad r = \frac{12}{N^2} = 0.0033, \tag{48}$$

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$$\xi = 5 \times 10^4 \sqrt{\lambda_h}. \tag{49}$$

- The tree level value $\lambda_h = 0.125$ implies $\xi \approx 18000$.

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- On the other hand, the **scale of inflation** in terms of the Higgs field is

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- This is problematic, since **we expect new physics at scales lower than the scale of inlation!**

Current research

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- In the inflationary regime, particle processes are suppressed by the large conformal factor and thus the RG running of λ_h **freezes in**.
- We have to do the analysis thoroughly, with **multiple fields**. Higgs inflation with multiple field is called **Higgs portal inflation**.

Exercise for students

1. Using the SM potential

$$U(h) = \frac{\lambda_h}{4}(h^2 - v^2)^2 \tag{53}$$

confirm that cosmological inflation is impossible either if the field rolls from $h = 0$ towards v or from $h \gg v$ towards v. Use the slow-roll analysis and the tree level parameters, $v = 246$ GeV, $\lambda_h = 0.13$ so that $m_h^2 = U''(h = v) = 2\lambda v^2$ and $m_h = 125$ GeV. Finally, the Planck mass is $M_{\mathrm{Pl}} = 2.4 \times 10^{18}$ GeV.

2. Using the potential

$$\hat{U}(\chi) = \frac{\lambda_h M_{\mathrm{Pl}}^4}{4\xi^2}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\chi}{M_{\mathrm{Pl}}}}\right) \tag{54}$$

of the Higgs inflation confirm the predictions shown in the lectures. Compute the field values h_{END} and h_{OBS} corresponding to the canonical fields χ_{END} and χ_{OBS}.