Cosmology

An introduction

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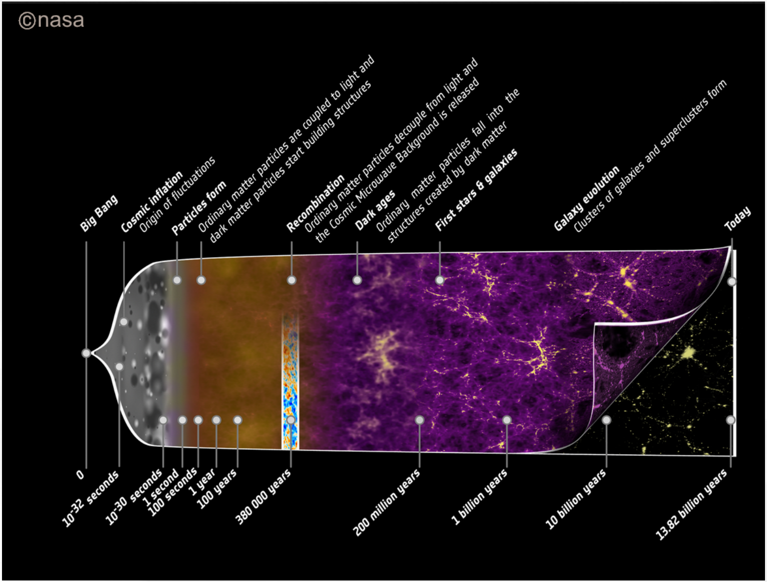
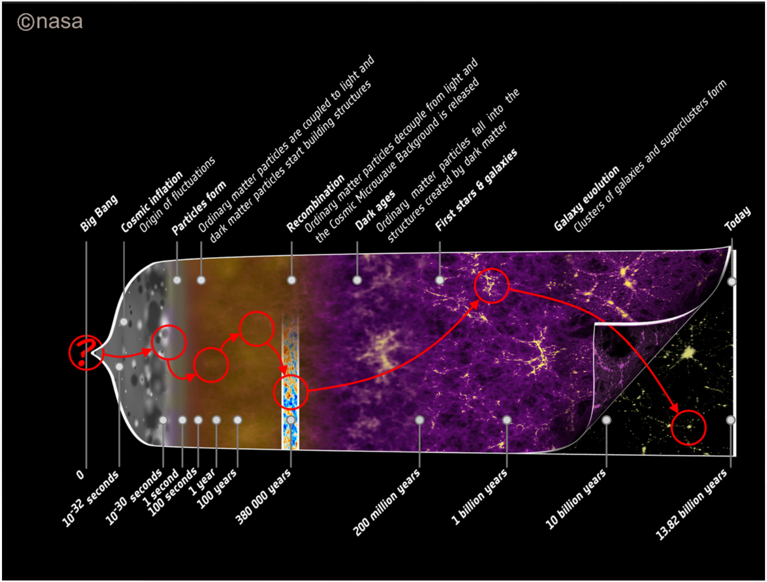
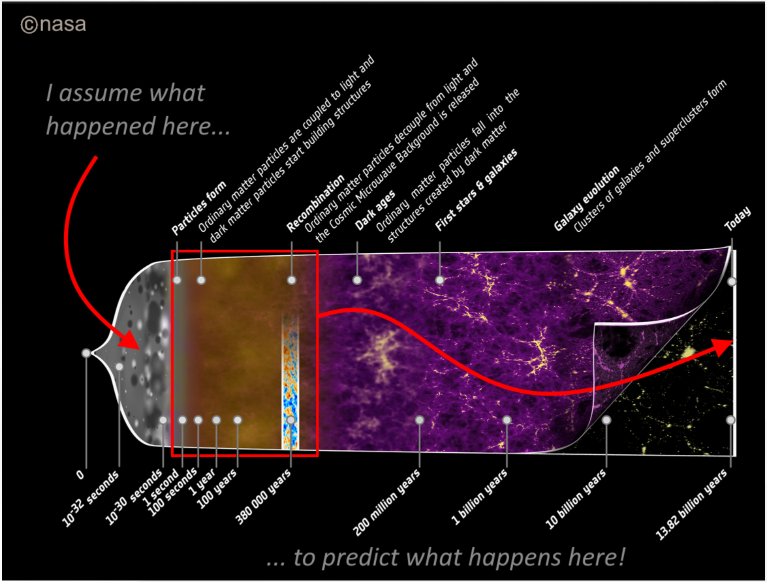
History of the Universe



A Game of Initial Conditions



A small part...


©nasa
I assume what happened here...
Particles form
Ordinary matter particles are coupled to light and dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
0
10^{-32} seconds
10^{-30} seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years
... to predict what happens here!

Einstein Equations

Geometry

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$$

Fit to Newtonian gravity

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Cosmological constant

Matter

The FLRW metric

Symmetric

$$\mathbf{g}(t,\vec{r}) = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{01} & g_{11} & g_{12} & g_{13} \\ g_{02} & g_{12} & g_{22} & g_{23} \\ g_{03} & g_{13} & g_{23} & g_{33} \end{pmatrix}$$

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Diagonalizable +
Homogeneity

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Isotropy

$$\mathbf{g}(t) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & a^2(t) & 0 & 0 \\ 0 & 0 & a^2(t) & 0 \\ 0 & 0 & 0 & a^2(t) \end{pmatrix}$$

FLRW metric: spatial part

- $a(t)$ is called the **scale factor**
- Line element squared:

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + \underbrace{a^2(t)\mathrm{d}\Sigma^2}_{\text{spatial part}}$$

- Spatial part can be...

$$\mathrm{d}\Sigma^2 = \begin{cases} \frac{\mathrm{d}r^2}{1-r^2} + r^2\mathrm{d}\Omega^2 & \text{Spherical geometry} \\ \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2 & \text{Flat geometry} \\ \frac{\mathrm{d}r^2}{1+r^2} + r^2\mathrm{d}\Omega^2 & \text{Hyperbolic geometry} \end{cases}$$

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- Due to observational evidence, we will use this

Friedmann equations

FLRW metric

$$\mathbf{g} = \mathrm{diag}\left[-1,\, a^2(t),\, a^2(t),\, a^2(t)\right]$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

$$G_{\mu\nu} + \cancel{\Lambda g_{\mu\nu}} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Negligible at early Universe

$$\mathbf{T} = \mathrm{diag}\left[\rho(t),\, a^2(t)p(t),\, a^2(t)p(t),\, a^2(t)p(t)\right]$$

Perfect fluid

Friedmann equations

- **Friedmann's 1st equation:**

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)} = \sqrt{\frac{8\pi G \rho(t)}{3}}$$

- **Friedmann's 2nd equation:**

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{3}\left[\rho(t) + 3p(t)\right]$$

Friedmann equations

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  - Definition of the Hubble rate
  - $H(t) > 0$ if $\rho(t) \neq 0$
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$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{3}\left[\rho(t) + 3p(t)\right]$$

  - Acceleration rate is negative for conventional matter

Dilution of matter

- Consider the following equation of state:

$$p(t) = w\rho(t), \quad \text{where} \quad w = \mathrm{const}.$$

- Depending on the value of w the time dependence of the scale factor is different:

$$a(t) \propto \begin{cases} t^{1/2}, & \text{if } w = 1/3, \text{ i.e. relativistic matter} \\ t^{2/3}, & \text{if } w = 0, \text{ i.e. non-relativistic matter} \\ \exp(t/t_0), & \textbf{if } w = -1, \text{i.e. Dark Energy} \end{cases}$$

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- These are slowing down...

Dilution of matter

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- ... but this one is getting faster!

Boltzmann equations - requirements

- But how to specify the content of the Universe, and how to introduce particle interactions?

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- But how to specify the content of the Universe, and how to introduce particle interactions?
- We would like a differential equation describing the evolution of the number density of a given particle, where
  1. we use statistical description,
  2. the number density always tends towards equilibrium,
  3. the changes in the number density are proportional to the ratio between the Hubble expansion (-) and the production rate (+).

Boltzmann equations - setup

- Statistical physics, out-of-equilibrium processes $\rightarrow$ Boltzmann equation
- Describes the evolution of the phase space density f in presence of some external forces

Collision term
particle processes

$$\hat{\mathbf{L}}[f] = \hat{\mathbf{C}}[f]$$

kinematics
Liouville operator

Definition:

$$\hat{\mathbf{L}}[f] \equiv p^\alpha \frac{\partial f}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial f}{\partial p^\alpha}$$

We use that...

1. in the FLRW metric, the Christoffel symbols are given by

$$\Gamma^i_{j0} = H(t)\delta^i_j \,, \quad \Gamma^0_{ij} = H(t) g_{ij} \,,$$

Boltzmann equations - Liouville operator

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2. due to **isotropy** the phase space density cannot depend on p_i, only on $|\mathbf{p}|$,
3. due to **homogeneity** the phase space density cannot depend on the spatial coordinates x^i.

Boltzmann equations - Liouville operator

$$
\begin{aligned}
\hat{\mathbf{L}}[f] &\equiv p^\alpha \frac{\partial f}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial f}{\partial p^\alpha} \\
&= E \frac{\partial f}{\partial t} + 0 - \Gamma^0_{ij} g_{ij} p^i p^j \frac{\partial f}{\partial E} + 0
\end{aligned}
$$

- Point 3. - homogeneity
- Point 1. - FLRW metric
- Point 2. - isotropy

Boltzmann equations - Liouville operator

$$
\begin{aligned}
\hat{\mathbf{L}}[f] &\equiv p^\alpha \frac{\partial f}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial f}{\partial p^\alpha} \\
&= E \frac{\partial f}{\partial t} - \Gamma^0_{ij} g_{ij}\, p^i p^j \frac{\partial f}{\partial E} \\
&= E \left[\frac{\partial f}{\partial t} - H(t) \frac{p^2}{E} \frac{\partial f}{\partial E} \right]
\end{aligned}
$$

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&= E \left[\frac{\partial f}{\partial t} - H(t) \frac{p^2}{E} \frac{\partial f}{\partial E} \right]
\end{aligned}
$$

Here we will be interested in the following definition of the **integrated Liouville term**:

$$
g \int \frac{\mathrm{d}^3 \mathbf{p}}{(2\pi)^3} \frac{\hat{\mathbf{L}}[f]}{E} = \dot{n} + 3H(t) n
$$

($n =$ number density, $g =$ degrees of freedom)

Boltzmann equations - Collision term for decays

Consider $a \to c + d$ decay:

$$g_a \int \frac{\mathrm{d}^3 \mathbf{p}_a}{(2\pi)^3} \frac{\hat{\mathbf{C}}[f]}{E_a} = \sum_{\text{spins}} \int \mathrm{d}\Pi_a \overbrace{\mathrm{d}\Pi_c \mathrm{d}\Pi_d (2\pi)^4 \delta^{(4)}(p_a - p_c - p_d)}^{\text{2-body phase space}}$$

$$\times \underbrace{|\mathcal{M}|^2}_{\substack{\text{Polarized} \\ \text{amplitude}}} \Big[\underbrace{f_c f_d (1 \pm f_a)}_{\text{Inverse decay}} - \underbrace{f_a (1 \pm f_c)(1 \pm f_d)}_{\text{Decay}} \Big],$$

$$\longrightarrow 1 \pm f = \begin{cases} 1 + f & \text{for bosons, } \textit{Bose enhancement} \\ 1 - f & \text{for fermions, } \textit{Pauli blocking} \end{cases}$$

Consider $a + b \to c + d$ annihilation (new terms w.r.t. decays in red):

$$g_a \int \frac{\mathrm{d}^3\mathbf{p}_a}{(2\pi)^3} \frac{\hat{\mathbf{C}}[f]}{E_a} = \sum_{\text{spins}} \int \mathrm{d}\Pi_a \mathrm{d}\Pi_b \overbrace{\mathrm{d}\Pi_c \mathrm{d}\Pi_d (2\pi)^4 \delta^{(4)}(p_{\text{in}} - p_{\text{out}})}^{\text{2-body phase space for the final states}}$$

$$\times \underbrace{|\mathcal{M}|^2}_{\substack{\text{Polarized}\\ \text{amplitude}}} \Big[\underbrace{f_c f_d (1 \pm f_a)(1 \pm f_b)}_{c+d \to a+b} - \underbrace{f_a f_b (1 \pm f_c)(1 \pm f_d)}_{a+b \to c+d} \Big].$$

$$\longrightarrow 1 \pm f = \begin{cases} 1 + f & \text{for bosons, } \textit{Bose enhancement} \\ 1 - f & \text{for fermions, } \textit{Pauli blocking} \end{cases}$$

Boltzmann equations - Approximations

1. Approximate all particle distributions with Maxwell-Boltzmann statistics, i.e.

$$f(\mathbf{p},\mu,m,T) = f_{\mathrm{MB}}(\mathbf{p},\mu,m,T) = \exp\left(-\frac{E(\mathbf{p},m)-\mu}{T}\right) .$$

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$$(1 \pm f_{\mathrm{MB}}) \simeq 1 \,.$$

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1. Approximate all particle distributions with Maxwell-Boltzmann statistics, i.e.
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2. By virtue of the MB statistics, neglect the Bose enhancement and Pauli blocking terms, i.e.
$$(1 \pm f_{\mathrm{MB}}) \simeq 1 .$$
3. In cosmology, the working assumption is that the particle species equilibrate to $\mu = 0$, we define the equilibrium statistics by
$$f^{\mathrm{eq}}_{\mathrm{MB}}(\mathbf{p}, \mu, m, T) = \exp\left(-\frac{E(\mathbf{p}, m)}{T}\right) .$$

Definition:

- For some quantity $\mathcal{A}$, the thermal average is given by

$$\langle \mathcal{A} \rangle_{\mathrm{MB}}(T) = \frac{\int \prod_{i=1}^{n} \mathrm{d}^3\mathbf{p}_i \, \mathcal{A} \, f_{\mathrm{MB}}^{\mathrm{eq}}(\mathbf{p}_i, m_i, T)}{\int \prod_{i=1}^{n} \mathrm{d}^3\mathbf{p}_i \, f_{\mathrm{MB}}^{\mathrm{eq}}(\mathbf{p}_i, m_i, T)} \, .$$

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- For decays ($n = 1$):

$$\langle \Gamma \rangle = \Gamma \frac{K_1(z)}{K_2(z)} \, , \quad z = \frac{m_{\mathrm{decay}}}{T}$$

  (center of mass frame vs cosmic rest frame)

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  (center of mass frame vs cosmic rest frame)
- For annihilation ($n = 2$):

$$\langle \sigma v_{\mathrm{Møl}} \rangle(T) = \frac{1}{8 m^4 T K_2^2\left(\frac{m}{T}\right)} \int_{4\mu^2}^{\infty} \mathrm{d}s \, \sigma(s) \left(s - 4m^2\right) \sqrt{s} \, K_1\left(\frac{\sqrt{s}}{T}\right) \, .$$

Boltzmann equations in Cosmology

- For decays:

$$\dot{n}_a + 3Hn_a = \langle\Gamma\rangle \left[\frac{n_c n_d}{n_c^{\mathrm{eq}} n_d^{\mathrm{eq}}} n_a^{\mathrm{eq}} - n_a\right]$$

- For annihilation:

$$\dot{n}_a + 3Hn_a = \langle\sigma v_{\text{Møl}}\rangle \left[\frac{n_c n_d}{n_c^{\mathrm{eq}} n_d^{\mathrm{eq}}} n_a^{\mathrm{eq}} n_b^{\mathrm{eq}} - n_a n_b\right]$$

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- These are just 1 if the final states are in equilibrium (e.g. SM particles)
- Not particularly clear how the 3rd requirement on slide 15 is satisfied

Boltzmann equations in Cosmology

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- In reality we may have a combination of the right hand sides for multiple decay and/or annihilation channels!

Way forward

- Rewrite the Boltzmann equations with temperature instead of time as the differential equation variable

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- Relevant processes and energy scales

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- Rewrite the Boltzmann equations with temperature instead of time as the differential equation variable
- Relevant processes and energy scales
- Coupled systems of differential equations
- How to actually solve them?

THANK YOU FOR YOUR ATTENTION!

Neutrino Dark Matter

in the Superweak model

Károly Seller

ELTE Department for Theoretical Physics

TABLE OF CONTENTS

What is dark matter?

- We don’t really know!

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- Constitutes about 26% of the total energy density of the Universe (Planck: $\Omega_{\mathrm{DM}} = 0.265$)

What is dark matter?

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- Technical definition in standard cosmology: any non-baryonic matter whose energy density dilutes as $\rho \propto a^{-3}$
- Constitutes about 26% of the total energy density of the Universe (Planck: $\Omega_{\mathrm{DM}} = 0.265$)
- Experimental evidence through its effects via gravity (so it has to be massive)

Dark matter production

- Assuming that dark matter exists and is in a form of a new particle species

Dark matter production

- There are a number of ways in which dark matter may be produced $\rightarrow$ mechanism depends on the underlying **particle physics model** and **parameters**!

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- **Idea:** dark matter species was subject to interactions at higher energies, but decoupled later, their lightest (stable) particles forming a relic density which we detect today
- We will look at these in more detail

Dark matter production: freeze in

- Requirements:
  1. Feeble coupling, $g_{\mathrm{x}} \lesssim 10^{-10}$
  2. Constrained initial conditions (initially 0 or close to 0 densities at T_{rh})
  3. Light dark matter particle

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    2. Constrained initial conditions (initially 0 or close to 0 densities at T_{rh})
    3. Light dark matter particle
- Nature of the mechanism:
    1. Dark matter species is unable to reach equilibrium with the cosmic plasma

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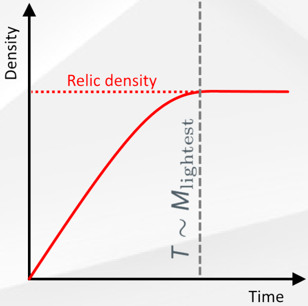

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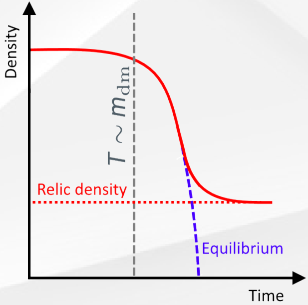

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- Assuming that dark matter particles are non-relativistic today,

$$\Omega_{\mathrm{DM}} = 611.53 \frac{m_{\mathrm{dm}}}{\text{keV}} \mathcal{Y}_\infty$$

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- Roughly:

$$\frac{\dot{T}}{T} = -H(T)\left[1+\delta(T)\right] \approx -H(T)$$

Boltzmann equations rewritten II

- For simplicity with equilibrium densities for the non-dark matter particles

Boltzmann equations rewritten II

- Decays:

Thermally averaged decay width

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Effective degrees of freedom

Model setup

- As an example, consider the $\mathrm{G}_{\mathrm{SM}} \otimes \mathrm{U}(1)_{\mathrm{x}}$ Superweak model (see talk by Z. Trócsányi)
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- Insertion of the particle physics model into cosmology $\rightarrow$ thermally averaged decay rates and cross sections of relevant processes

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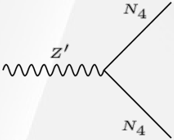
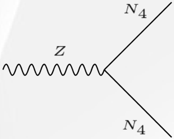
Superweak model: vertices and decay rates I.



$$\propto g_{\rm x} \longrightarrow \Gamma(Z' \to N_4 N_4) = \frac{m_{Z'}}{96\pi} g_{\rm x}^2 \left(1 - \frac{4m_4^2}{m_{Z'}^2}\right)^{\frac{3}{2}}$$



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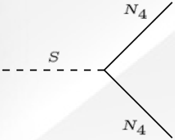

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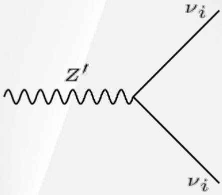
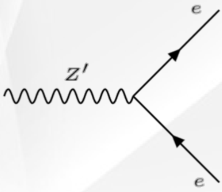
Superweak model: vertices and decay rates II.



$$\propto g_x \longrightarrow \sum_{i\in\{e,\mu,\tau\}} \Gamma(Z' \to \nu_i\nu_i) = \frac{m_{Z'}}{32\pi} g_x^2$$



$$\propto g_x \longrightarrow \Gamma(Z' \to ee) = \frac{m_{Z'}}{48\pi} g_x^2 \left[4\left(1-\frac{k}{2}\right)\cos^2\theta_W - 1\right]$$

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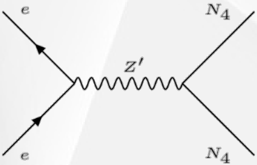

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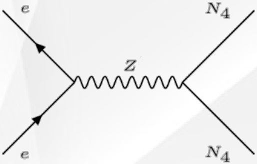

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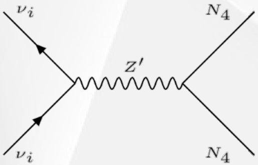

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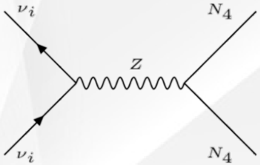

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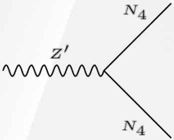

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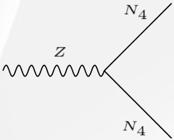

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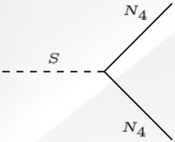

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- How to produce Z'? $\rightarrow$ use inverse decays of electrons and SM neutrinos (equilibrium densities!)

Boltzmann equation for freeze in

- Schematically we have to solve:

$$\frac{\mathrm{d}\mathcal{Y}_{\mathrm{Z}'}}{\mathrm{d}z} = \text{production via inverse decay from electrons in equilibrium}$$
$$+ \text{ production via inverse decay from SM neutrinos in equilibrium}$$
$$- \text{ depletion via decay into right-handed neutrinos}$$

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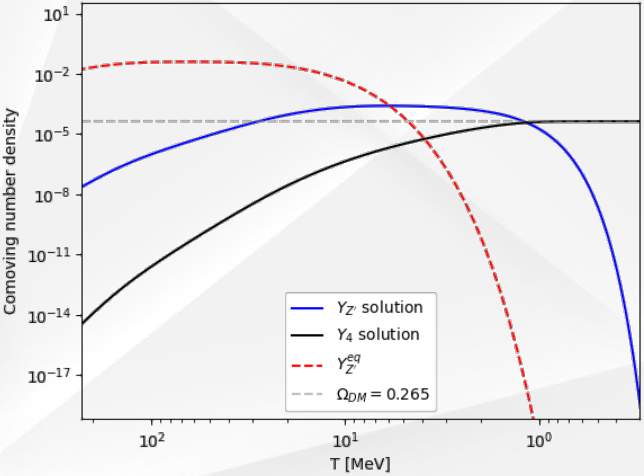
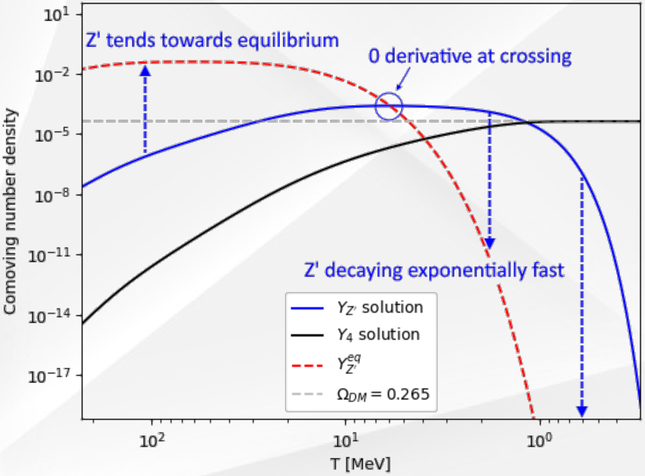
- Note:
  - N_4 density is strictly increasing, since inverse decays from N_4 into other particles is negligible due to $\mathcal{Y}_4 \ll \mathcal{Y}_4^{\mathrm{eq}}$
  - The second equation is decoupled from the first in a sense, that given a solution for the evolution of Z', the evolution of N_4 can be calculated

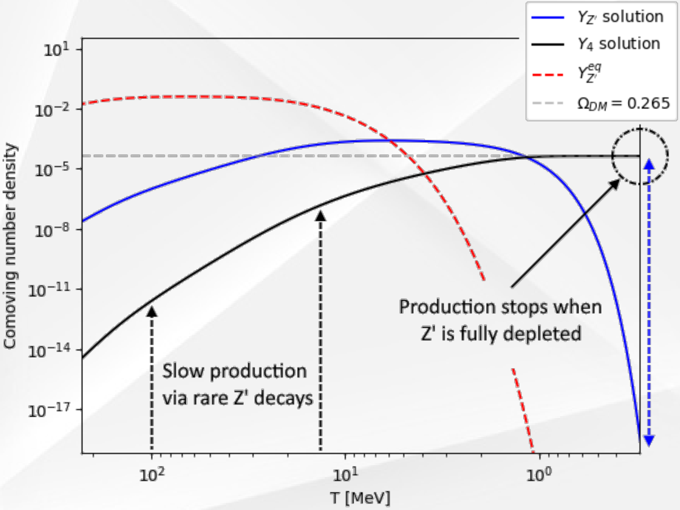
Freeze in example solution



Freeze in example solution: Z'



Freeze in example solution: Neutrino



Freeze out mechanism: setup

- Relevant processes: **annihilations**
- Coupling: has to be around $g_{\mathrm{x}} \sim 10^{-4} - 10^{-6}$
- Masses: To exploit resonant production, $m_{\mathrm{Z}'} \gtrsim 2m_4$, while $m_{\mathrm{Z}'} \ll m_{\mathrm{Z}}$
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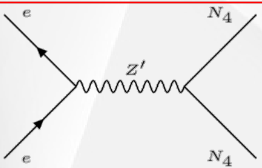
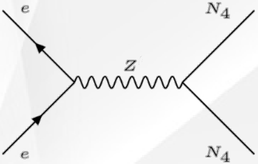


$\propto g_x^2$

$\longrightarrow$ $$\sigma(ee \to Z' \to N_4 N_4) = \frac{1}{12\pi}\sqrt{1-\frac{4m_4^2}{s}}\,\frac{s}{(s-m_{Z'}^2)^2+m_{Z'}^2\Gamma_{Z'}^2}\,g_X^4(k\cos^2\theta_W-1)^2$$



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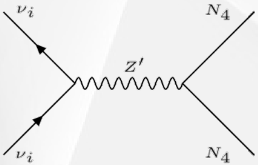
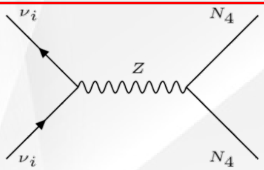


$\propto g_x^2$

$\longrightarrow$ $\sigma(\nu_i\nu_i \to Z' \to N_4N_4) = \frac{N_\nu}{48\pi}\sqrt{1-\frac{4m_4^2}{s}}\frac{s}{(s-m_{Z'}^2)^2+m_{Z'}^2\Gamma_{Z'}^2}g_X^4$



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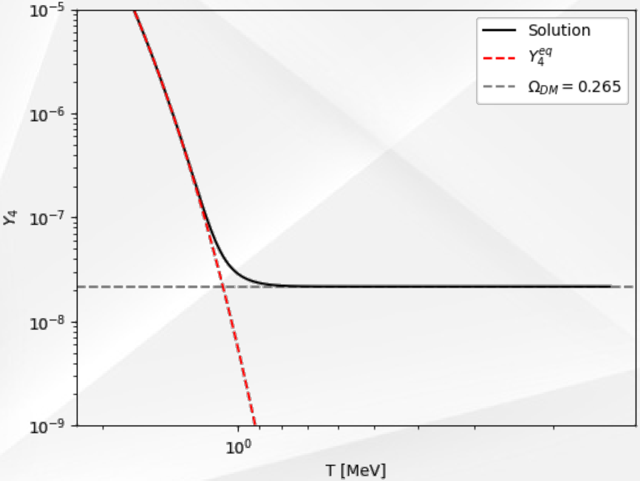
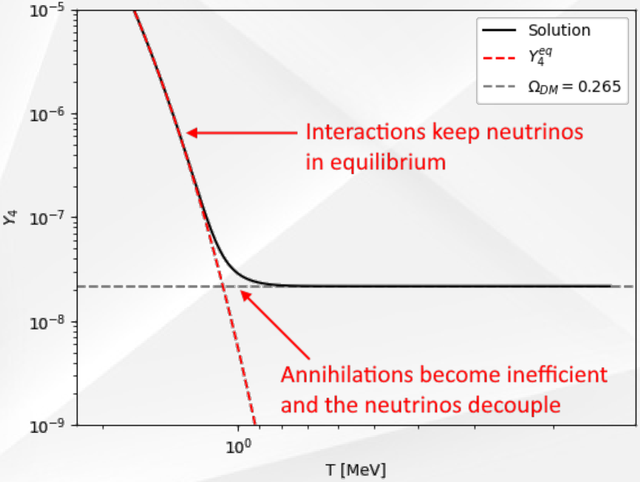
- Difficulty: integration of the thermally averaged cross section (has to be done numerically)

Freeze in example solution



Freeze in example solution: Neutrino



Thermally averaged cross section

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$$\langle \sigma v_{\text{Møl}} \rangle(T) = \frac{1}{8 m_4^4 T K_2^2\left(\frac{m_4}{T}\right)} \int_{4m_4^2}^{\infty} \mathrm{d}s\, \sigma(s) \left(s - 4m_4^2\right) \sqrt{s}\, K_1\left(\frac{\sqrt{s}}{T}\right) .$$

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- On the figure shown, $m_4 = 20\,\text{MeV}$, and decoupling happens around $T = 1\,\text{MeV}$, here $K_1(40) \approx 10^{-18}$ already.

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- Freeze in requires very small couplings, making the experimental check near impossible today
- Freeze out needs couplings of order $10^{-5} - 10^{-6}$, but at the expense of resonant production, i.e. the masses have to be tuned so that $2m_4 \lesssim m_{\mathrm{Z}'}$
- Freeze out is better constrained by experiments, and the preferred region of parameter space cannot be fully ruled out

Thank You for Your Attention!