Neutrino mass

ELFT Winter School

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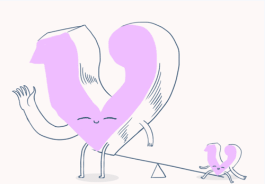
3.2.21



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  - Loop-generated neutrino mass

Motivation: Why neutrino masses are important?

- Present bounds from [PLANCK 2018] and [KATRIN 2019]:

$$\sum_{\text{active}} m_\nu \leq 0.12\ \text{eV}, \quad m(\nu_e) < 1.1\ \text{eV}$$

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- ν is the most common matter particle in the universe. Its mass gives a small effect on evolution of large-scale structure of the universe:

$$\frac{n_\nu}{n_\gamma} = \frac{3}{11}, \quad \frac{\Omega_\nu}{\Omega_m} \leq 1\,\%$$

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- Several BSM/GUT's predict massive neutrinos and ν_R: seesaw, left-right symmetry, SO(10)...

$$\mathbf{16} = \begin{pmatrix} \nu_L & d^c_R & d^c_G & d^c_B & u_R & u_G & u_B & e^+ \\ e^- & u^c_R & u^c_G & u^c_B & d_R & d_G & d_B & (\nu_L)^c \end{pmatrix}$$

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$$\psi = \underbrace{\frac{1}{2}(I - \gamma_5)\psi}_{\equiv P_L} + \underbrace{\frac{1}{2}(I + \gamma_5)\psi}_{\equiv P_R} \equiv (P_L + P_R)\psi \equiv \psi_L + \psi_R$$

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- P_L and P_R are chiral projection operators, and ψ_L and ψ_R are eigenstates of chirality.

- Helicity operator for spin-$\frac{1}{2}$ particles is defined

$$h = \frac{\mathbf{\Sigma} \cdot \mathbf{p}}{|\mathbf{p}|}, \quad \mathbf{\Sigma} = \gamma^5 \gamma^0 \boldsymbol{\gamma} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix}$$

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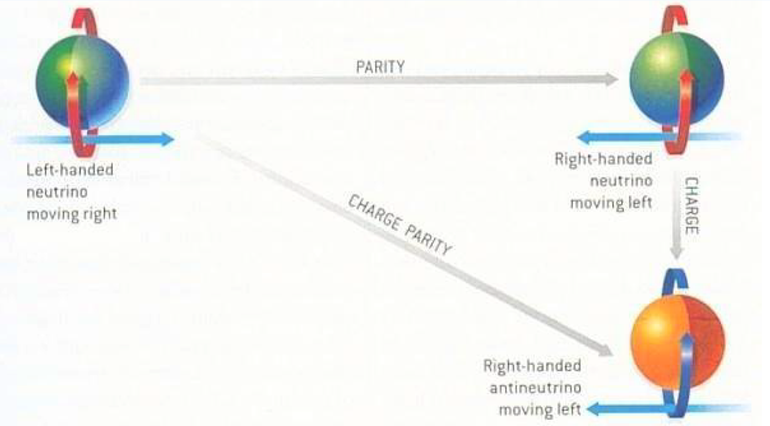
- In general this is not true! For low mass limit $m \ll E$

$$\psi_L \approx \psi_- + \frac{m}{E}\psi_+$$
$$\psi_R \approx \psi_+ + \frac{m}{E}\psi_-$$

Weak interaction is maximally parity violating

Right-handed (RH) fermion has $\mathbf{p} \uparrow\uparrow \mathbf{S}$.
Left-handed (LH) fermion has $\mathbf{p} \uparrow\downarrow \mathbf{S}$.



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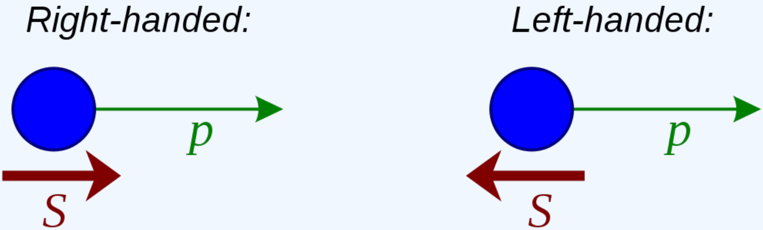

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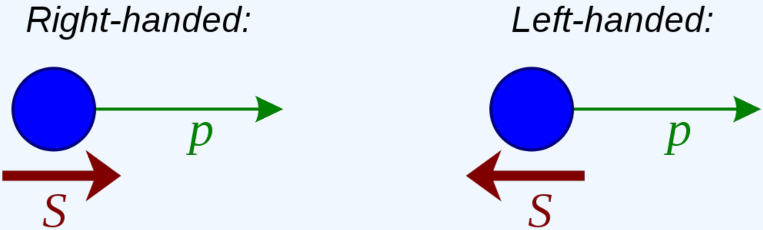

- Since neutrinos are massive, we may Lorentz boost from a frame where neutrino is LH to a frame where it is RH. For massless particles this helicity flip is impossible.
- To write the neutrino mass terms, we introduce *six* neutrino spinor fields

$$\nu_L \equiv \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}, \quad N_R = \begin{pmatrix} N_{1R} \\ N_{2R} \\ N_{3R} \end{pmatrix}$$

- Charge conjugated neutrino fields are defined as

$$(\nu_L)^c \equiv C\overline{\nu_L}^T, \quad (N_R)^c \equiv C\overline{N_R}^T,$$

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$$P_L(N_R)^c = (N_R)^c, \quad P_R(\nu_L)^c = \nu_L^c$$
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- Therefore we only need ν_L, N_R and their charge conjugates to write the neutrino mass terms.

Dirac mass terms

$$\mathcal{L} = -\overline{L_L} Y_\nu H' N_R + \text{h.c.}, \quad L_L = \begin{pmatrix} \ell_L \\ \nu_L \end{pmatrix}, \quad H' = i\sigma_2 H^*$$

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- After spontaneous symmetry breaking $SU(2)_L \otimes U(1)_Y \to U(1)_Q$,

$$\mathcal{L}' = -\frac{Y_\nu v}{\sqrt{2}} \overline{\nu_L} N_R + \text{h.c.} \equiv -\overline{\nu_L} M_D N_R + \text{h.c.}, \quad v \approx 246.22 \text{ GeV}$$

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- M_D is the Dirac mass matrix, which can be diagonalized via a biunitary transform: $V^\dagger M_D U = M_{\text{diag}}$.

Dirac mass terms

- In the new basis (mass basis),

$$\mathcal{L}' = -\overline{\nu'_L} M_{\text{diag}} N'_R + \text{h.c.}, \quad \nu'_L = V^\dagger \nu_L, \quad N'_R = U^\dagger N_R$$

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- $0\nu\beta\beta$ not possible: $(A, Z) \to (A, Z+2) + 2e^-$ violates L.

Fermion mass terms in the Standard Model

$$Y^{\ell}\overline{L}_{L\alpha}H\ell_{R\alpha} + Y^{u}\overline{Q}_{Li}Hu_{Ri} + Y^{d}\overline{Q}_{Li}H'd_{Ri}$$

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$\Downarrow$ SSB

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$\Downarrow$ mass matrices

$$M^\ell = \frac{v}{\sqrt{2}} Y^\ell, \quad M^u = \frac{v}{\sqrt{2}} Y^u, \quad M^d = \frac{v}{\sqrt{2}} Y^d$$

- Yukawa couplings in the SM spans six orders of magnitude without explanation. (Do we need explanation?)

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- Exotic way out: N_R wave function leaks to extra dimensions?
  [Dienes *et al.* 1999, Arkani-Hamed *et al.* 2002]

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- A symmetric matrix M_L may be diagonalized via a unitary transformation: $U^T M_L U = M_{\text{diag}}$.

Majorana mass term

- In the new basis (mass basis again)

$$\mathcal{L} = -\frac{1}{2}\overline{\nu_L'} M (\nu_L')^c + \text{h.c.}, \quad \nu_L' = U^T \nu_L, \quad (\nu_L')^c \equiv C\overline{\nu_L'}^T$$

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- Defining a Majorana spinor $\nu' \equiv \nu'_L + (\nu'_L)^c = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$, which satisfies Majorana condition $(\nu')^c = \nu'$, that is, neutrinos are their own antiparticles (Majorana, 1937).

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- Factor $\frac{1}{2}$ accounts for the fulfillment of Dirac equation for ν_L.

In a minimal model, to only add the neutrino masses and nothing else:

	Dirac	**Majorana**
Neutrino oscillations	Yes	Yes
$0\nu\beta\beta$	No	Yes
CP violating phases	1	3
Sterile neutrinos	Yes	No
Gauge invariance broken	No	Yes!
Lepton number broken	No	Yes, $\Delta L = 2$
Lepton flavour broken	Yes	Yes
Renormalizable	Yes	Yes

$d = 5$ effective operator (Weinberg, 1979)

$$\frac{\mathcal{L}^{(5)}}{\Lambda} = \frac{1}{2} f \overline{L_{L\alpha}} H' (H')^T (L_{L\beta})^c + \text{h.c.}, \quad H' = i\sigma_2 H^*$$

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- The only gauge invariant dimension-5 operator which can be constructed from SM fields produces the Majorana mass term after spontaneous symmetry breaking:

$$\frac{\mathcal{L}'}{\Lambda} = -\frac{1}{2} \overline{\nu_L} M_L (\nu_L)^c + \text{h.c.}, \quad M_L = \frac{1}{2} f v^2$$

Λ is the cutoff scale and f neutrino coupling matrix.

$d = 5$ EFFECTIVE OPERATOR (WEINBERG, 1979)

$$\frac{\mathcal{L}^{(5)}}{\Lambda} = \frac{1}{2} f \overline{L_{L\alpha}} H' (H')^T (L_{L\beta})^c + \text{h.c.}, \quad H' = i\sigma_2 H^*$$

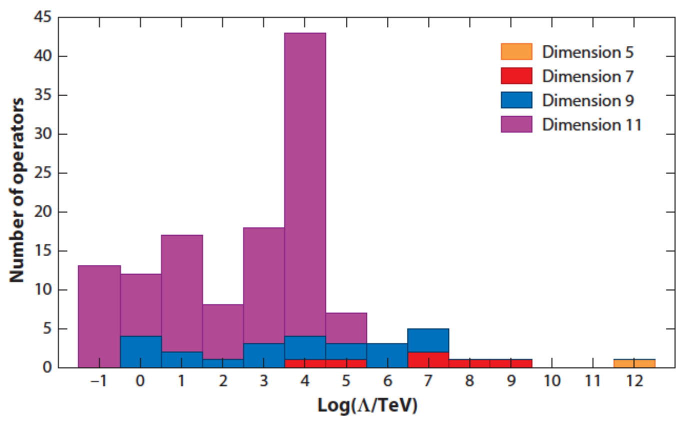
- The only gauge invariant dimension-5 operator which can be constructed from SM fields produces the Majorana mass term after spontaneous symmetry breaking:

$$\frac{\mathcal{L}'}{\Lambda} = -\frac{1}{2} \overline{\nu_L} M_L (\nu_L)^c + \text{h.c.}, \quad M_L = \frac{1}{2} f v^2$$

  Λ is the cutoff scale and f neutrino coupling matrix.
- Higher-dimensional operators exist for neutrino masses, but will not be covered here. Why?

There are $\mathcal{O}(100)$ different possiblities!

Number of possible effective operators increases if higher dimensions are allowed. Assume coupling matrix $f_{ij} = \mathcal{O}(1)$.



Picture: André De Gouvêa, Annu. Rev. Nucl. Part. Sci. 2016. 66:197217.

When we include Weinberg operator...

	Dirac	**Majorana**
Neutrino oscillations	Yes	Yes
$0\nu\beta\beta$	No	Yes
CP violating phases	1	3
Sterile neutrinos	Yes	No
Gauge invariance broken	No	No
Lepton number broken	No	Yes, $\Delta L = 2$
Lepton flavour broken	Yes	Yes
Renormalizable	Yes	No!

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Seesaw mechanisms work at tree level

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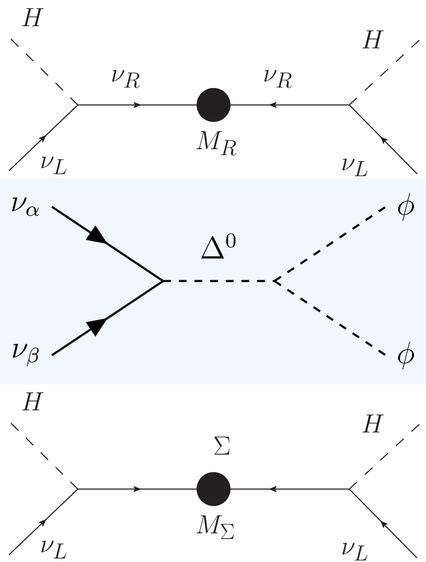
- Type III: add fermion triplet $\Sigma = (\Sigma^{+}, \Sigma^{0}, \Sigma^{-}) \sim (\mathbf{3}, 0)$



Type I seesaw mechanism

[Fritzsch, Gell-Mann, Minkowski 1975; Minkowski 1977; Yanagida 1979; Glashow 1979; Gell-Mann, Ramond, Slansky 1979; Mohapatra and Senjanoviç 1979

- Use both Dirac and Majorana mass terms and N_R.

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- After SSB, we may construct the full 6×6 mass matrix:

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- Once we block-diagonalize the mass matrix, we obtain in leading order

$$m_\nu = -M_D M_R^{-1} M_D^T = -\frac{v^2}{2} Y_\nu M_R^{-1} Y_\nu^T, \quad M_N = M_R$$

- If we avoid flavour problem ($Y_{ij} \sim 1$), then we need the RH neutrinos N_R to be extremely heavy: $M_N = \mathcal{O}(10^{15})$ GeV, near GUT scale.
  - $\Rightarrow$ no hope for direct detection

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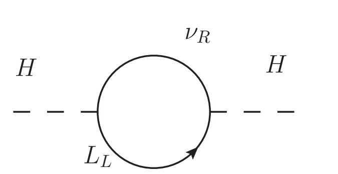
- Hierarchy problem gets worse: Higgs mass gets a large one-loop correction.

$$\delta M_H^2 = -\frac{\text{eig}(Y_\nu)^2}{8\pi^2}\left(\Lambda^2 + M_{N_i}^2 \ln \frac{M_{N_i}^2}{\Lambda^2}\right)$$



Low-scale Type-I seesaw mechanism

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- This value acts as an approximate lower bound for the simplest seesaw case.

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Different seesaw scales for different problems

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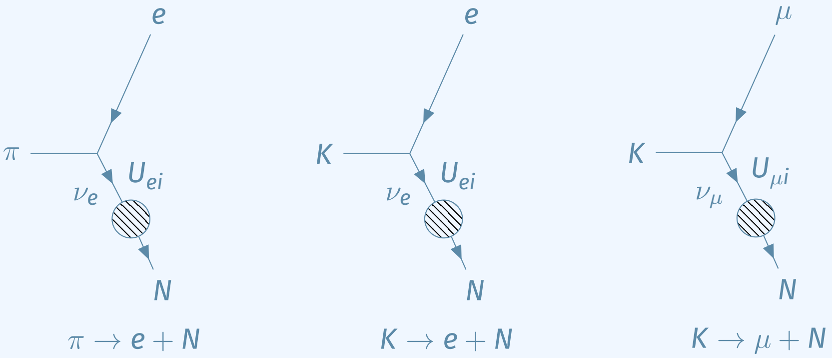
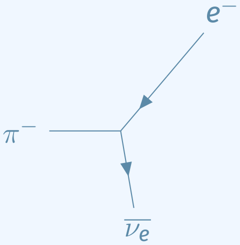
eV	Neutrino oscillation anomalies.

Sterile neutrino production

If $M_{\text{meson}} > M_N + m_\ell$, sterile neutrino N is produced like an active neutrino, but the rate is suppressed by $|U_{\ell i}|^2$.



$$\Gamma(\pi^- \to e^- \overline{\nu_e}) = \frac{G_F^2 f_\pi^2 \cos^2\theta_c m_\pi^3}{8\pi} \left(\frac{m_e}{m_\pi}\right)^2 \left(1 - \frac{m_e^2}{m_\pi^2}\right)$$

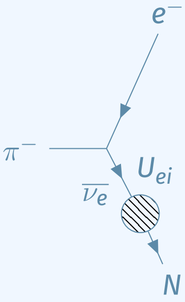


Pion has spin 0, so e^- and $\overline{\nu_e}$ must have opposite spins. At limit $m_\nu \approx 0$, $\overline{\nu_e}$ is purely RH and therefore has positive helicity, so e^- must be LH. Due to conservation of angular momentum, e^- must also have positive helicity, but

$$\psi_L \approx \psi_- + \frac{m_e}{E}\psi_+$$

Helicity suppression.

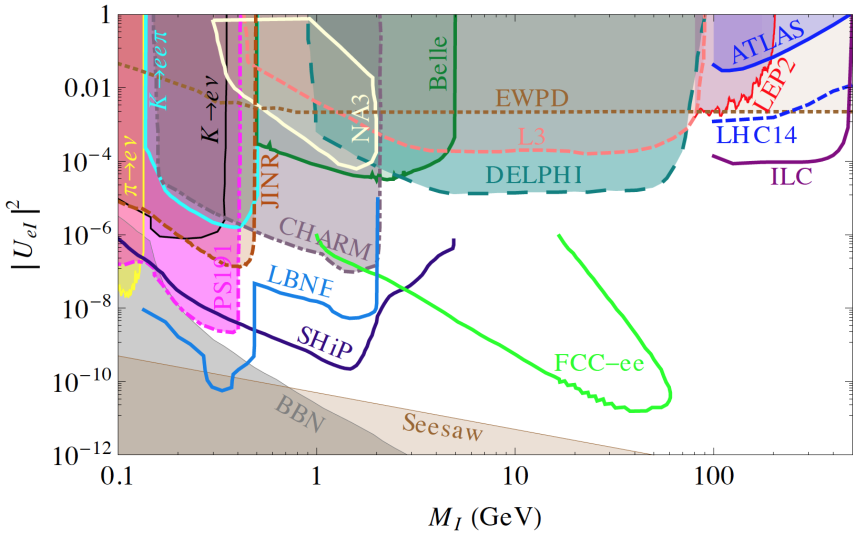
EXAMPLE: $\pi^- \to e^- N$ AT REST



$$\Gamma(\pi^- \to e^- N) = \Gamma(\pi^- \to e^- \overline{\nu_e}) |U_{ei}|^2 \left(\frac{M_N}{m_e} \right)^2$$

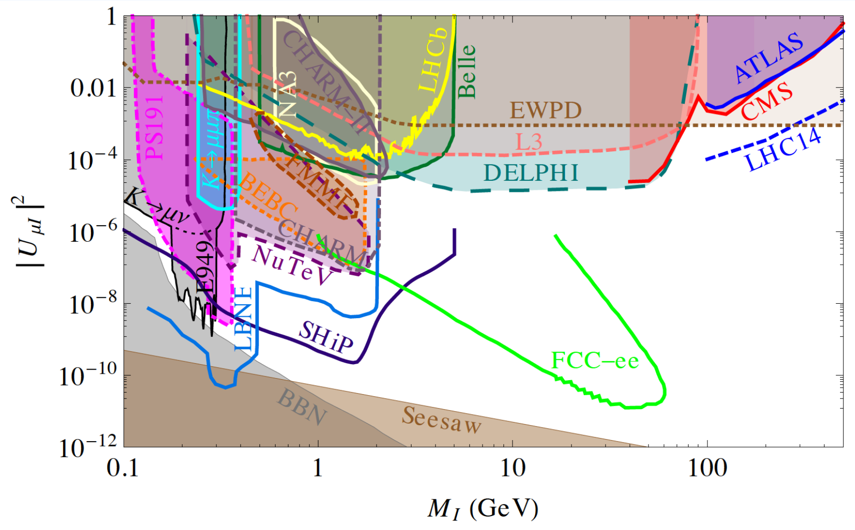
Being massive, sterile neutrino has both helicities. Therefore the electron will not be helicity suppressed, and there will be a peak on the pion decay spectrum. Similar searches can be done with kaon decays.

Bounds on ν_e-sterile mixing strength



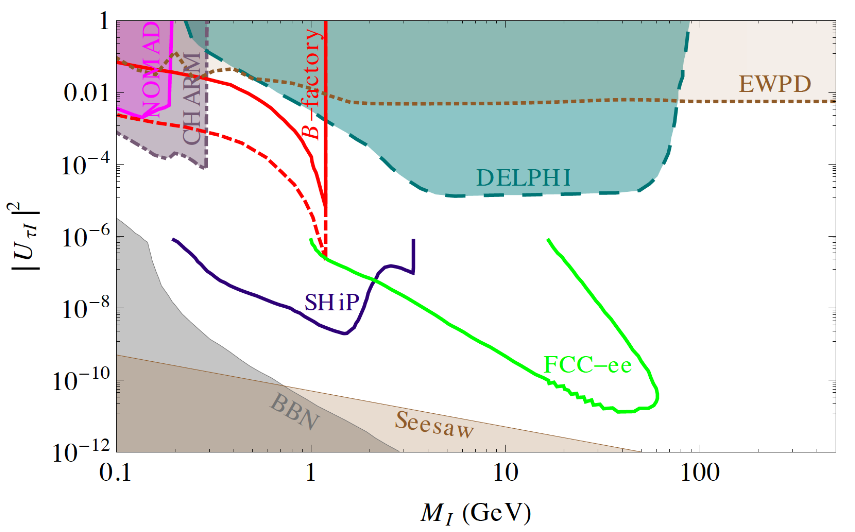
Plot: Alekhin *et al.*, Rep. Prog. Phys. 79 (2016) 124201

Bounds on ν_μ-sterile mixing strength



Plot: Alekhin *et al.*, Rep. Prog. Phys. 79 (2016) 124201

Bounds on ν_τ-sterile mixing strength



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- Neutrinos are weakly interacting, very light, spin-$\frac{1}{2}$, flavour-oscillating elementary particles.
- Chirality and helicity coincide for massless particles.
- Dirac and Majorana mass terms can be combined to produce light neutrino masses via seesaw mechanism, which utilizes gauge-singlet heavy sterile Majorana neutrinos.
- Low-scale seesaw implies sterile neutrinos at mass range available for several different experimental approaches.

Thank you for you attention!