Neutrino Oscillations and the non-unitarity of the PMNS

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Part I

Neutrino Oscillation

Experiments & Theory

Motivation

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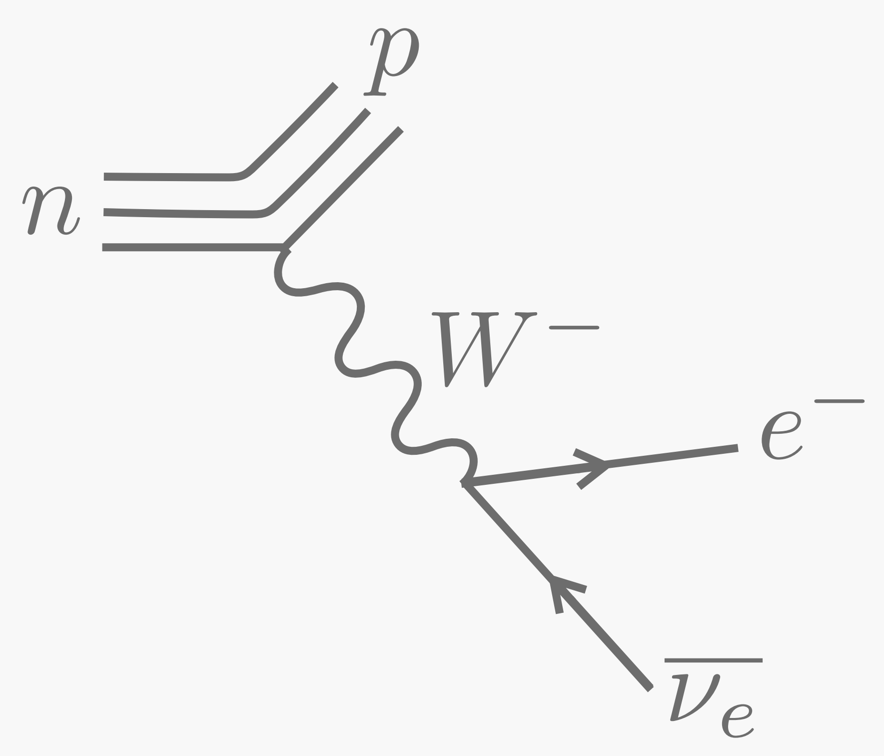
- Neutrinos are neutral leptons: no charge & no color

  $\Rightarrow$ they only interact via Weak Interactions

- They also have very light masses

their interactions are suppressed

Introduced by Pauli in 1930 to save E and $\vec{p}$ conservation in $\beta-$decays



Motivation

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Detection of neutrinos:

- 1956: $\overline{\nu_e}$ discovery from reactors
- 1962: $\overline{\nu_\mu}$ discovery from π decays
- 2002: $\overline{\nu_\tau}$ discovery by DONUT collaboration

Motivation

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Detection of neutrinos:

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NP 2002: Davis & Koshiba for detecting solar and supernova ν

NP 2015: Kajita & McDonald for the discovery of ν oscillations

Motivation

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The discovery of neutrino oscillations imply:

- neutrinos have mass
- non-zero leptonic mixing

→ necessary extension of the SM

This extension could be much richer than simply mirroring the quark pattern

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Active area of particle physics, many things to be studied:

– neutrino mass
– neutrino mass hierarchy
– CPV in the leptonic sector?
– Majorana character of neutrinos?

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They could hold the key to understanding some other SM open problems

- Dark matter candidate → lecture by Károly Seller

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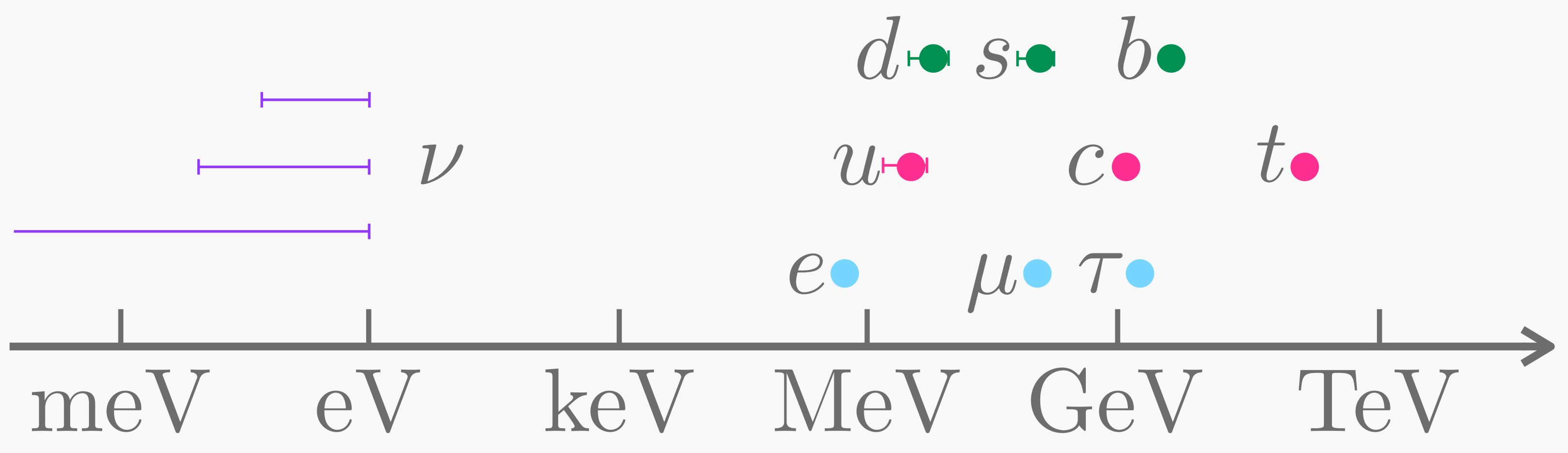

no SM explanation for Yukawa ordering

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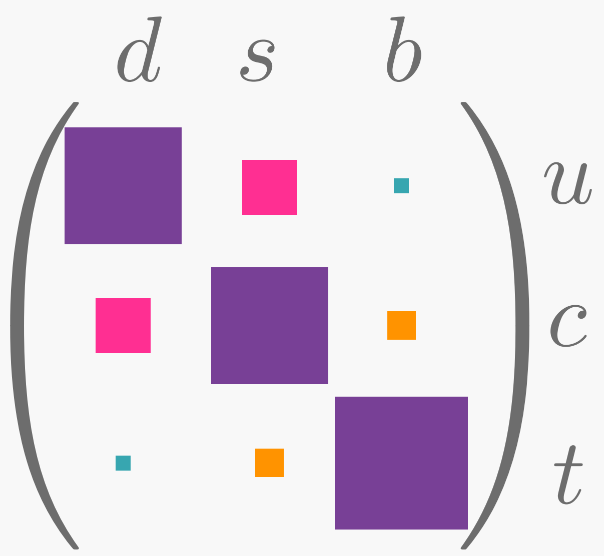
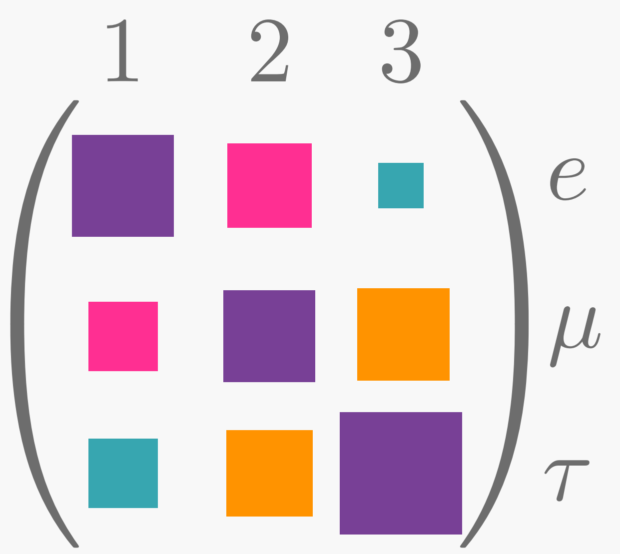


$U_{\text{PMNS}} =$


dissimilar pattern of quark and lepton mixings

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- Dark matter candidate → lecture by Károly Seller
- Flavor puzzle
- Matter-antimatter asymmetry of the Universe (Baryogenesis through Leptogenesis)

  → lecture by Zsolt Szép

Neutrino Oscillations: Introduction

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If neutrinos are massive, it will be a misalignment between the mass and flavor eigenstates

$$\nu_\alpha = U_{\alpha i}\nu_i$$

mass neutrino
(propagation)
$i = 1, 2, 3$

flavor neutrino
(production/detection)
$\alpha = e, \mu, \tau$

Neutrino Oscillations: Introduction

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$U_{\alpha i}$ unitary leptonic mixing matrix that diagonalizes the ν mass matrix.
Equivalent to the CKM quark mixing matrix.

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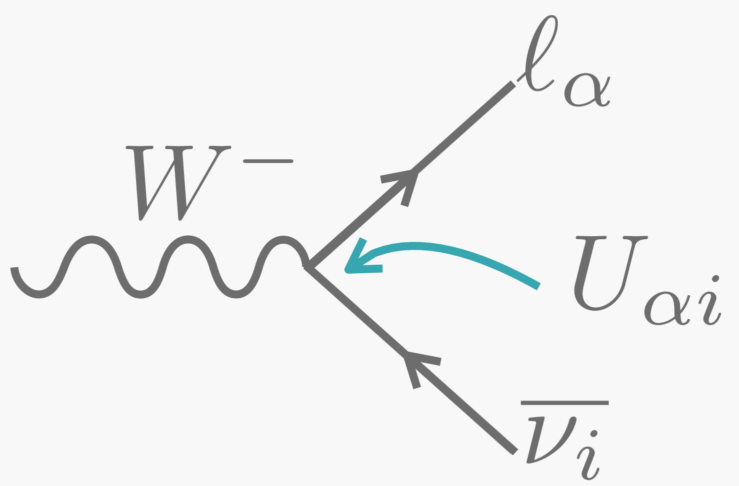
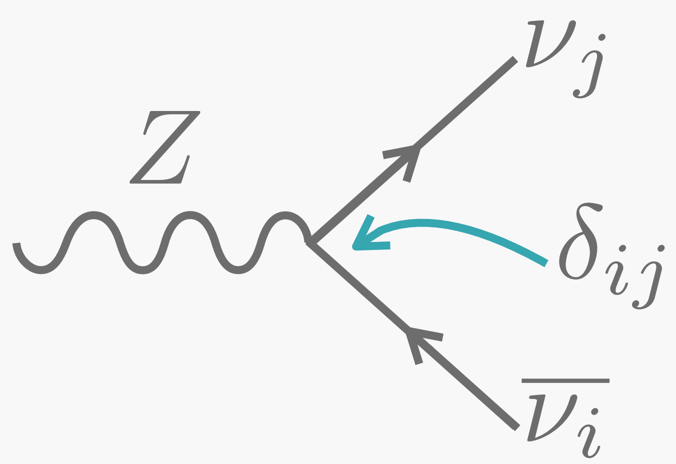
Equivalent to the CKM quark mixing matrix.
It appears in the leptonic charged current (CC) interactions

$$\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}} \overline{\ell_\alpha} \gamma^\mu P_L U_{\alpha i} \nu_i W^-_\mu + \text{h.c.} \qquad \mathcal{L}_{\rm NC} = \frac{g}{2 c_{\rm W}} \left(\overline{\nu_i} \gamma^\mu P_L \nu_i - \overline{\ell_\alpha} \gamma^\mu P_L \left(1 - 2 s^2_{\rm W}\right) \ell_\alpha \right) Z_\mu$$





Neutrino Oscillations: Introduction

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A $n \times n$ unitary matrix is parametrized by

$$\frac{n}{2}(n-1) \quad \text{angles} \qquad\qquad \frac{n}{2}(n+1) \quad \text{phases}$$

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But phases can be reabsorbed in field redefinitions that leave the Lagrangian invariant

$$\begin{aligned} \ell_\alpha &\to \ell'_\alpha e^{i\theta_\alpha} \\ \nu_i &\to \nu'_i e^{i\theta_i} \end{aligned} \quad \Rightarrow$$

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n phases absorbed in the charged leptons

$n-1$ phases absorbed in the neutrinos

$$\frac{n}{2}(n+1) - 2n + 1 \quad \text{physical phases}$$

Neutrino Oscillations: Introduction

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However, if neutrinos are Majorana particles → lecture by Timo Kärkkäinen

- Dirac ν mass

  $m_D\left(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L\right)$

- Majorana ν mass

  $m_M \overline{\nu_L^c} \nu_L$ with $\nu^c = i\gamma_0\gamma_2\overline{\nu_L}^t \equiv C\overline{\nu_L}^t$

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  $e^{i\theta}$ $e^{i\theta}$

  not invariant under phase redefinition

  $(n-1)$ additional phases become physical (Majorana phases)

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Summary

	$\frac{n}{2}(n-1)$ angles	$\frac{n}{2}(n+1)-2n+1$ Dirac phases	$(n-1)$ Majorana phases
$n=2$	1	0	1
$n=3$	3	1	2

Neutrino Oscillations: 2 family approx. Vacuum

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Let's assume two families: $\alpha = e, \mu$ and $i = 1, 2$

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \quad \Rightarrow \quad \begin{aligned} |\nu_e\rangle &= \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle \\ |\nu_\mu\rangle &= -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle \end{aligned}$$

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Imagine that I produce a ν_μ from π decay

$$\pi^+ \to \mu^+ + \nu_\mu$$

If I try to detect it as ν_e

$$\langle \nu_e|\nu_\mu\rangle = -sc\,\langle \nu_1|\nu_1\rangle + sc\,\langle \nu_2|\nu_2\rangle = 0 \qquad s \equiv \sin\theta \quad c \equiv \cos\theta$$

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The flavor neutrinos (ν_e and ν_μ) are produced and detected in CC interactions;

but the mass neutrinos (ν_1 and ν_2) are eigenstates of the free Hamiltonian

$$H\,|\nu_i\rangle = E_i\,|\nu_i\rangle = \sqrt{p_i^2 + m_i^2}\,|\nu_i\rangle$$

Neutrino Oscillations: 2 family approx. Vacuum

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Therefore, if I let ν_μ propagate a distance (baseline) L

$$|\nu_\mu(t)\rangle = -se^{-iE_1 t}|\nu_1\rangle + ce^{-iE_2 t}|\nu_2\rangle$$

$$\simeq -se^{-i\sqrt{p^2+m_1^2}t}|\nu_1\rangle + ce^{-i\sqrt{p^2+m_2^2}t}|\nu_2\rangle$$

same momentum approx (coherence)

H. J. Lipkin, Phys. Lett. **B642** (2006) 366–371.

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The probability of detecting ν_μ as ν_e

$$P_{\nu_\mu\to\nu_e}(t) = |\langle\nu_e|\nu_\mu(t)\rangle|^2 = \left|-sc\left(e^{-i\sqrt{p^2+m_1^2}t} - e^{-i\sqrt{p^2+m_2^2}t}\right)\right|^2$$

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Using that

$$E, p \gg m_\nu \quad \Rightarrow \quad \sqrt{p^2+m_i^2} \simeq p + \frac{m_i^2}{2p} \simeq p + \frac{m_i^2}{2E}$$

$$\text{Relativistic } \nu \quad \Rightarrow \quad t \simeq L$$

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Neutrino Oscillations: 2 family approx. Vacuum

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$$P_{\nu_\alpha \to \nu_\beta}(L) \equiv P_{\alpha\beta}(L) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \qquad \Delta m^2 \equiv m_2^2 - m_1^2$$

Neutrino oscillation happens only if $\theta \neq 0$ and $m_1 \neq m_2$.

$2\nu \Rightarrow$ no phases $\Rightarrow P_{\alpha\beta} = \overline{P_{\alpha\beta}}$. 3ν needed for $\not{C}\not{P}$.

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$$P_{\alpha\alpha} = 1 - P_{\alpha\beta}$$

Neutrino Oscillations: 2 family approx. Vacuum

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$P_{\alpha\beta}$: appearance channel

$P_{\alpha\alpha}$: disappearance or survival channel

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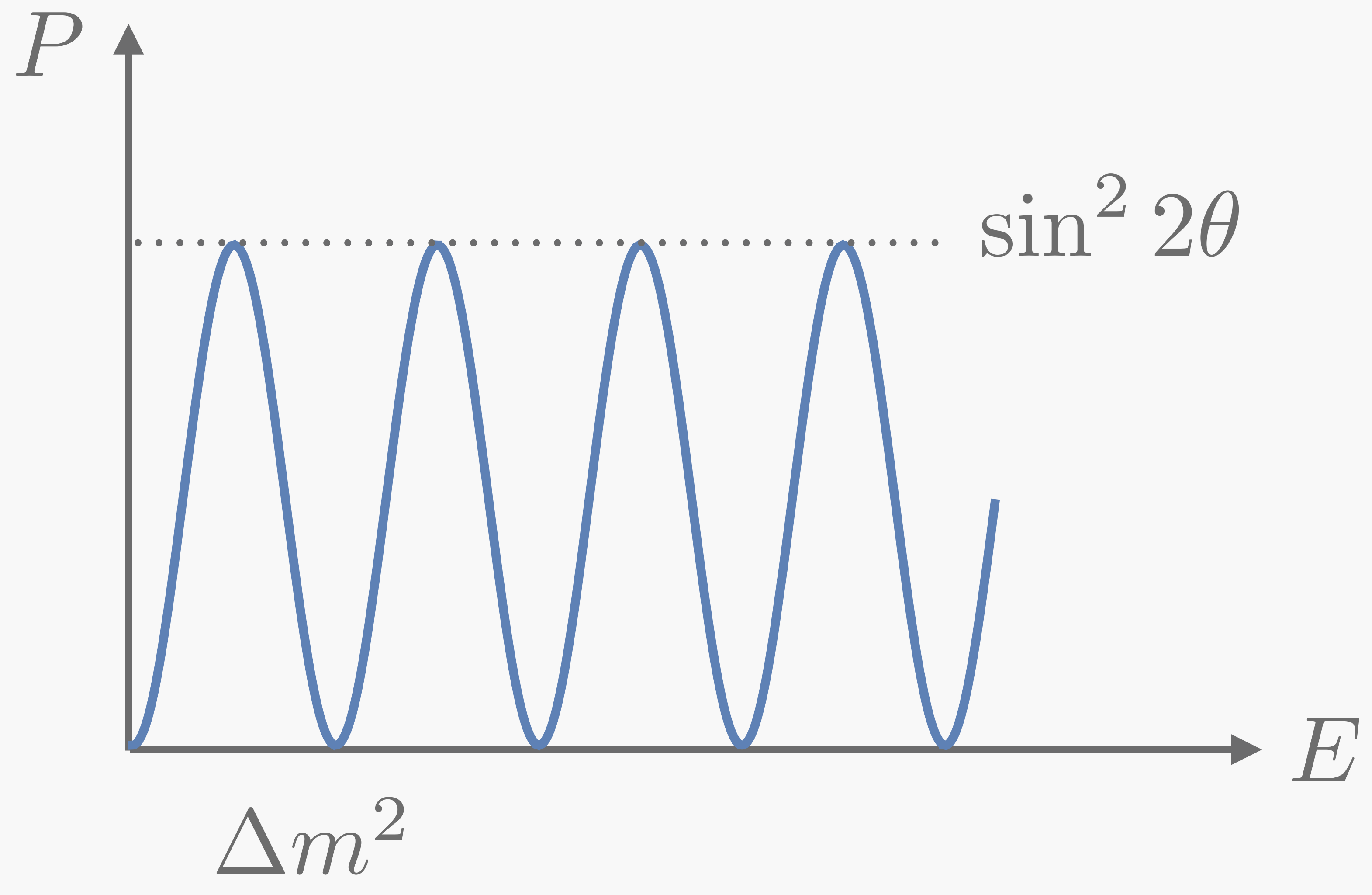

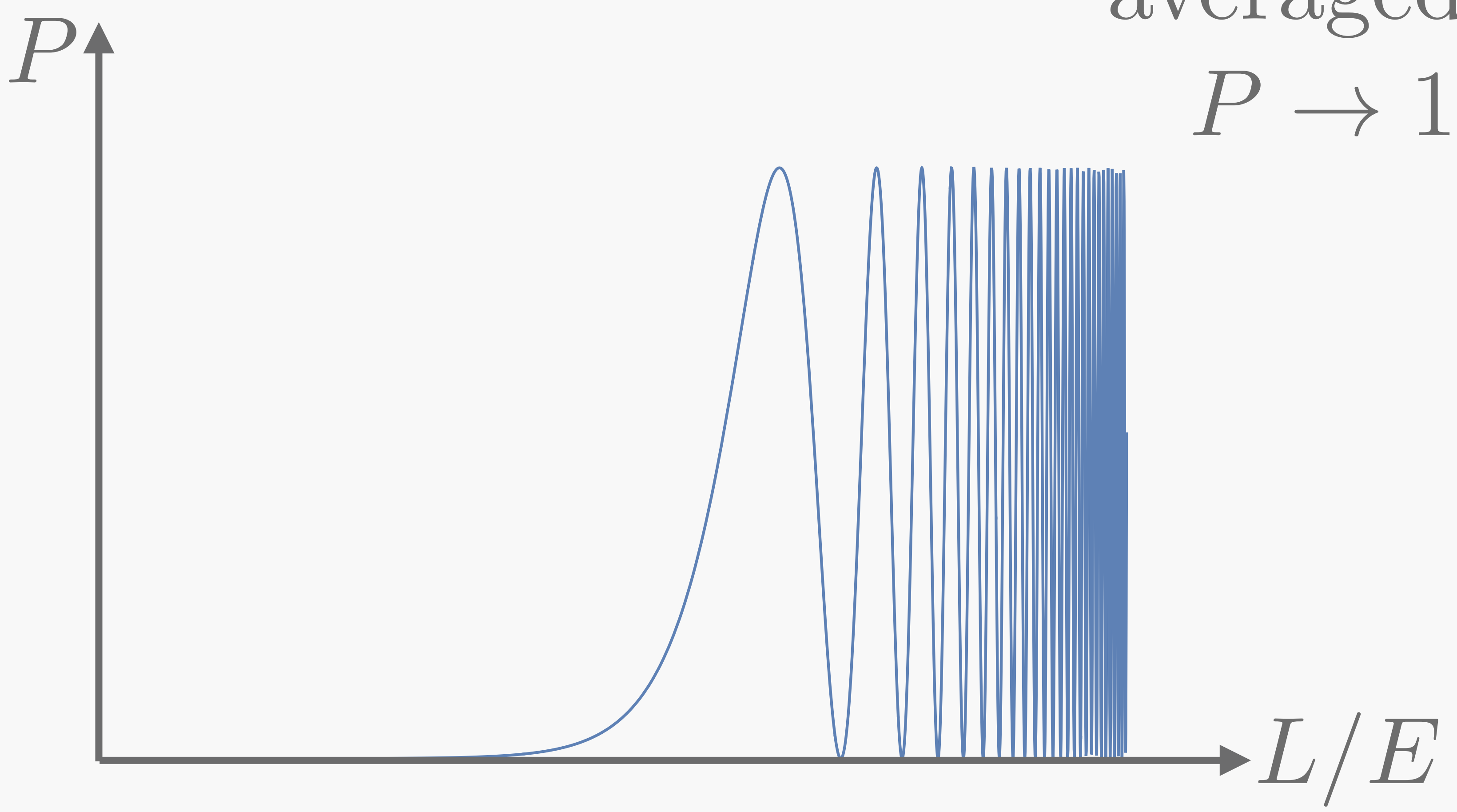

averaged-out

$P \to 1/2$

first oscillation maximum

$$\frac{\Delta m^2 L}{4E} = 1.27 \frac{\Delta m^2}{\mathrm{eV}^2} \frac{L}{\mathrm{km}} \frac{\mathrm{GeV}}{E}$$

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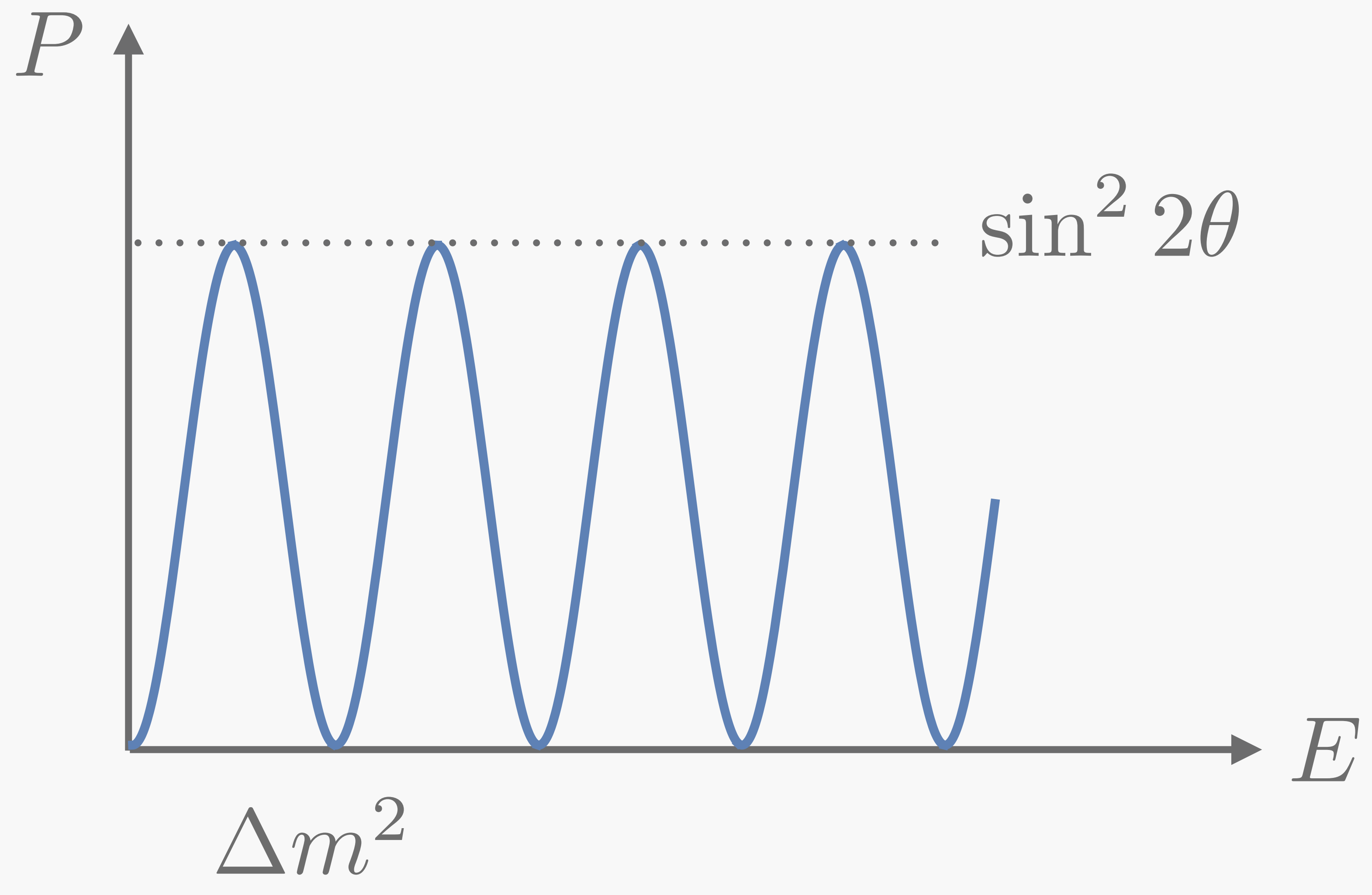
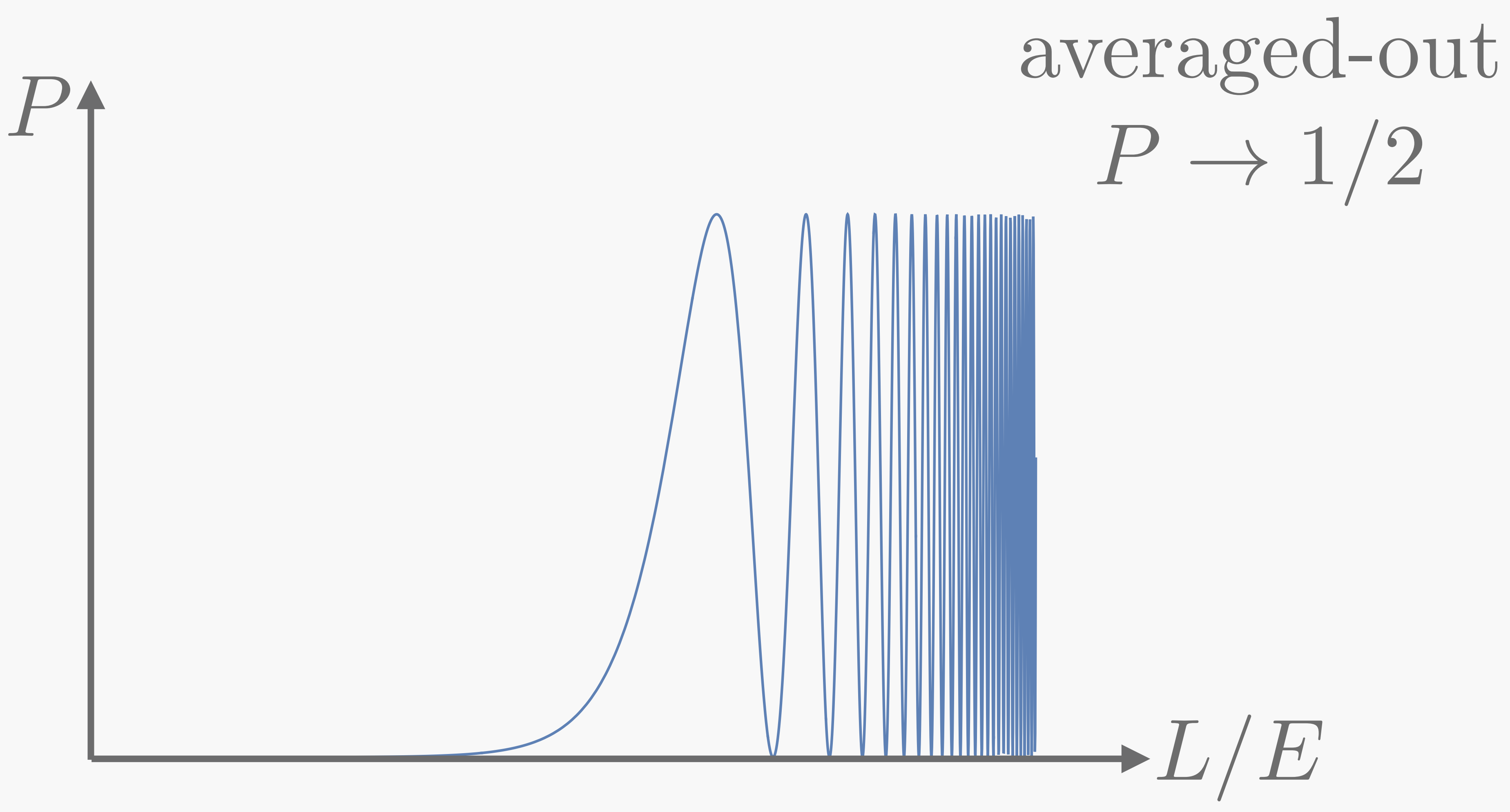
no sensitivity to sign of Δm^2





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Neutrino Oscillations: 2 family approx. Matter

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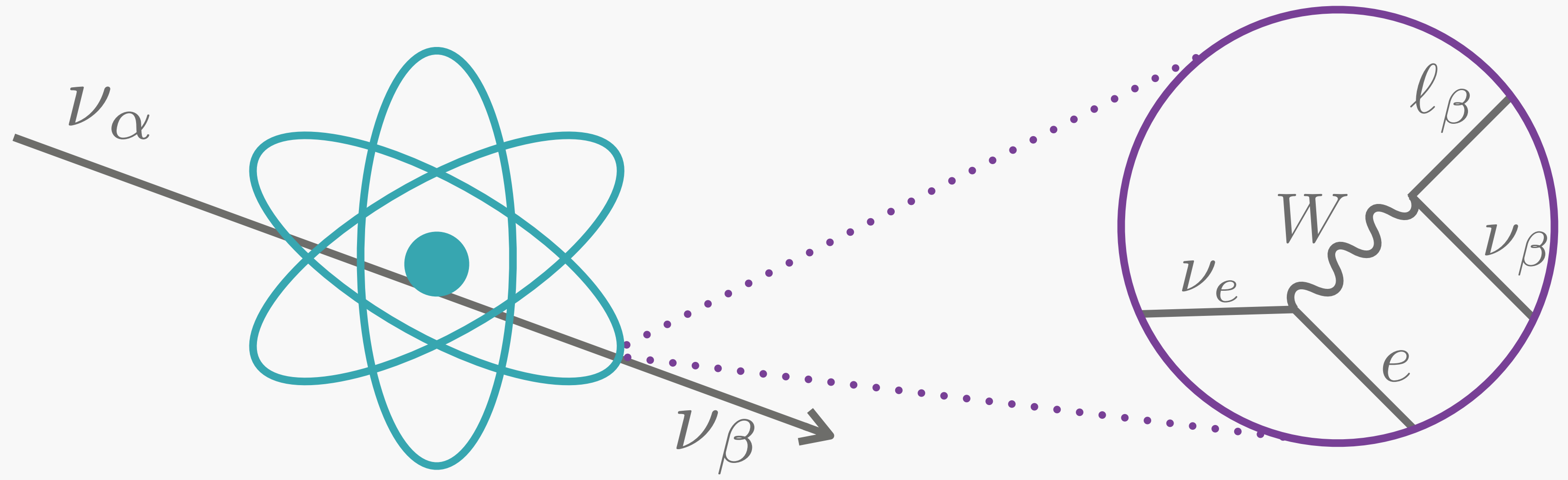
The sensitivity to the sign of Δm^2 comes from matter effects.

- Coherent scattering of ν_e with e via W exchange



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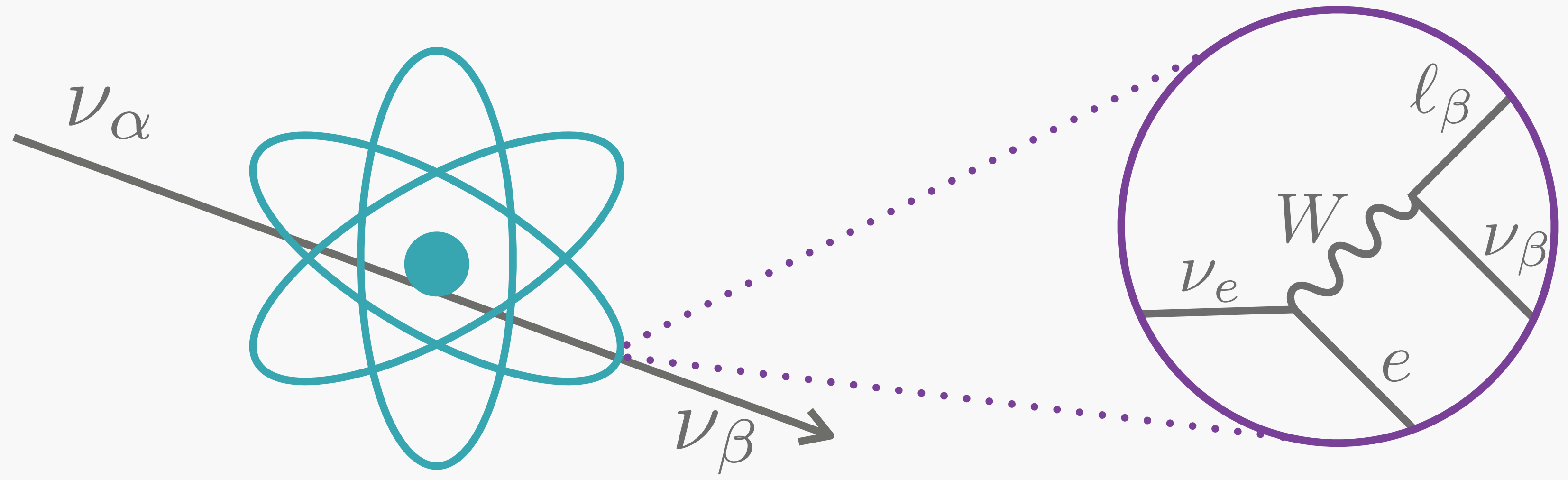

$$V_{\rm CC} = \pm\sqrt{2} G_F n_e$$

G_F : Fermi constant

n_e : electron density

$$V_{\rm eff} = \begin{pmatrix} V_{\rm CC} & 0 \\ 0 & 0 \end{pmatrix}$$

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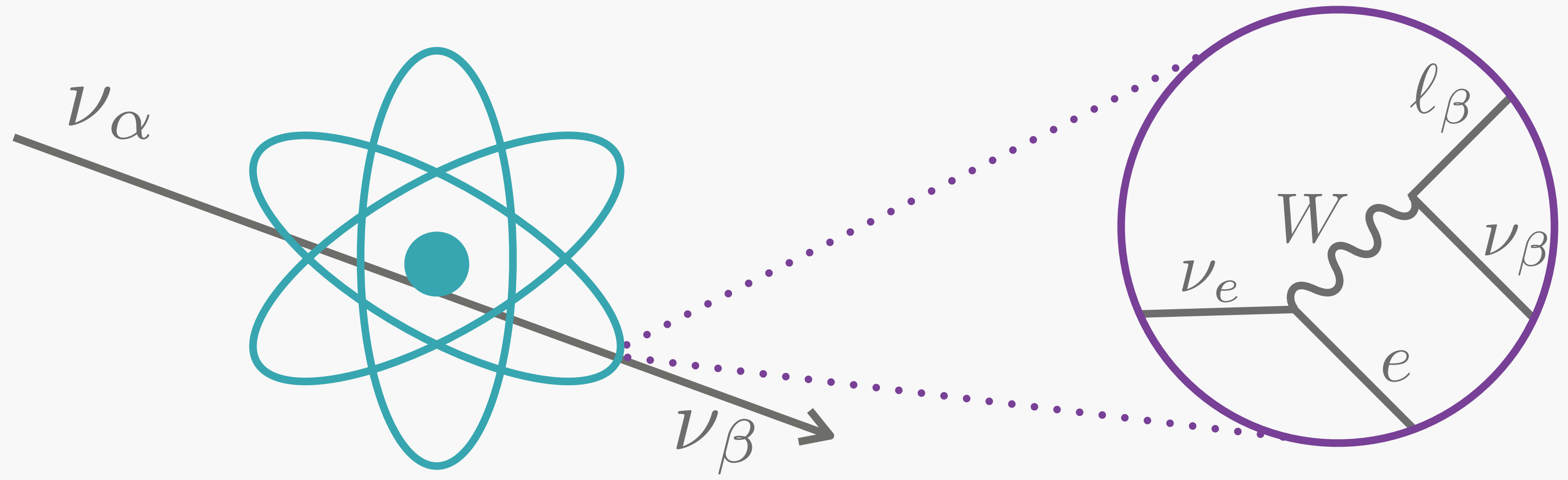

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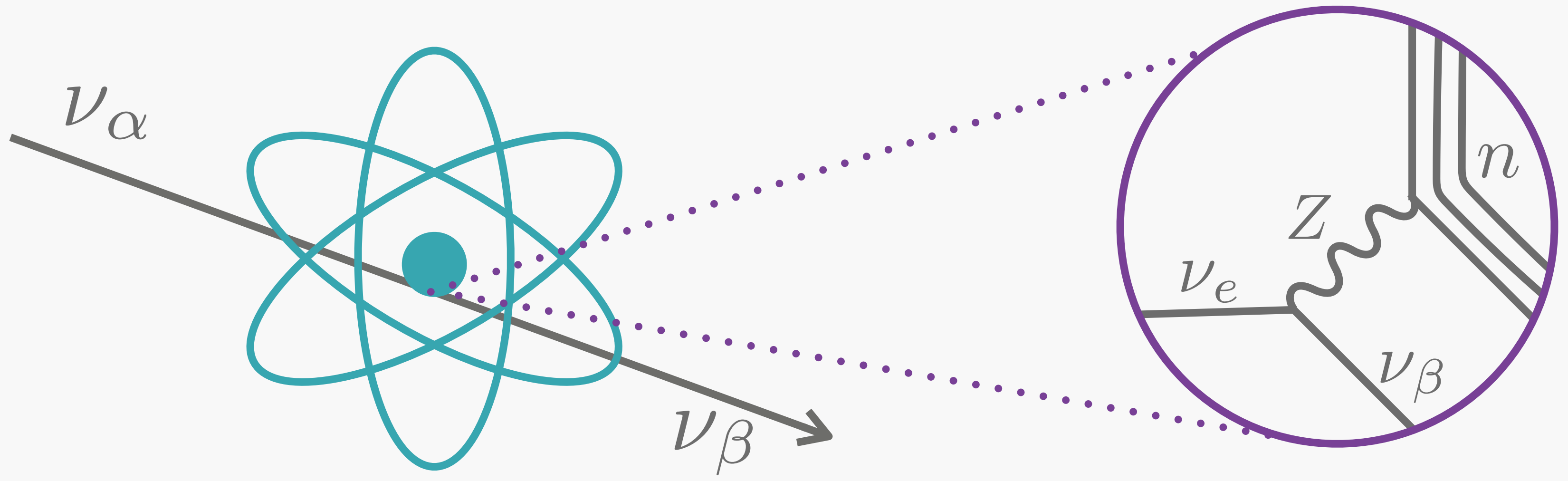
- NC interaction of ν_α with n via Z exchange



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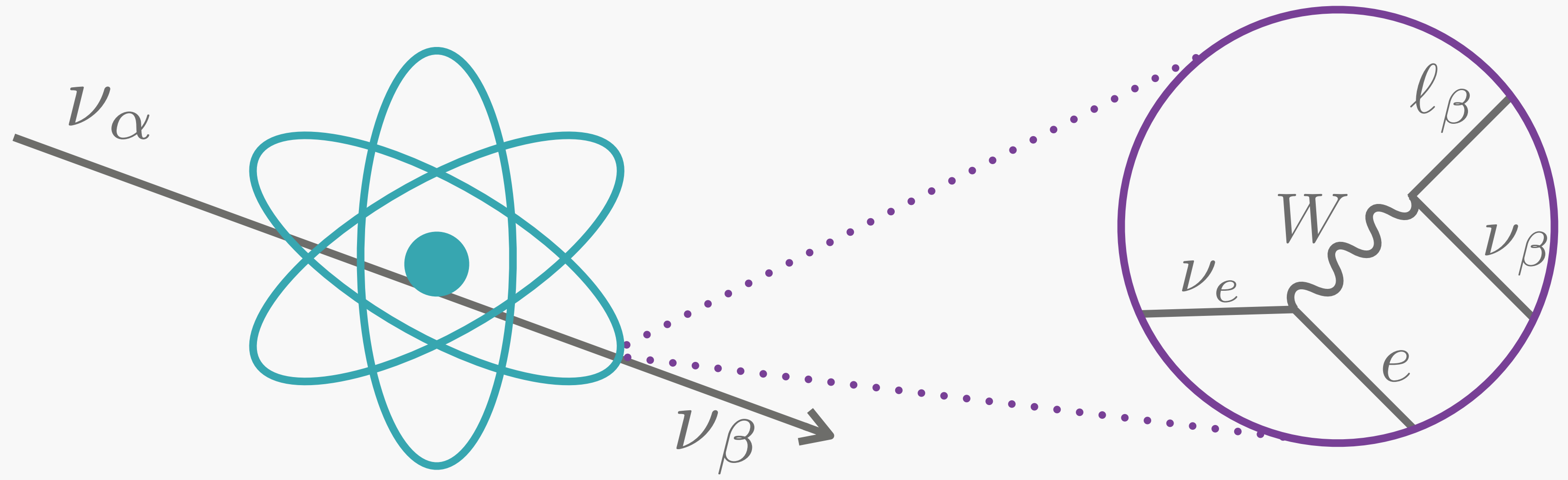

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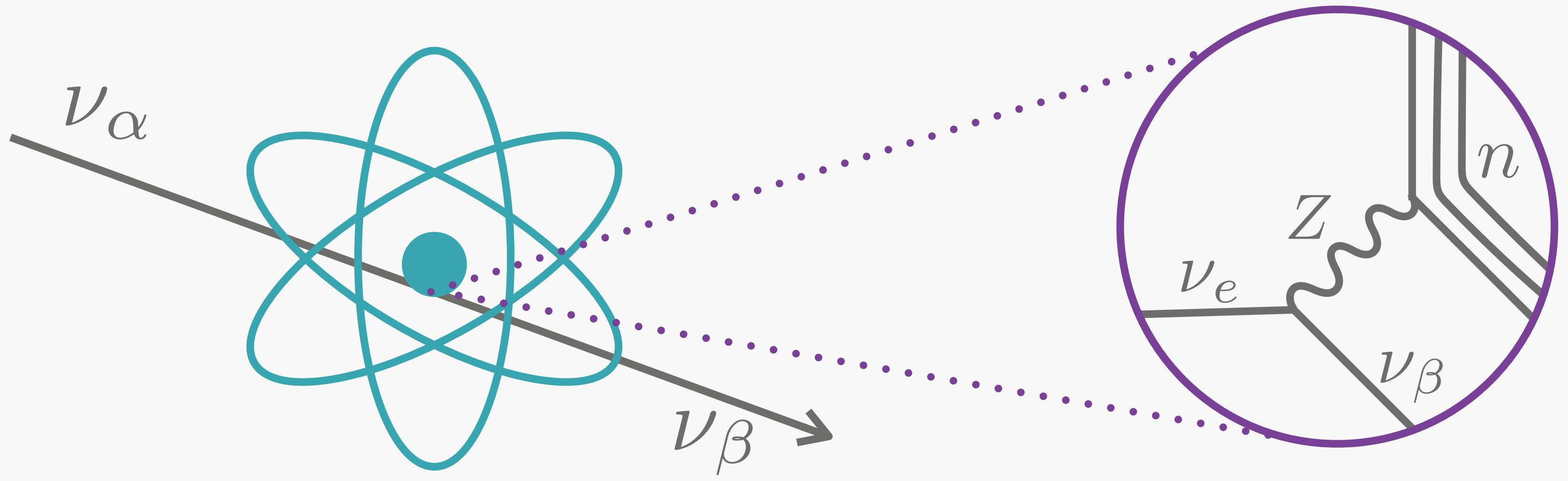

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$$V_{\text{eff}} = \begin{pmatrix} V_{\text{CC}} + V_{\text{NC}} & 0 \\ 0 & V_{\text{NC}} \end{pmatrix}$$

$$V_{\text{NC}} = \mp \frac{1}{\sqrt{2}} G_F n_n$$

for neutral matter

n_n : neutron density

Neutrino Oscillations: 2 family approx. Matter

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The Schrödinger equation will be modified

$$i\frac{d}{dt}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = H^{\mathrm{m}} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

$$H^{\mathrm{m}} = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} p + \frac{m_1^2}{2E} & 0 \\ 0 & p + \frac{m_2^2}{2E} \end{pmatrix} \begin{pmatrix} c & -s \\ s & c \end{pmatrix} + \begin{pmatrix} V_{\mathrm{CC}} + V_{\mathrm{NC}} & 0 \\ 0 & V_{\mathrm{NC}} \end{pmatrix}$$

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$$+ \begin{pmatrix} p + \frac{m_1^2 + m_2^2}{4E} + V_{\mathrm{NC}} & 0 \\ 0 & p + \frac{m_1^2 + m_2^2}{4E} + V_{\mathrm{NC}} \end{pmatrix}$$

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$$+ \begin{pmatrix} p + \frac{m_1^2 + m_2^2}{4E} + V_{\mathrm{NC}} & 0 \\ 0 & p + \frac{m_1^2 + m_2^2}{4E} + V_{\mathrm{NC}} \end{pmatrix}$$

global phase ⇒ cancels when computing P

Neutrino Oscillations: 2 family approx. Matter

zoom.us video

The Schrödinger equation will be modified

$$i\frac{d}{dt}\begin{pmatrix}|\nu_e\rangle\\|\nu_\mu\rangle\end{pmatrix}=H^{\mathrm{m}}\begin{pmatrix}|\nu_e\rangle\\|\nu_\mu\rangle\end{pmatrix}$$

$$H^{\mathrm{m}}=\begin{pmatrix}c & s\\-s & c\end{pmatrix}\begin{pmatrix}p+\frac{m_1^2}{2E} & 0\\0 & p+\frac{m_2^2}{2E}\end{pmatrix}\begin{pmatrix}c & -s\\s & c\end{pmatrix}+\begin{pmatrix}V_{\mathrm{CC}}+V_{\mathrm{NC}} & 0\\0 & V_{\mathrm{NC}}\end{pmatrix}$$

$$=\begin{pmatrix}c & s\\-s & c\end{pmatrix}\begin{pmatrix}\frac{m_1^2-m_2^2}{4E} & 0\\0 & \frac{m_2^2-m_1^2}{4E}\end{pmatrix}\begin{pmatrix}c & -s\\s & c\end{pmatrix}+\begin{pmatrix}V_{\mathrm{CC}} & 0\\0 & 0\end{pmatrix}$$

$$+\begin{pmatrix}p+\frac{m_1^2+m_2^2}{4E}+V_{\mathrm{NC}} & 0\\0 & p+\frac{m_1^2+m_2^2}{4E}+V_{\mathrm{NC}}\end{pmatrix}$$

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$-\Delta$ $\qquad$ Δ

Neutrino Oscillations: 2 family approx. Matter

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Thus, we have

$$i\frac{d}{dt}\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \left[\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}\Delta \pm \begin{pmatrix} V_{\mathrm{CC}} & 0 \\ 0 & 0 \end{pmatrix}\right]\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}$$

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$$V_{\mathrm{CC}} = \sqrt{2}G_F n_e(t)$$

Depending on the matter potential, it can be very difficult to solve.

We can focus on two simple and useful cases:

- $n_e(t) =$ constant
- $n_e(t)$ changes very slowly (adiabatically)

Neutrino Oscillations: 2 family approx. Matter

zoom.us video

- $n_e(t) = \text{constant}$

Can be a reasonable approximation inside the Earth.

If $n_e = \text{constant}$, H^{m} can be diagonalized with $U = \begin{pmatrix} \cos\theta_{\text{m}} & \sin\theta_{\text{m}} \\ -\sin\theta_{\text{m}} & \cos\theta_{\text{m}} \end{pmatrix}$ and

$$\sin^2 2\theta_{\text{m}} = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta \mp V_{\text{CC}}/2\Delta)^2}$$

$$\Delta m^2_{\text{m}} = \Delta m^2 \sqrt{\sin^2 2\theta + (\cos 2\theta \mp V_{\text{CC}}/2\Delta)^2}$$

$$P^{\text{m}}_{\alpha\beta} = \delta_{\alpha\beta} - \sin^2 2\theta_{\text{m}} \sin^2\left(\frac{\Delta m^2_{\text{m}} L}{4E}\right)$$

Neutrino Oscillations: 2 family approx. Matter

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$$P^{\rm m}_{\alpha\beta} = \delta_{\alpha\beta} - \sin^2 2\theta_{\rm m} \sin^2\left(\frac{\Delta m^2_{\rm m} L}{4E}\right)$$

  – Vacuum limit:

    If $V_{\rm CC} \ll 2\Delta \cos 2\theta \Rightarrow \theta_{\rm m} \simeq \theta$. Vacuum solution recovered.

Neutrino Oscillations: 2 family approx. Matter

zoom.us video

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  - Vacuum limit:

    If $V_{\text{CC}} \ll 2\Delta\cos 2\theta \Rightarrow \theta_{\text{m}} \simeq \theta$. Vacuum solution recovered.

  - Matter domination limit:

    If $V_{\text{CC}} \gg 2\Delta\cos 2\theta \Rightarrow$ matter effects dominate and the transition probability is suppressed. The system evolves to the initial flavor.

Neutrino Oscillations: 2 family approx. Matter

zoom.us video

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  - Resonance:

    If $V_{\mathrm{CC}} = 2\Delta\cos 2\theta \Rightarrow \sin^2 2\theta_{\mathrm{m}} = 1 \Rightarrow$ maximal oscillation $\theta_{\mathrm{m}} = \pi/4$ inside matter even if $\theta \ll 1$. Mikheyev-Smirnov-Wolfenstein (MSW) resonance.

Neutrino Oscillations: 2 family approx. Matter

zoom.us video

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$$\rightarrow \quad P^{\text{m}}_{\alpha\beta} = \delta_{\alpha\beta} - \sin^2 2\theta_{\text{m}} \sin^2\left(\frac{\Delta m^2_{\text{m}} L}{4E}\right)$$

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    Resonance if $\begin{array}{l} \Delta m^2 > 0 \text{ and } V_{\text{CC}} > 0 \ (\nu) \\ \Delta m^2 < 0 \text{ and } V_{\text{CC}} < 0 \ (\overline{\nu}) \end{array} \Rightarrow \begin{array}{l} \text{if } P_{\alpha\beta} \text{ enhanced over } \overline{P_{\alpha\beta}} \Rightarrow \Delta m^2 > 0 \\ \text{if } \overline{P_{\alpha\beta}} \text{ enhanced over } P_{\alpha\beta} \Rightarrow \Delta m^2 < 0 \end{array}$

Neutrino Oscillations: 2 family approx. Matter

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- $n_e\,(t)$ changes very slowly (adiabatically)

  Good approximation for neutrinos produced in the Sun.

  If the matter potential changes very slowly

  $$\left|\frac{dV_{\mathrm{CC}}}{dt}\right| \ll |E_2 - E_1| \qquad \text{(adiabatic condition)}$$

  the ν has time to adapt to the change of the potential, and the solution at time t

  $$|\nu(t)\rangle = \alpha\,|\nu_1(t)\rangle + \beta\,|\nu_1(t)\rangle \qquad \text{with} \qquad H(t)\,|\nu_i(t)\rangle = E_i(t)\,|\nu_i(t)\rangle$$

Neutrino Oscillations: 2 family approx. Matter

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  In the center of the Sun, ν_e are produced via nuclear fusion and $n_e \gg \Rightarrow V_{\rm CC} \gg \Delta \Rightarrow$

  $$H(t=0) \simeq \begin{pmatrix} V_{\rm CC} & 0 \\ 0 & 0 \end{pmatrix} \quad \Rightarrow \quad |\nu_e\rangle \text{ is an eigenstate of H!}$$

  $\Rightarrow |\nu_e(t \gg)\rangle$ out of the Sun must also be an eigenstate.

Neutrino Oscillations: 2 family approx. Matter

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- $n_e\,(t)$ changes very slowly (adiabatically)

  Since $|\nu_e(t=0)\rangle$ is the ν with the largest eigenvalue $\Rightarrow$

  $|\nu_e\rangle$ emerges as the ν with the largest eigenvalue in vacuum: $|\nu_2\rangle$

$$|\nu_e\rangle \xrightarrow[\text{conversion}]{\text{adiabatic}} |\nu_2\rangle$$

  The Sun produces $|\nu_2\rangle$. It is not really an oscillation.

Neutrino Oscillations: 3 families

zoom.us video

We generalize for 3 families

$$|\nu_\alpha\rangle = U^*_{\alpha i}\,|\nu_i\rangle \quad \Rightarrow \quad |\nu_\alpha(L)\rangle = e^{-ipL}e^{-i\frac{m_i^2 L}{2E}}U^*_{\alpha i}\,|\nu_i\rangle \quad \Rightarrow \quad \langle\nu_\beta|\nu_\alpha(L)\rangle = e^{-ipL}e^{-i\frac{m_i^2 L}{2E}}U_{\beta i}U^*_{\alpha i}$$

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Thus, we have

$$\begin{aligned} P_{\alpha\beta}\left(\overline{P_{\alpha\beta}}\right) &= \sum_{i,j} U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}e^{i\frac{\Delta m^2_{ij}L}{2E}} \\ &= \delta_{\alpha\beta} - 4\sum_{i>j}\mathrm{Re}\left[U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right]\sin^2\left(\Delta_{ij}\right) \\ &\quad + (-)\,2\sum_{i>j}\mathrm{Im}\left[U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right]\sin\left(2\Delta_{ij}\right) \end{aligned}$$

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$$P_{\alpha\beta}\left(\overline{P_{\alpha\beta}}\right) = \sum_{i,j} U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}e^{i\frac{\Delta m_{ij}^2 L}{2E}}$$

$$\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$$

$$= \delta_{\alpha\beta} - 4\sum_{i>j}\mathrm{Re}\left[U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right]\sin^2\left(\Delta_{ij}\right)$$

$$\Delta_{ij} \equiv \frac{\Delta m_{ij}^2 L}{4E}$$

$$+(-)\,2\sum_{i>j}\mathrm{Im}\left[U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right]\sin\left(2\Delta_{ij}\right)$$

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$\not{C}\not{P}$ Jarlskog invariant (measure of CPV in ν) "GIM" cancellation

Neutrino Oscillations: 3 families

zoom.us video

For 3 families, the unitary rotation that diagonalizes the mass matrix is given by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix.

Neutrino Oscillations: 3 families

zoom.us video

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3 families ⇒ parametrized by 3 angles θ_{ij}, 1 Dirac phase δ, and 2 Majorana phases α_i

$$U_{\mathrm{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & e^{-i\frac{\alpha_3}{2}} \end{pmatrix}$$

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"atmospheric" angle

$\theta_{23} \simeq 45^{\mathrm{o}}$

NEUTRINO OSCILLATIONS: 3 FAMILIES

zoom.us video

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"atmospheric" angle
$\theta_{23} \simeq 45^{\mathrm{o}}$

"reactor" angle
$\theta_{13} \simeq 8.5^{\mathrm{o}}$
δ Dirac phase

Neutrino Oscillations: 3 families

zoom.us video

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$\theta_{23} \simeq 45^{\rm o}$

"reactor" angle
$\theta_{13} \simeq 8.5^{\rm o}$
δ Dirac phase

"solar" angle
$\theta_{12} \simeq 33^{\rm o}$

Neutrino Oscillations: 3 families

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"atmospheric" angle
$\theta_{23} \simeq 45^\circ$

"reactor" angle
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$\theta_{12} \simeq 33^\circ$

Majorana phases do not participate in ν oscillations (only in $\not{L}$ processes)

Neutrino Oscillations: 3 families

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Neutrino Oscillations: 3 families

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With these values of the mixing angles, the amount of $\not{CP}$ in the leptonic sector

$$J \equiv \mathrm{Im}\left[U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right] = \frac{1}{8}\sin(2\theta_{12})\sin(2\theta_{13})\sin(2\theta_{23})\cos\theta_{13}\sin\delta$$

$\simeq 0.03\sin\delta$

$\rightarrow$ lecture by Zsolt Szép

Neutrino Oscillations: 3 families

zoom.us video

Moreover, 3 families $\Rightarrow$ 2 mass differences

$\Delta m^2_{21} \simeq 7.5 \cdot 10^{-5}\ \mathrm{eV}^2$ "solar" mass spliting $(\Delta m^2_{\mathrm{sol}})$ (> 0 known sign!)

$\left|\Delta m^2_{31}\right| \simeq 2.4 \cdot 10^{-3}\ \mathrm{eV}^2$ "atmospheric" mass spliting $(\Delta m^2_{\mathrm{atm}})$

Neutrino Oscillations: 3 families

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Since

$\theta_{13} \ll \theta_{12}, \theta_{23}$

$\Delta m^2_{21} \ll \left|\Delta m^2_{31}\right|$

→ the simple 2-family approximation recovered in almost all the regimens

$$P_{\alpha\beta} = \delta_{\alpha\beta} - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

NEUTRINO OSCILLATIONS: 3 FAMILIES

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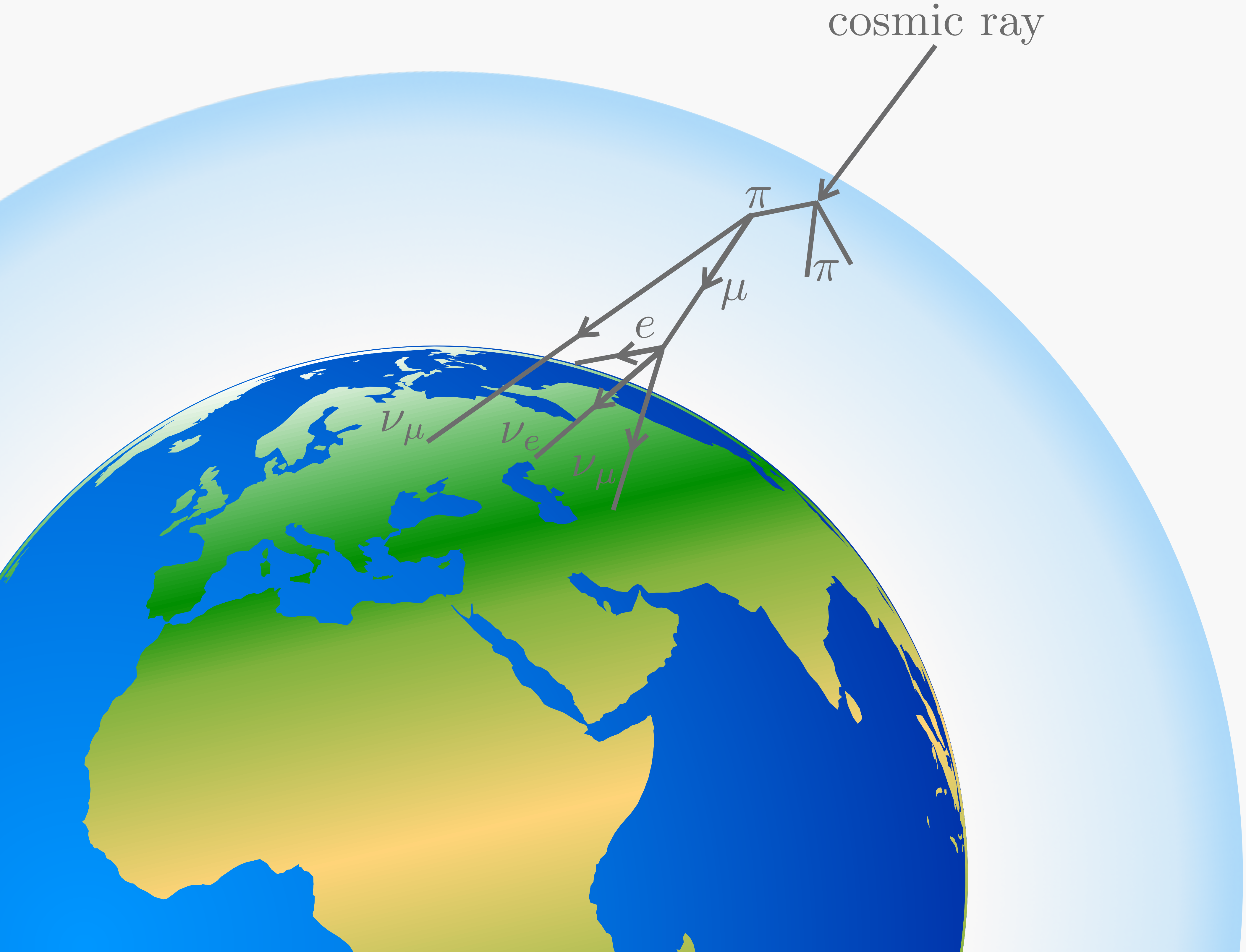
- Atmospheric regime

Atmospheric ν produced in the atmosphere by collision of high-energy cosmic rays



$$\pi \to \mu + \nu_\mu$$

$$\hookrightarrow e + \nu_e + \nu_\mu \longrightarrow \text{two } \nu_\mu \text{ per } \nu_e$$

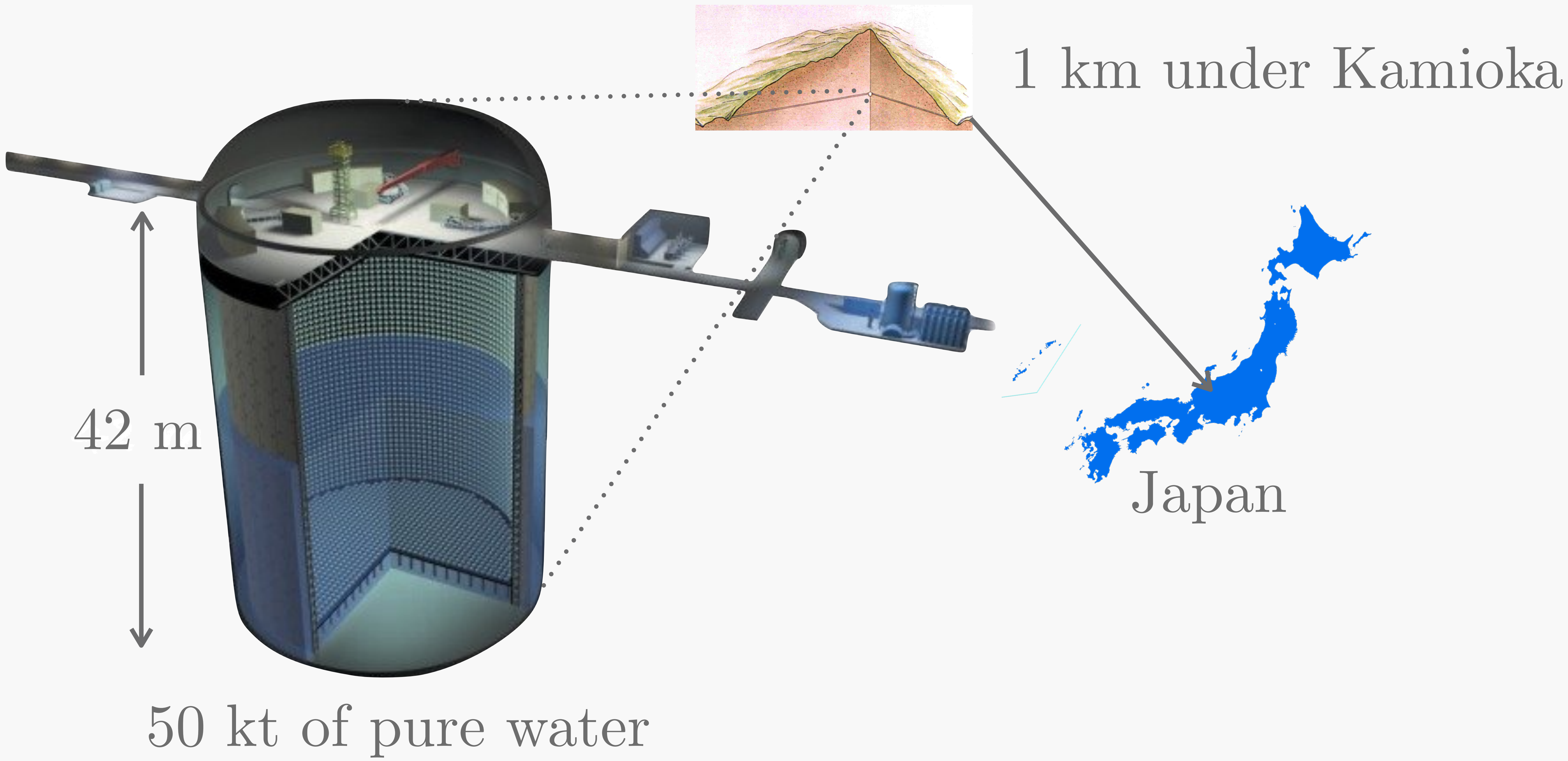
These ν_μ and ν_e are detected by SuperKamiokande (SK) in Japan.

Neutrino Oscillations: 3 families

zoom.us video

- Atmospheric regime

  SuperKamiokande is a water-Cherenkov detector.



Neutrino Oscillations: 3 families

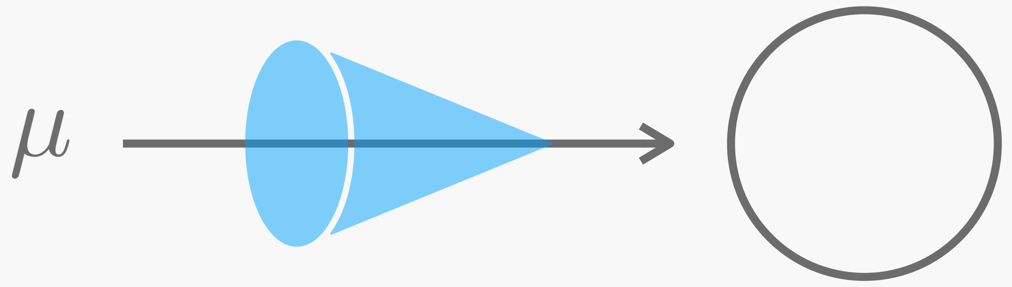
zoom.us video

- Atmospheric regime

  SuperKamiokande is a water-Cherenkov detector.

  The incoming ν_α produces a ℓ_α via CC interaction in SK

$$v_\ell \simeq c > c_{\text{water}} \Rightarrow \text{Cherenkov radiation}$$



→ well defined ring in the event display → μ-like event

Neutrino Oscillations: 3 families

zoom.us video

- Atmospheric regime

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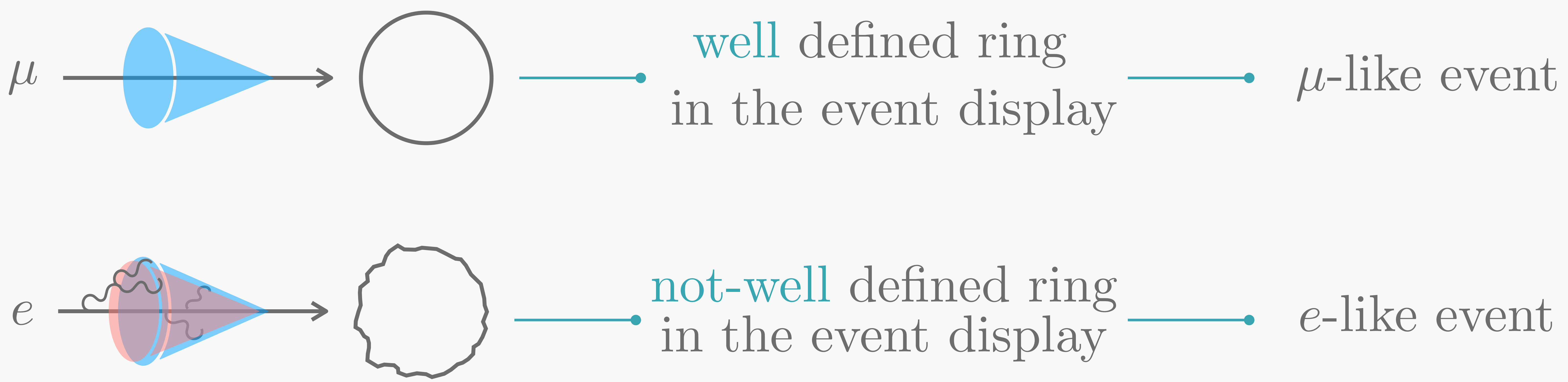

Neutrino Oscillations: 3 families

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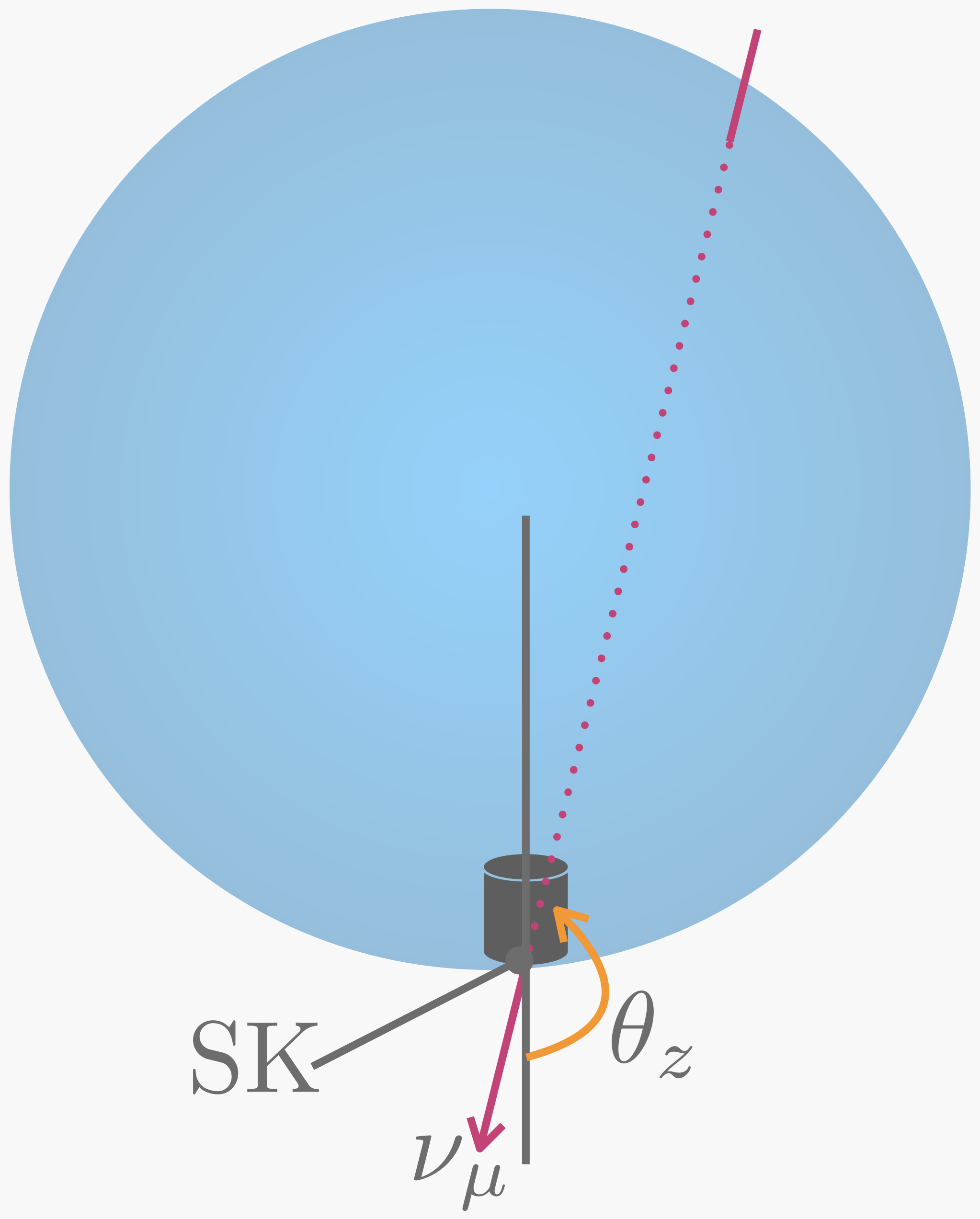
- Atmospheric regime

They count number of e-like and μ-like events as a function of zenit angle θ_z



NEUTRINO OSCILLATIONS: 3 FAMILIES

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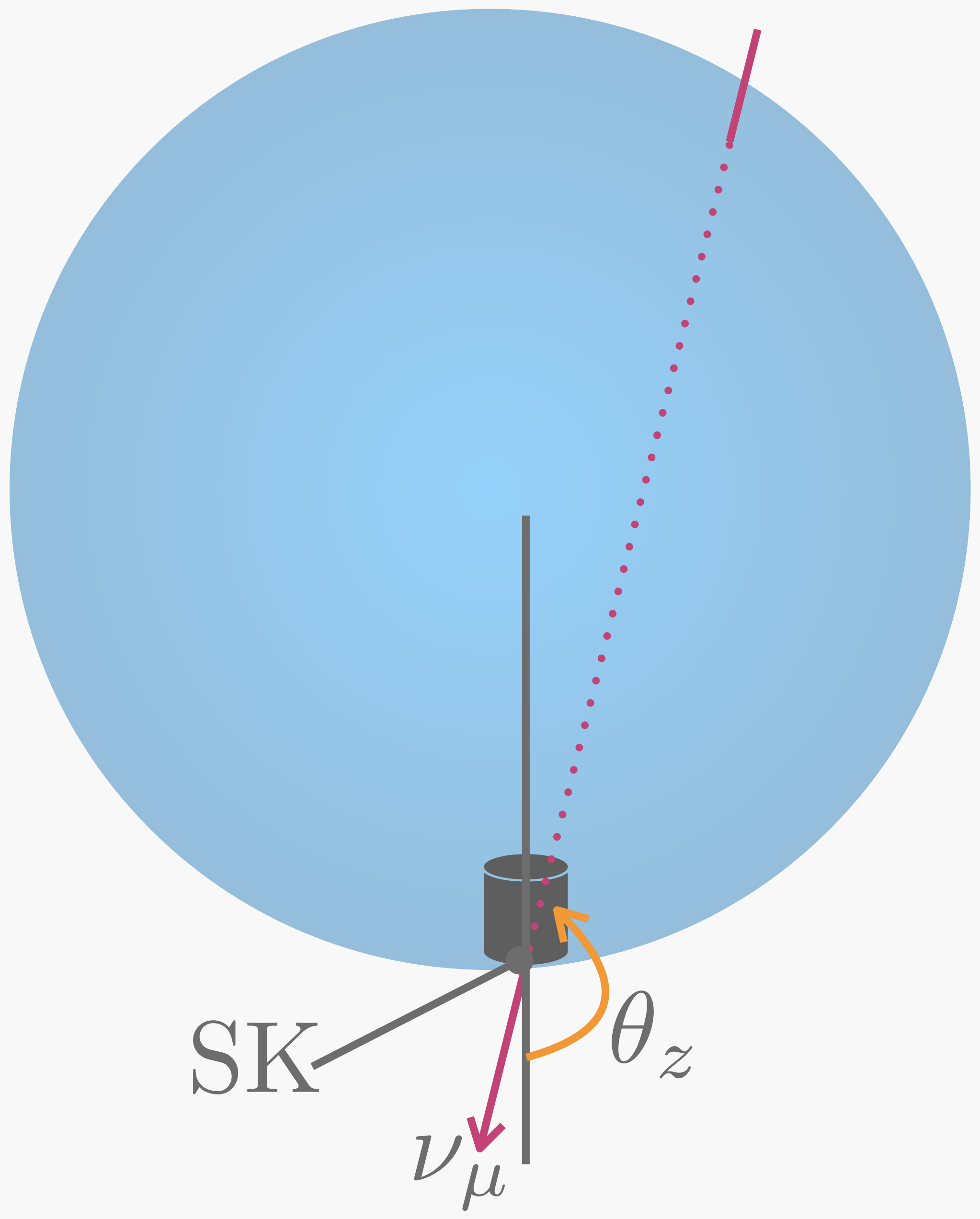
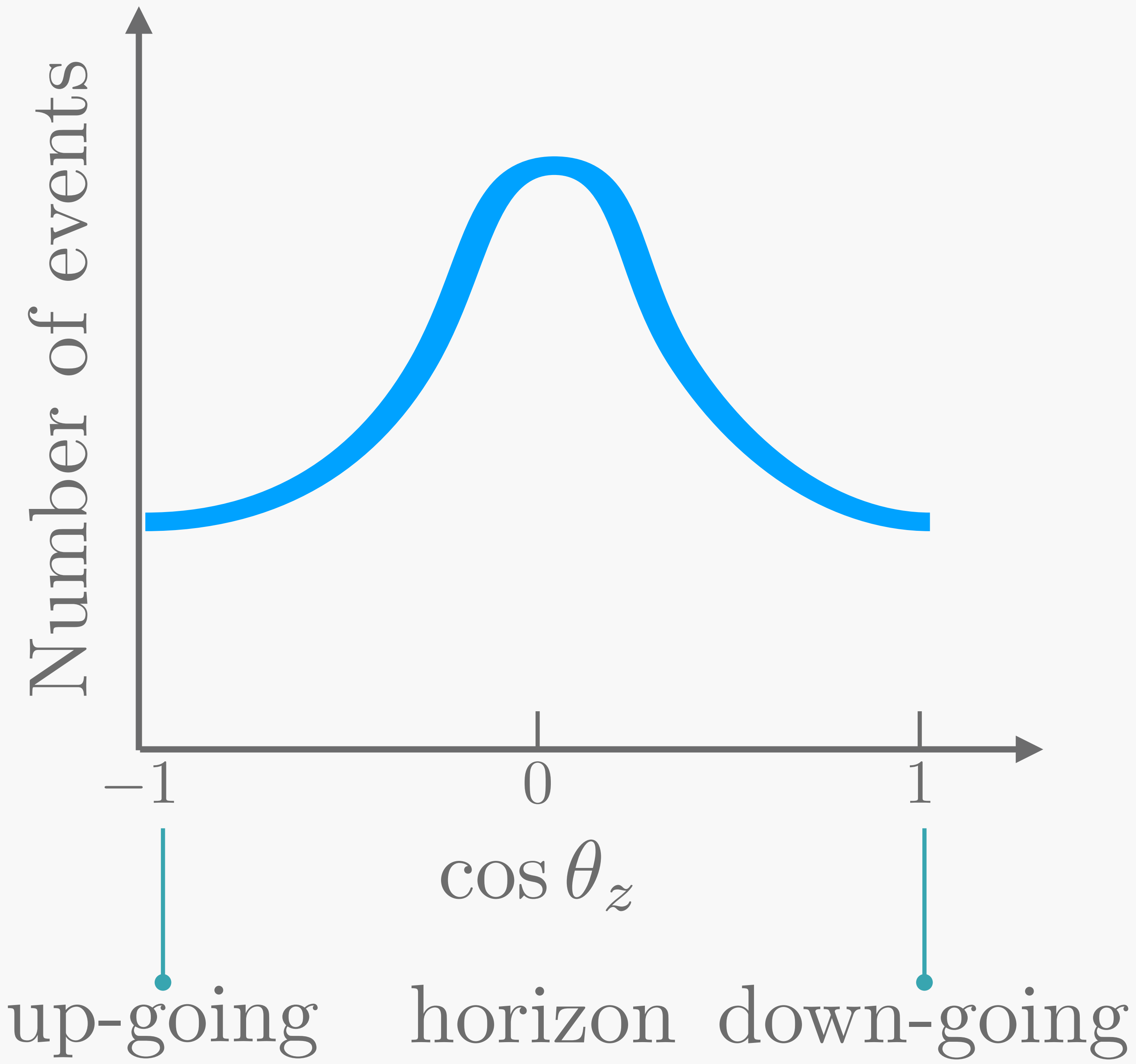




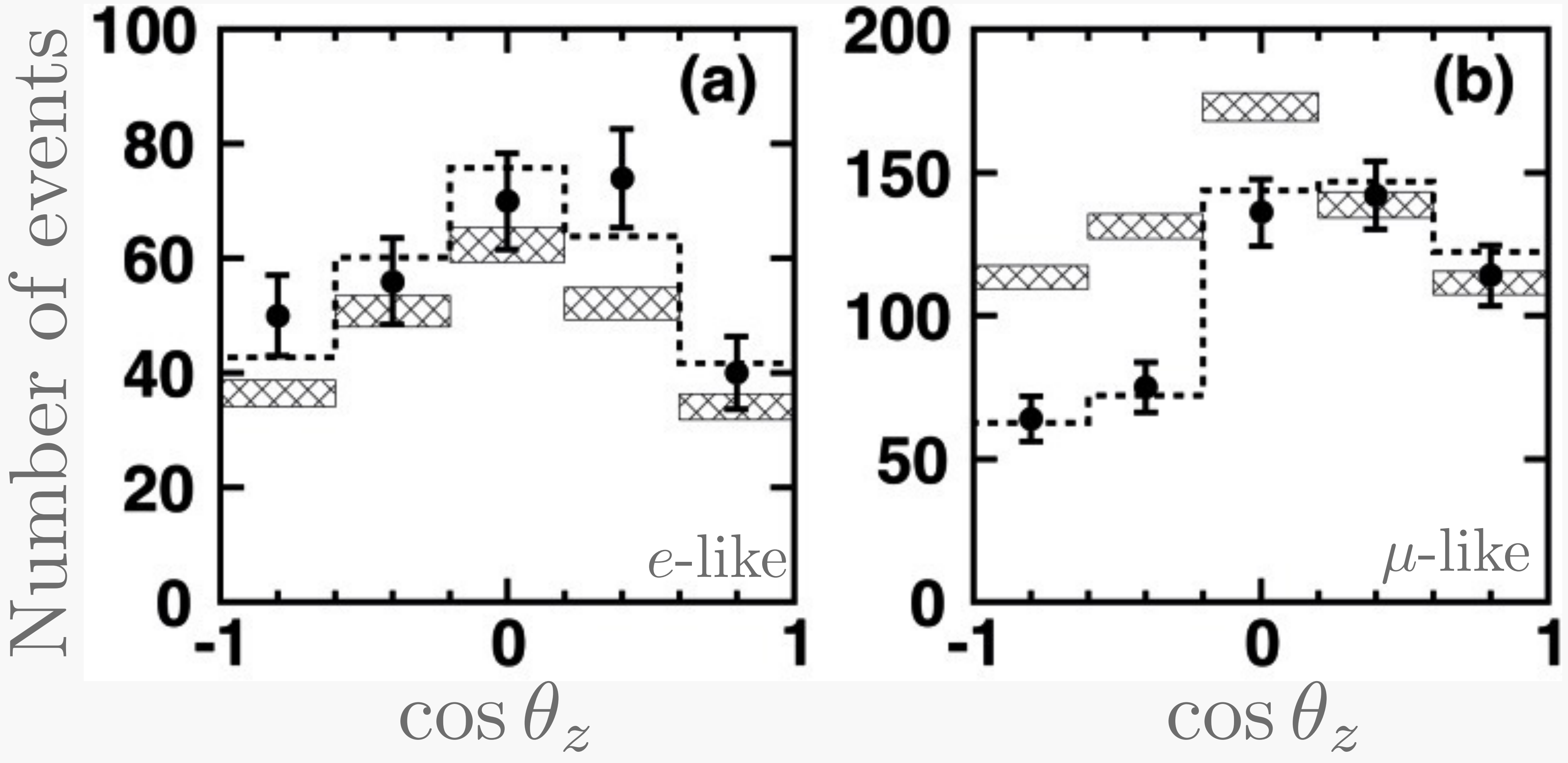
assuming no oscillations

Neutrino Oscillations: 3 families

zoom.us video

- Atmospheric regime

How it started



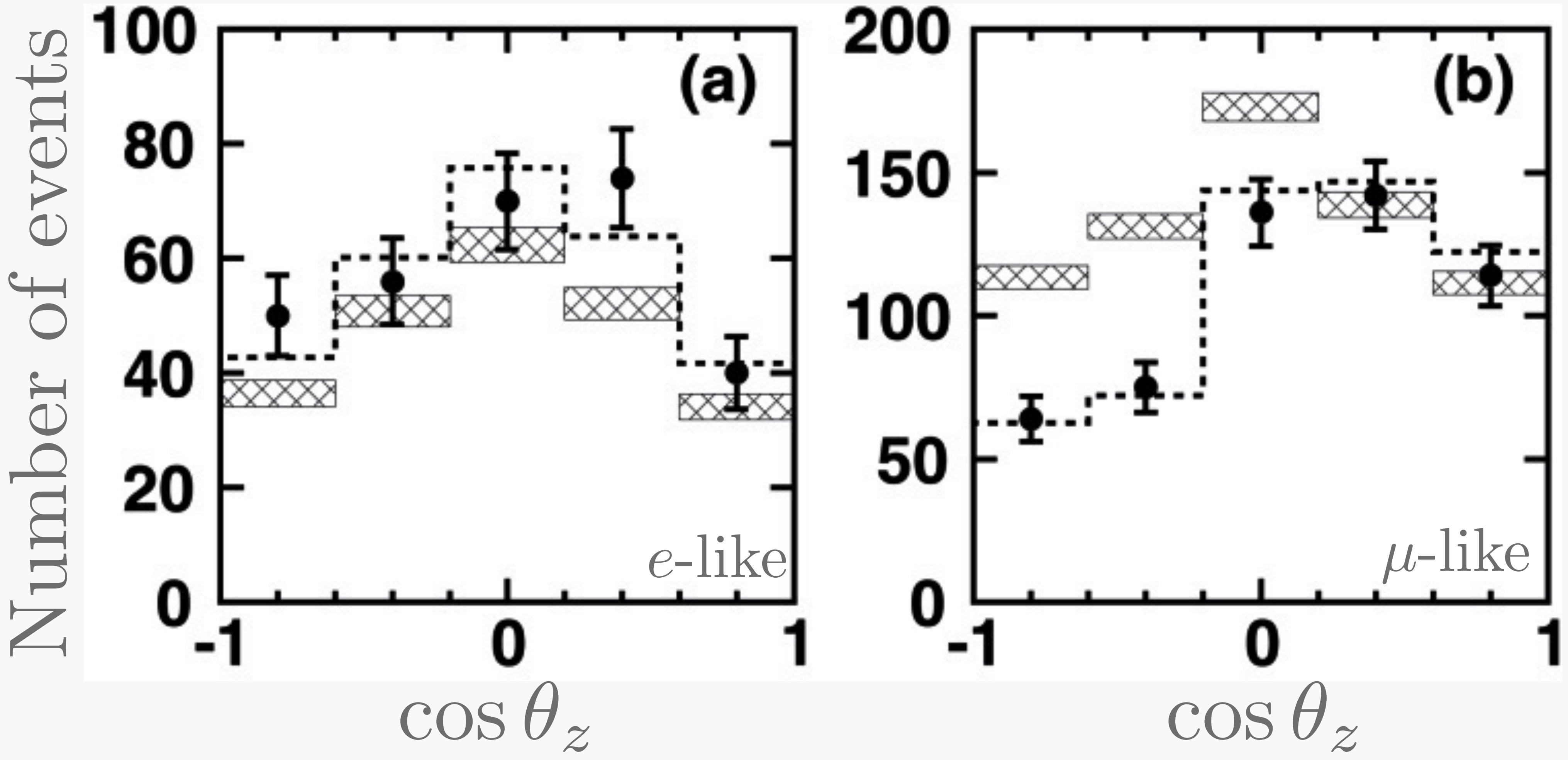
SK (535 days)

Neutrino Oscillations: 3 families

zoom.us video

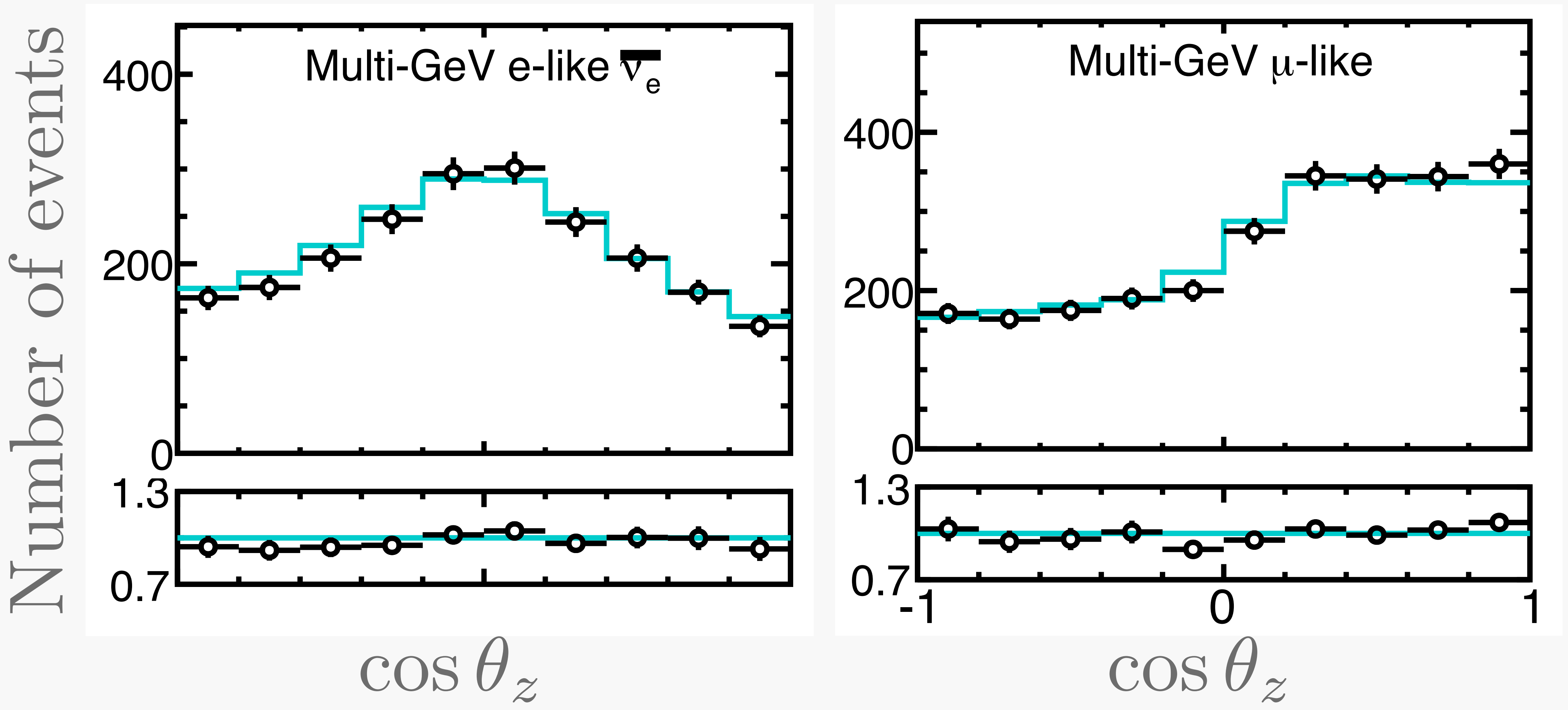
- Atmospheric regime

How it started



SK (535 days)

How it's going



SK I-IV (5326 days)

Neutrino Oscillations: 3 families

zoom.us video

- Atmospheric regime

  Characterized by $E \sim$ few GeV, and $L \sim 1000$ km.

$$\frac{\Delta m^2_{21} L}{4E} \simeq 0 \ll \frac{\Delta m^2_{32} L}{4E} \simeq \frac{\Delta m^2_{31} L}{4E}$$

Neutrino Oscillations: 3 families

zoom.us video

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$$\frac{\Delta m_{21}^2 L}{4E} \simeq 0 \ll \frac{\Delta m_{32}^2 L}{4E} \simeq \frac{\Delta m_{31}^2 L}{4E}$$

$$P_{\nu_\mu \to \nu_\tau} \simeq \sin^2(2\theta_{23}) \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right)$$

$$P_{\nu_\mu \to \nu_e} \simeq 0 \quad (\text{since } (U_{\text{PMNS}})_{e3} \simeq 0)$$

Neutrino Oscillations: 3 families

zoom.us video

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$$P_{\nu_\mu \to \nu_\mu} \simeq 1 - P_{\nu_\mu \to \nu_\tau} \simeq 1 - \sin^2(2\theta_{23}) \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

Neutrino Oscillations: 3 families

zoom.us video

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Measured by SuperKamiokande. Confirmed by accelerator LBL experiments

– K2K ~ 250 km – MINOS ~ 735 km – T2K ~ 295 km

Neutrino Oscillations: 3 families

zoom.us video

- Solar regime ($E \sim$ few MeV)

  Inside the Sun, hydrogen is converted into helium during the nuclear fusion

$$4p \rightarrow {}^4\mathrm{He} + 2e^+ + 2\nu_e + 2\gamma$$

Neutrino Oscillations: 3 families

zoom.us video

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Neutrino Oscillations: 3 families

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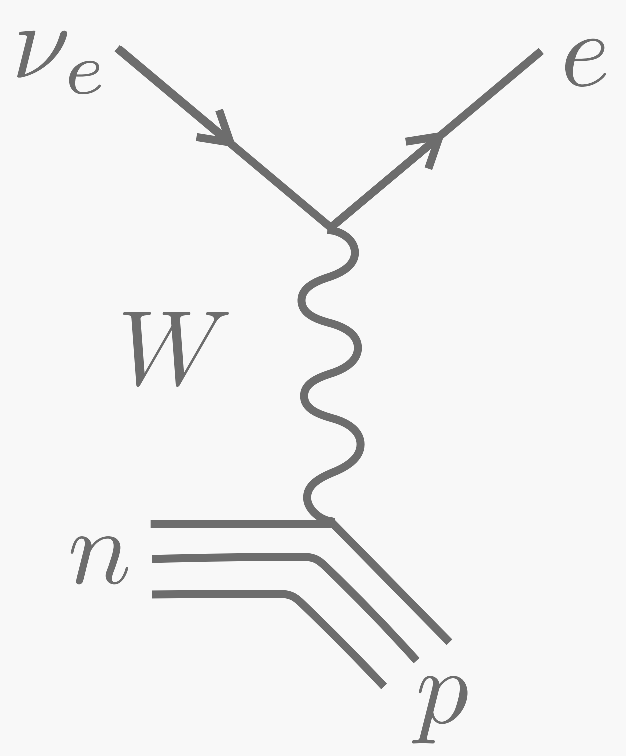
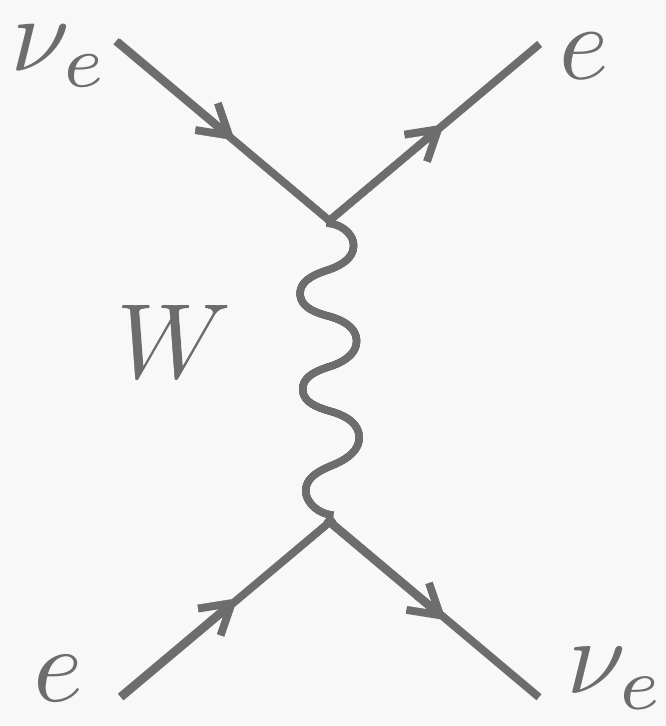
  Finally, ν_2 will be detected in SuperKamiokande as ν_e via

CC



ES



Neutrino Oscillations: 3 families

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- Solar regime ($E \sim$ few MeV)

  Therefore, the probability of measuring the ν_e produced in the Sun goes like

$$P^{\text{Sun}}_{\nu_e \to \nu_e} \simeq |\langle \nu_e | \nu_2 \rangle|^2 = |(U_{\text{PMNS}})_{e2}|^2 \simeq \cos^2 \theta_{12}$$

Neutrino Oscillations: 3 families

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  By observing the resonant behaviour, they also deduced that $\Delta m^2_{21} > 0$, establishing the hierarchy between m_1 and m_2.

Neutrino Oscillations: 3 families

zoom.us video

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  Measured by SuperKamiokande, SNO and Borexino.
  Confirmed by KamLAND, a reactor experiment measuring $\overline{\nu_e}$ disappearance

$$P^{\text{K-LAND}}_{\overline{\nu_e} \to \overline{\nu_e}} \simeq 1 - \sin^2(2\theta_{12}) \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$$

$E \sim$ few MeV
$L \sim 100$ km

Neutrino Oscillations: 3 families

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- Reactor regime ($E \sim$ few MeV and $L \sim 1$ km)

  At least two detectors; one close to the nuclear reactor and the other about 1 km away are used to measure the $\overline{\nu_e}$ flux.

Neutrino Oscillations: 3 families

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$$P^{\text{reactor}}_{\overline{\nu_e}\to\overline{\nu_e}} \simeq 1 - \sin^2(2\theta_{13})\sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

Neutrino Oscillations: 3 families

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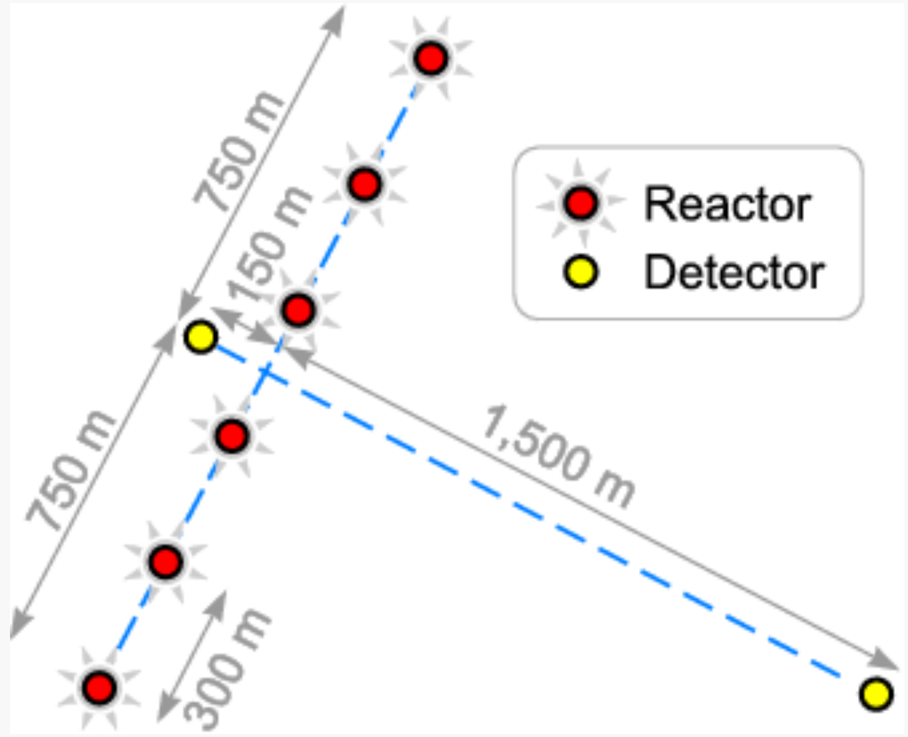
  Measured by Daya Bay, RENO and Double CHOOZ. Discovery of $\theta_{13} \neq 0$ in 2012!



Scheme of the RENO experiment (South Korea)

Neutrino Oscillations: 3 families

zoom.us video

Summary of the dependence on the neutrino oscillation parameters

Neutrino oscillation experiment	Leading dependence	Subleading dependence
Solar experiments [1]	θ_{12}	$\Delta m^2_{\rm sol}$ & θ_{13}
Reactor LBL [2]	$\Delta m^2_{\rm sol}$	θ_{12} & θ_{13}
Reactor MBL [3]	θ_{13}	$\lvert\Delta m^2_{\rm atm}\rvert$
Atmospheric experiments [4]	θ_{23}	$\Delta m^2_{\rm atm}\,, \theta_{13}$ & δ
Acc. LBL ν_μ & $\overline{\nu}_\mu$ disappearance [5]	$\lvert\Delta m^2_{\rm atm}\rvert$	θ_{23} & θ_{13}
Acc. LBL ν_e appearance [6]	θ_{13}	$\Delta m^2_{\rm atm}\,, \delta$ & θ_{23}

[1] SNO, Borexino, Gallex, SK
[2] KamLAND
[3] Daya Bay, Reno, Double-Chooz
[4] SK, MINOS, IceCUBE
[5,6] T2K, MINOS, NOνA

Neutrino Oscillations: 3 families

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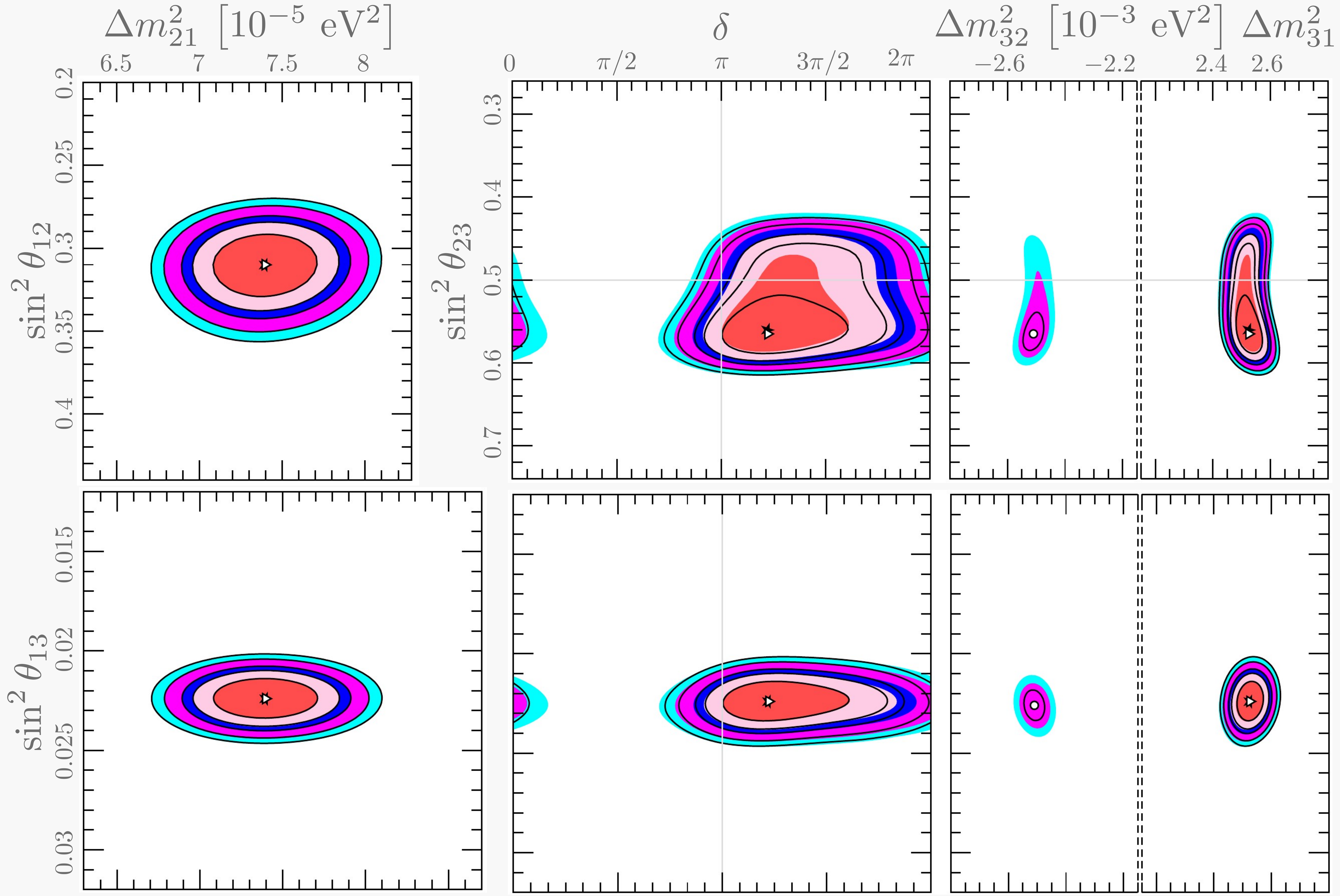
Present values obtained through a global-fit to a complete set of ν oscillation experiments



NuFIT 4.1
www.nu-fit.org

Neutrino Oscillations: 3 families

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The present best-fit values of the neutrino oscillation parameters are

$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$
$\sin^2\theta_{23}$	$0.558^{+0.020}_{-0.033}$
$\sin^2\theta_{13}$	$0.02141^{+0.00066}_{-0.00065}$
Δm^2_{sol}	$7.39^{+0.21}_{-0.20}\cdot 10^{-5}\text{eV}^2$
$\lvert\Delta m^2_{\text{atm}}\rvert$	$2.523^{+0.032}_{-0.030}\cdot 10^{-3}\text{eV}^2$

NuFIT 4.1
www.nu-fit.org

In the global-fit, the normalization of reactor fluxes is left free while data from short-baseline reactor experiments are included. The values of $\sin^2\theta_{23}$ and $\sin^2\theta_{13}$ are obtained assuming normal ordering.

NEUTRINO OSCILLATIONS: 3 FAMILIES

zoom.us video

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www.nu-fit.org

However, there are still some unknown values

– δ (maximal?) $\Rightarrow$ $\not{CP}$?

– θ_{23} octant (maximal mixing?)

– $\Delta m^2_{\rm atm}$ sign

Neutrino Oscillations: 3 families

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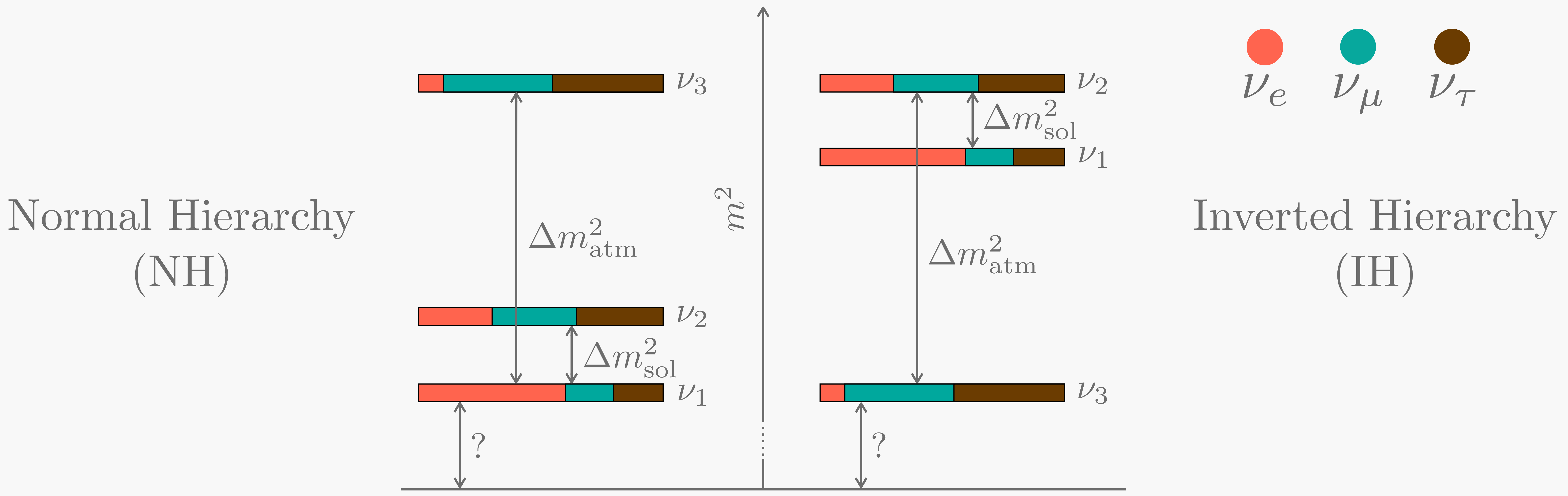
The sign of $\Delta m^2_{\mathrm{atm}}$ gives rise to



Neutrino Oscillations: 3 families

zoom.us video

The best window to measure mass ordering and $\cancel{CP}$ in the leptonic sector is through $\nu_\mu \to \nu_e$ disappearance channel at accelerator LBL ν experiments.

Neutrino Oscillations: 3 families

zoom.us video

The best window to measure mass ordering and $\not{CP}$ in the leptonic sector is through $\nu_\mu \to \nu_e$ disappearance channel at accelerator LBL ν experiments.

The probability of the golden channel goes like

$$
\begin{aligned}
P_{\nu_\mu \to \nu_e} \simeq\ & c_{23}^2 \sin^2(2\theta_{12}) \sin^2(\Delta_{21}) \\
& + s_{23}^2 \sin^2(2\theta_{13}) \sin^2(\Delta_{31}) \\
& + c_{13} \sin(2\theta_{12}) \sin(2\theta_{13}) \sin(2\theta_{23}) \cos(\pm\delta - \Delta_{31}) \sin(\Delta_{31}) \sin(\Delta_{21})
\end{aligned}
$$

Neutrino Oscillations: 3 families

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The probability of the golden channel goes like

$$P_{\nu_\mu \to \nu_e} \simeq c_{23}^2 \sin^2(2\theta_{12}) \sin^2(\Delta_{21})$$ — solar regime

$$+s_{23}^2 \sin^2(2\theta_{13}) \sin^2(\Delta_{31})$$ — atmospheric regime

$$+c_{13} \sin(2\theta_{12}) \sin(2\theta_{13}) \sin(2\theta_{23}) \cos(\pm\delta - \Delta_{31}) \sin(\Delta_{31}) \sin(\Delta_{21})$$ — CP interference

By comparing ν_μ and $\overline{\nu_\mu}$ disappearance channels, they could measure δ.

Neutrino Oscillations: 3 families

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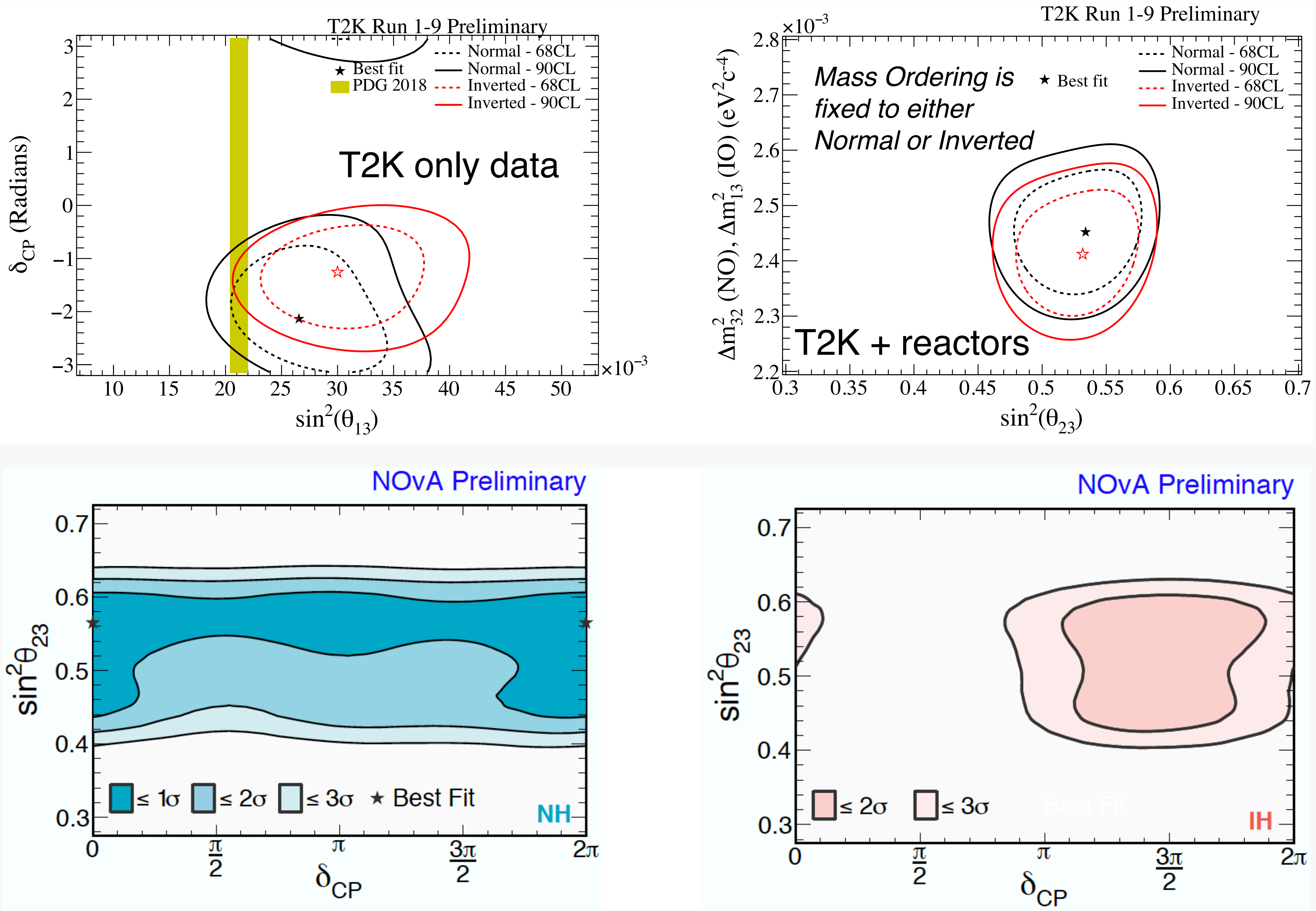
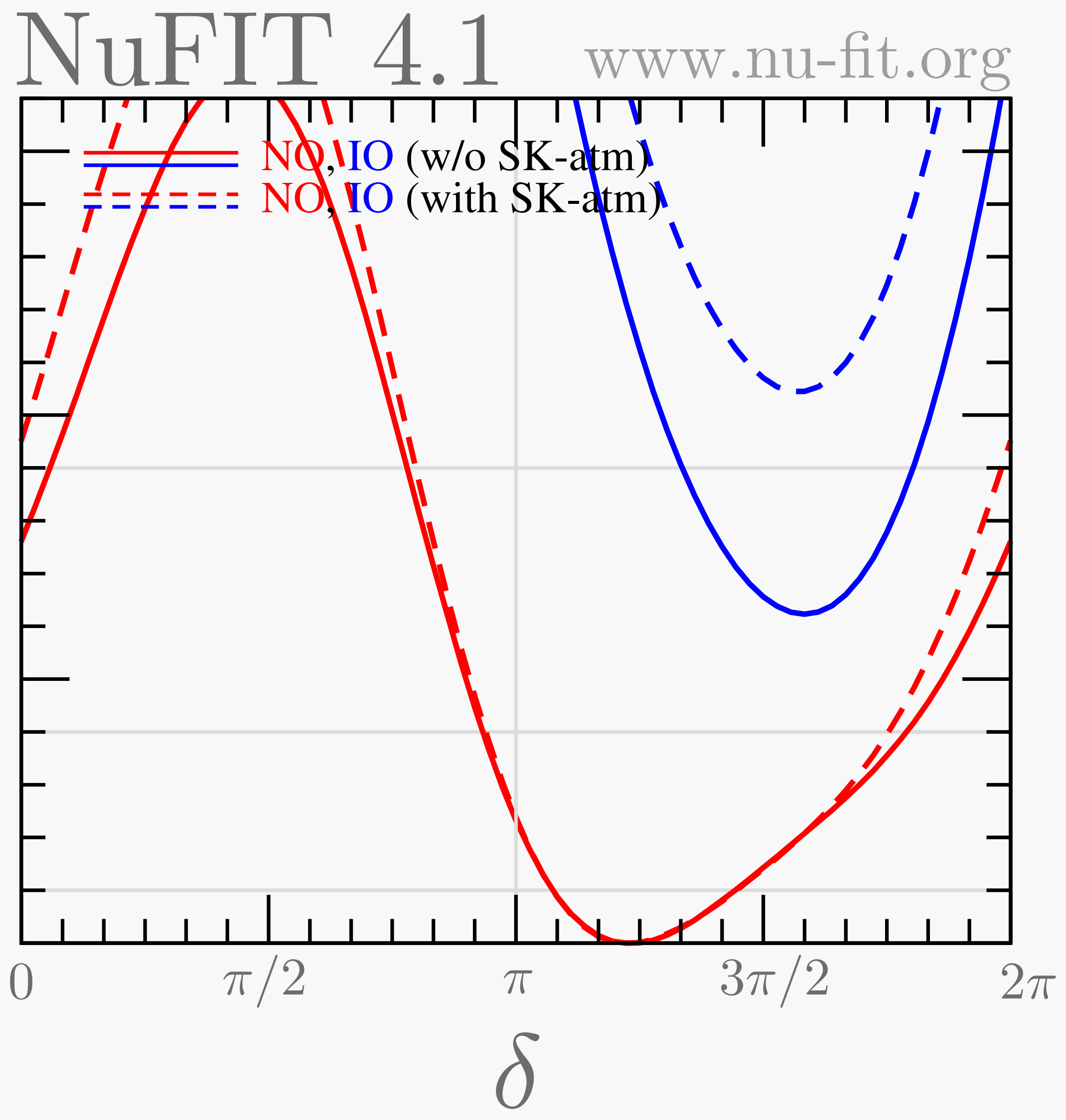
T2K ($L \simeq 295$ km, $E \simeq 1$ GeV) and NOνA ($L \simeq 810$ km, $E \simeq 3$ GeV) preliminary results





Mild preference for

NH + $\delta \simeq 3\pi/2$

NEUTRINO OSCILLATIONS: 3 FAMILIES

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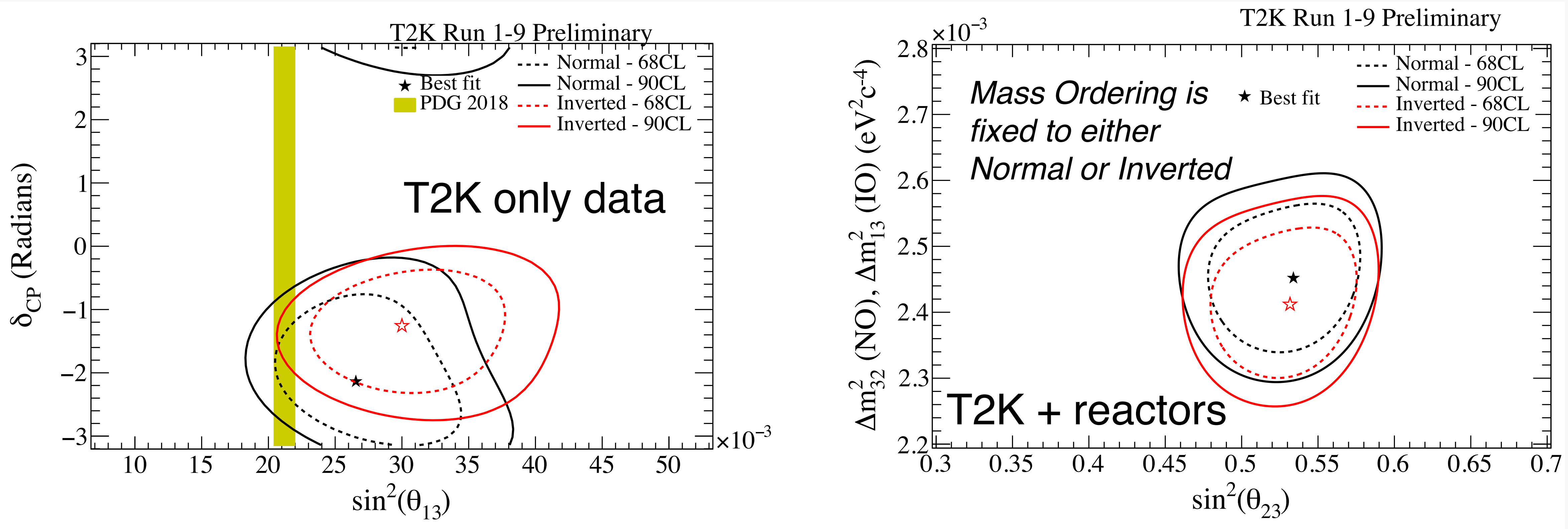
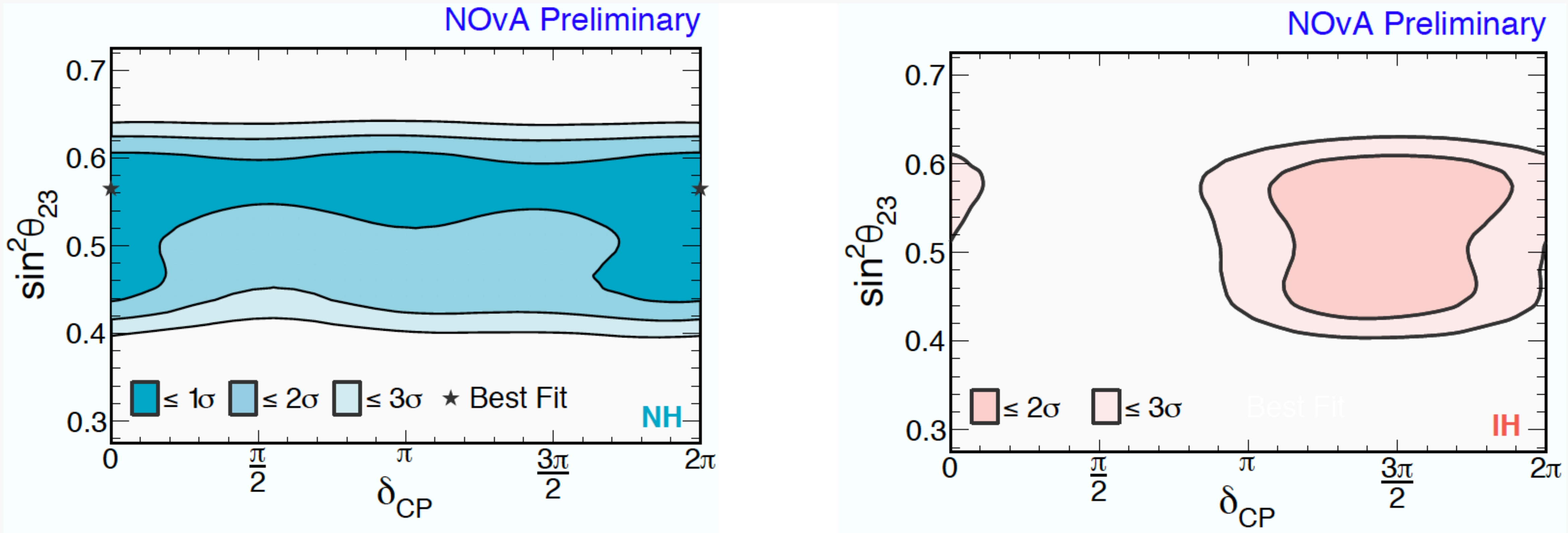
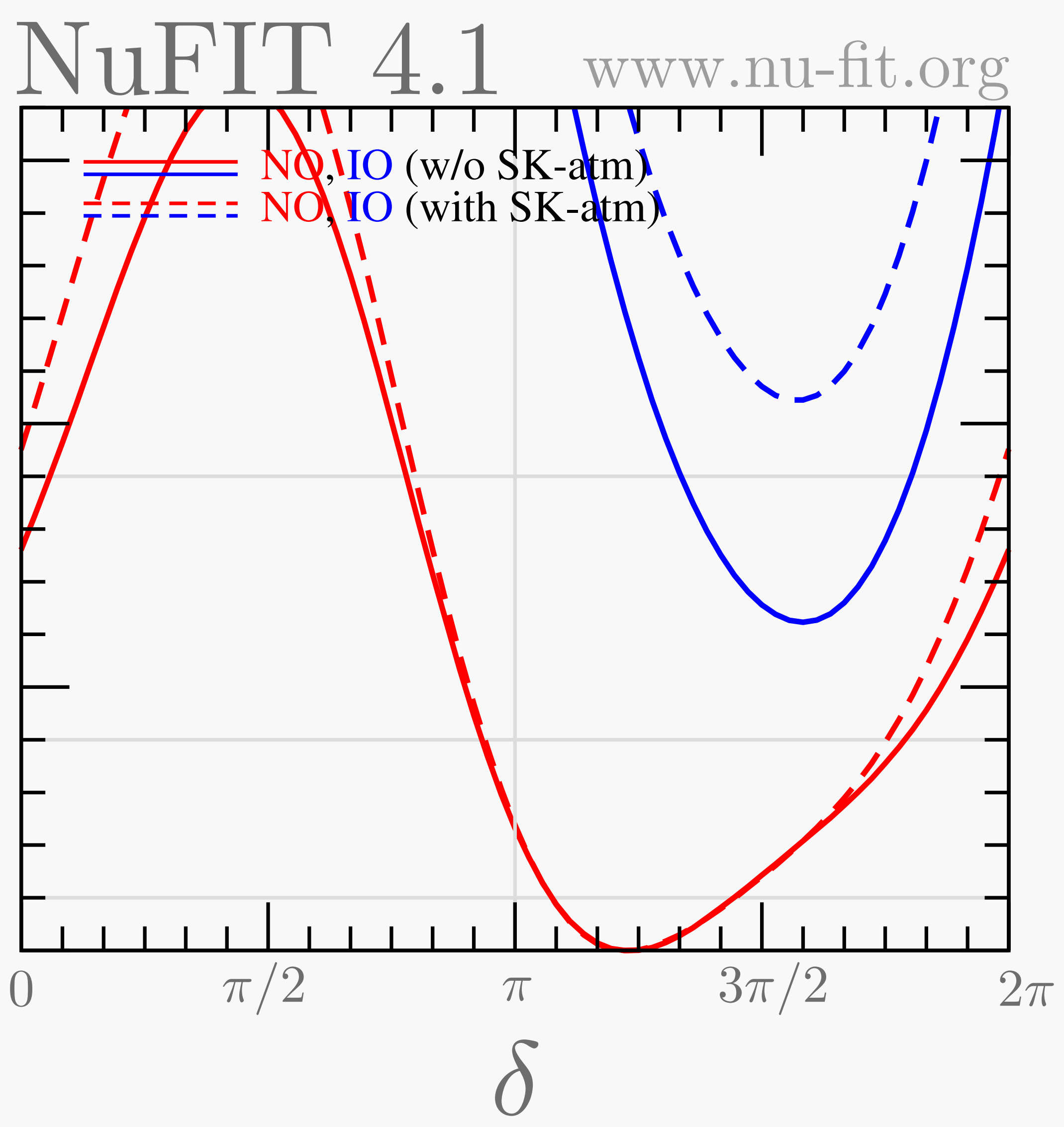






Mild preference for

$\text{NH} + \delta \simeq 3\pi/2$

Will be confirmed by future ν oscillation experiments: DUNE & T2HK

zoom.us video

Part II

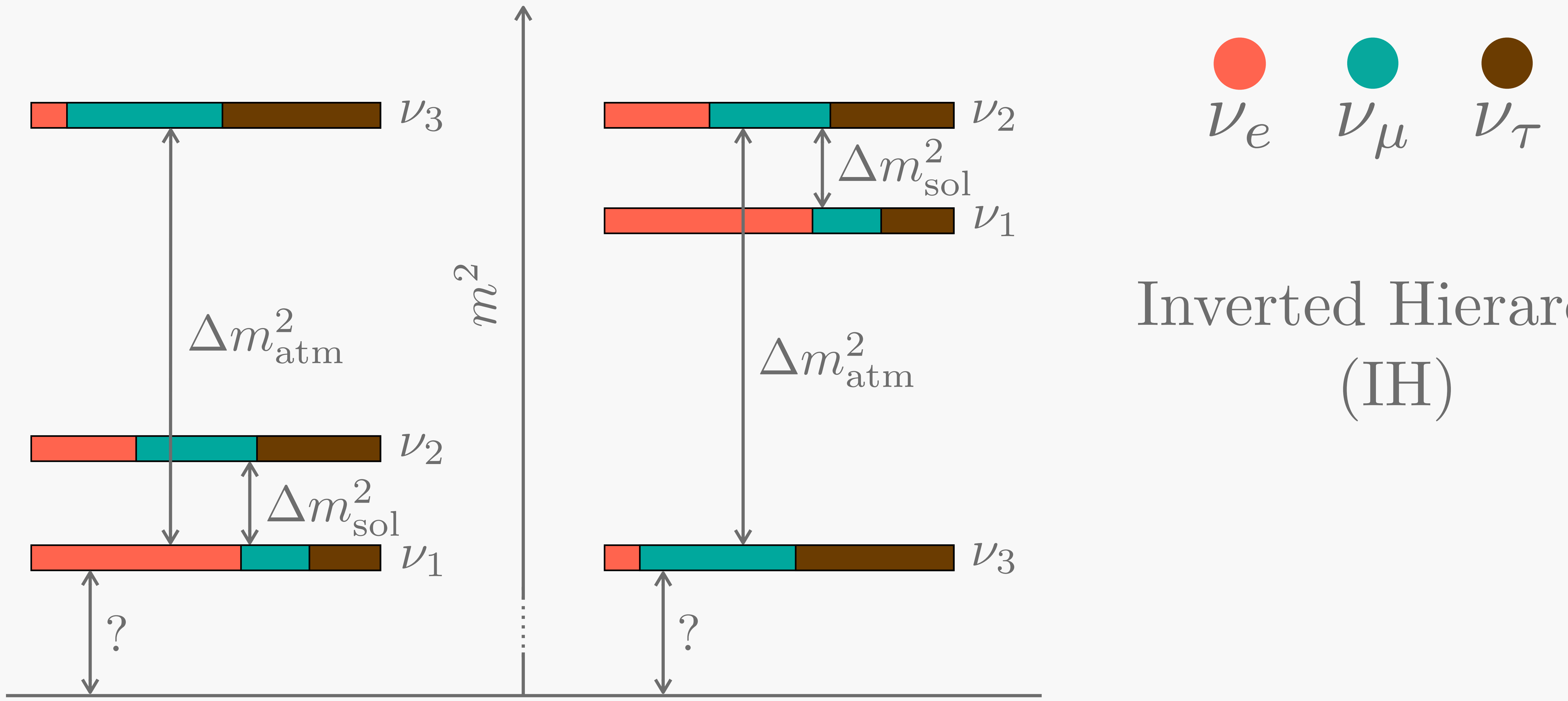
Effective approach to neutrino interactions, absolute mass scale of neutrinos and leptogenesis

Neutrino mass scale from β-decay

zoom.us video

Remember that...

Normal Hierarchy (NH)



Inverted Hierarchy (IH)

The neutrino mass scale is not known.

Neutrino mass scale from β-decay

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The best way to measure the absolute neutrino mass scale in a model independent way is through single β-decay experiments.

Neutrino mass scale from β-decay

zoom.us video

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Tritium β-decay

$$T \to {}^{3}\mathrm{He}^{+} + e^{-} + \overline{\nu_e} \qquad (T_{1/2} = 12.3 \text{ years})$$

Superallowed β transition $\Rightarrow$

- constant nuclear matrix element
- spectrum determined by phase space

Neutrino mass scale from β-decay

zoom.us video

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Superallowed β transition $\Rightarrow$
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- spectrum determined by phase space

The differential decay rate goes like

$$\frac{d\Gamma^\beta}{dE_e} \propto |p_e|\, E_e\, |p_\nu|\, E_\nu$$

$$\simeq \sqrt{2m_e T_e}\,(m_e + T_e)\,\sqrt{(E_0 - T_e)^2 - {m_\nu}^2}\,(E_0 - T_e)$$

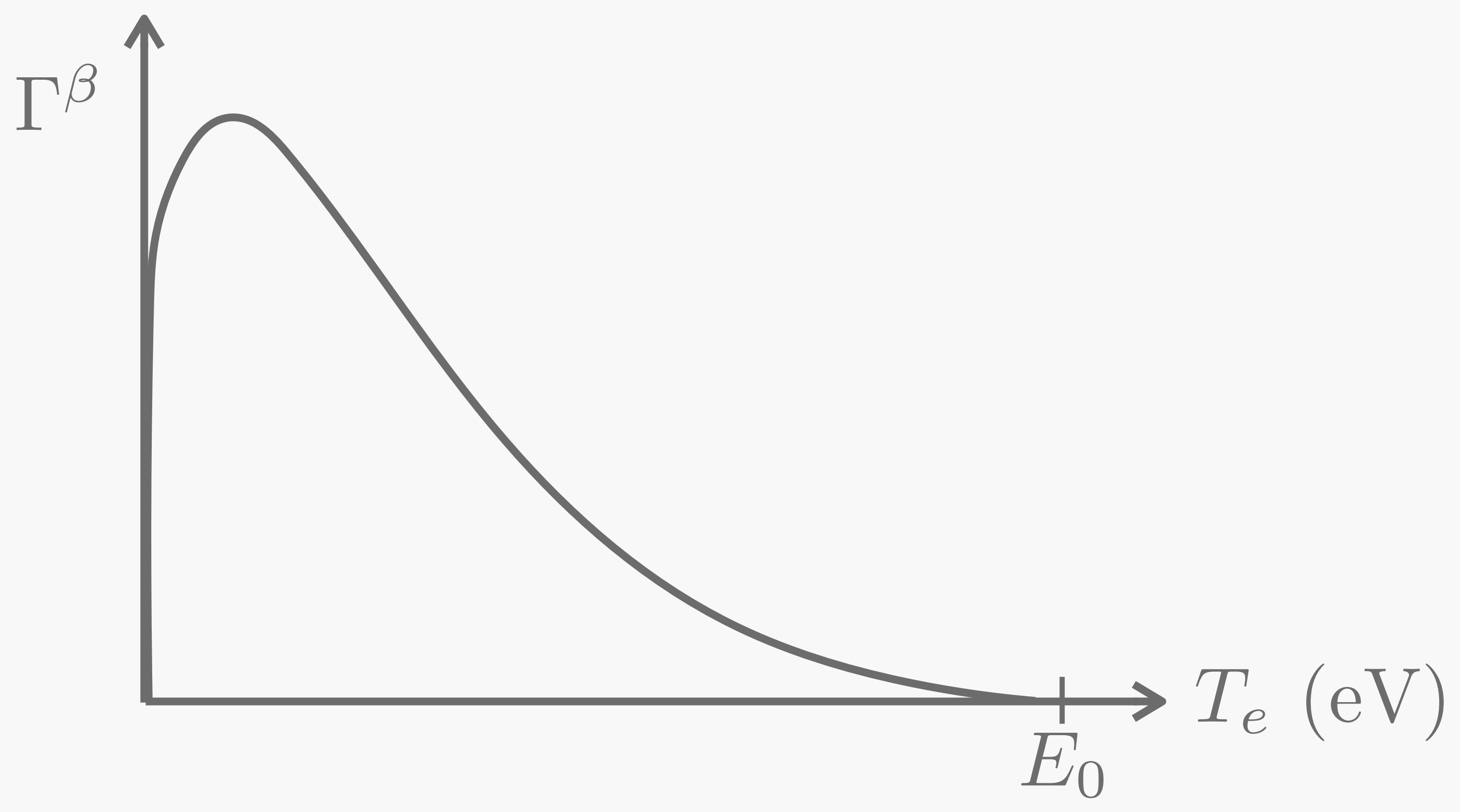
e kinetic energy

with $E_0 = 18.575$ keV the energy released in the decay.

Neutrino mass scale from β-decay

zoom.us video

They study the shape of the β spectrum close to the end point



Neutrino mass scale from β-decay

zoom.us video

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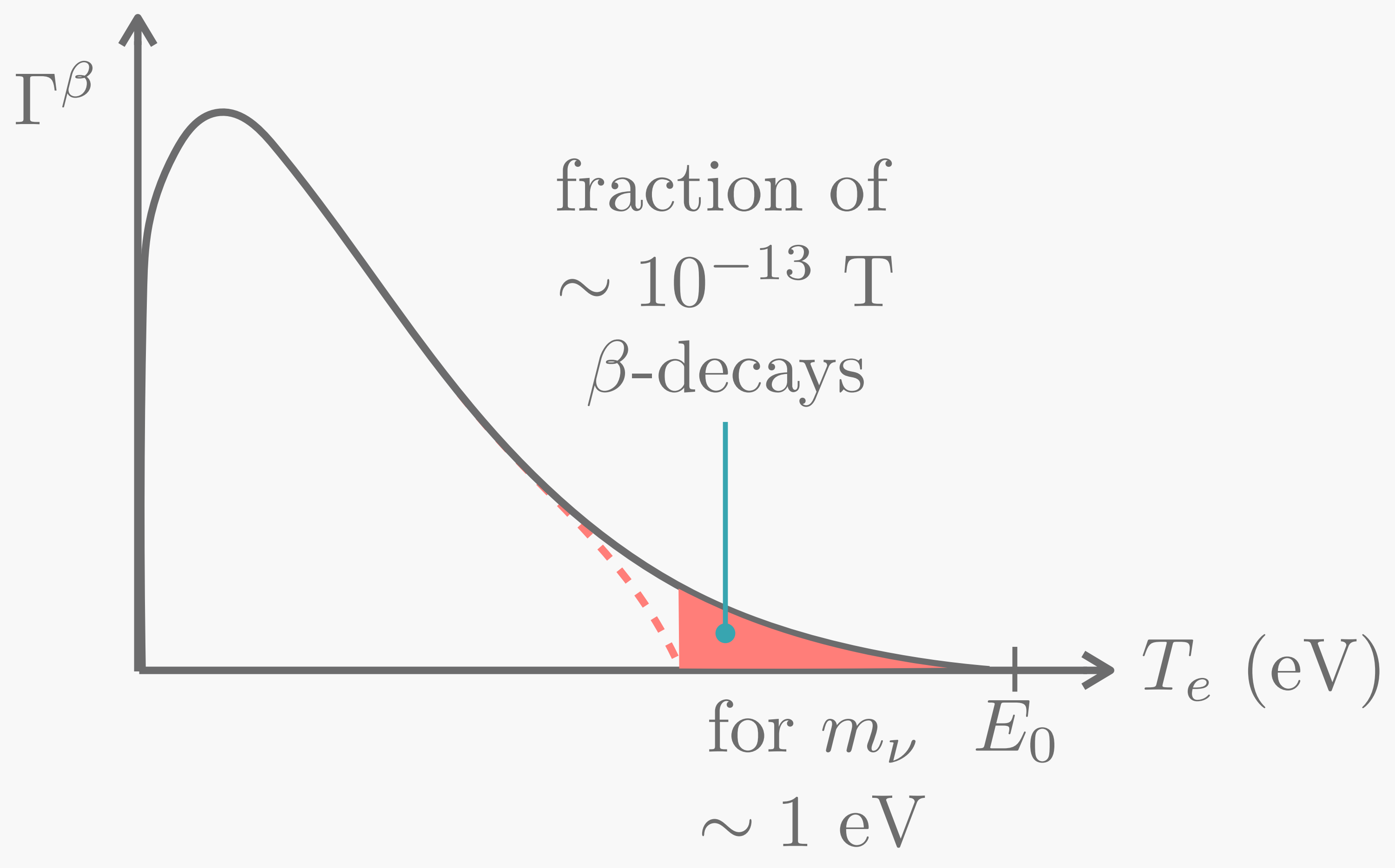

Neutrino mass scale from β-decay

zoom.us video

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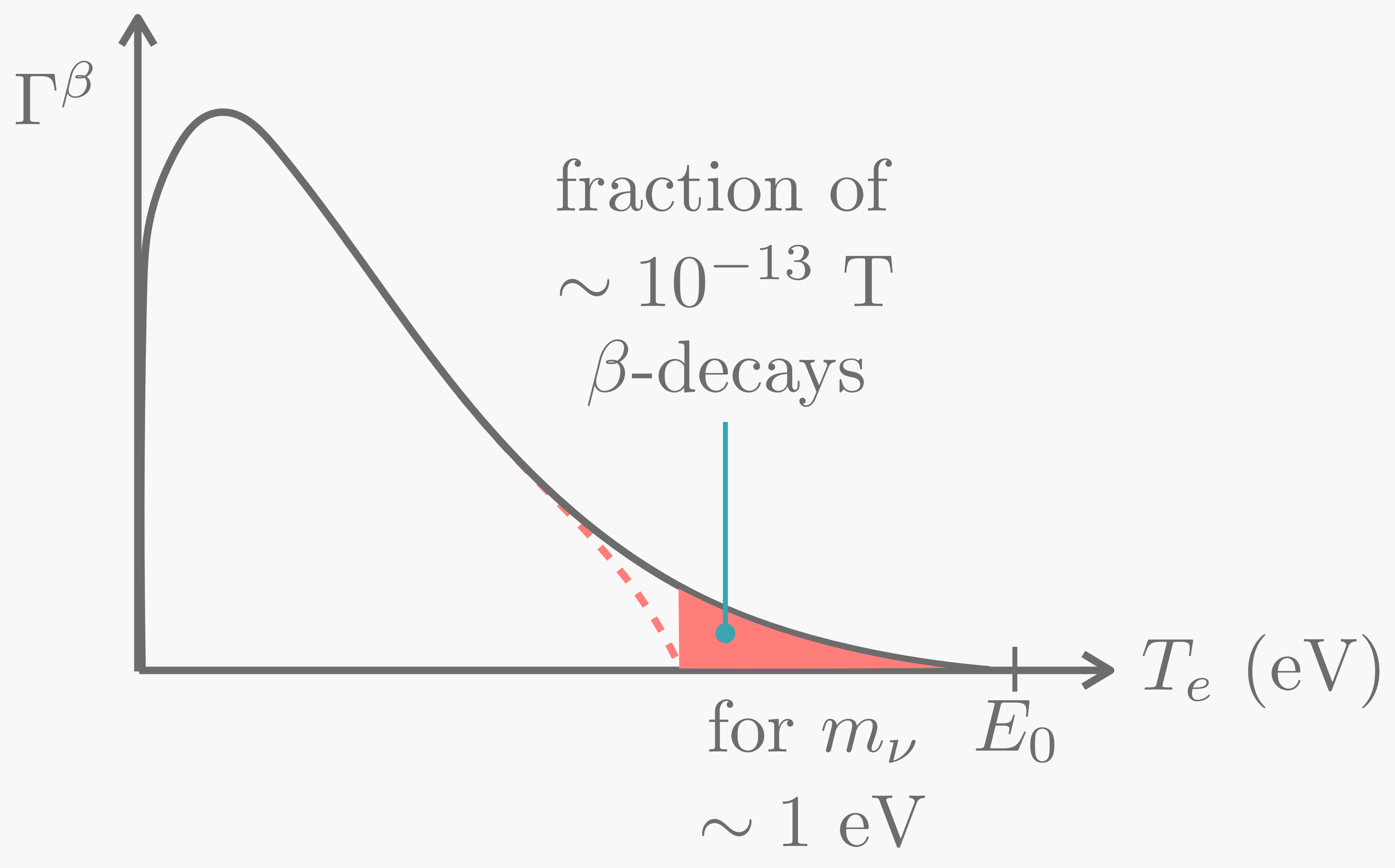

to measure the effective electron neutrino mass defined by

$$\left(m_{\nu_e}^{\text{eff}}\right)^2 \equiv \frac{\sum_i \left|(U_{\text{PMNS}})_{ei}\right|^2 m_i^2}{\sum_i \left|(U_{\text{PMNS}})_{ei}\right|^2} = \sum_i \left|(U_{\text{PMNS}})_{ei}\right|^2 m_i^2$$

Neutrino mass scale from β-decay

zoom.us video

Present upper bounds from

– Mainz: $m_{\nu_e}^{\text{eff}} < 2.3$ eV at 95%CL

– Troitsk: $m_{\nu_e}^{\text{eff}} < 2.1$ eV at 95%CL

– KATRIN: $m_{\nu_e}^{\text{eff}} < 1.1$ eV at 90%CL

4 weeks of data!

Neutrino mass scale from β-decay

zoom.us video

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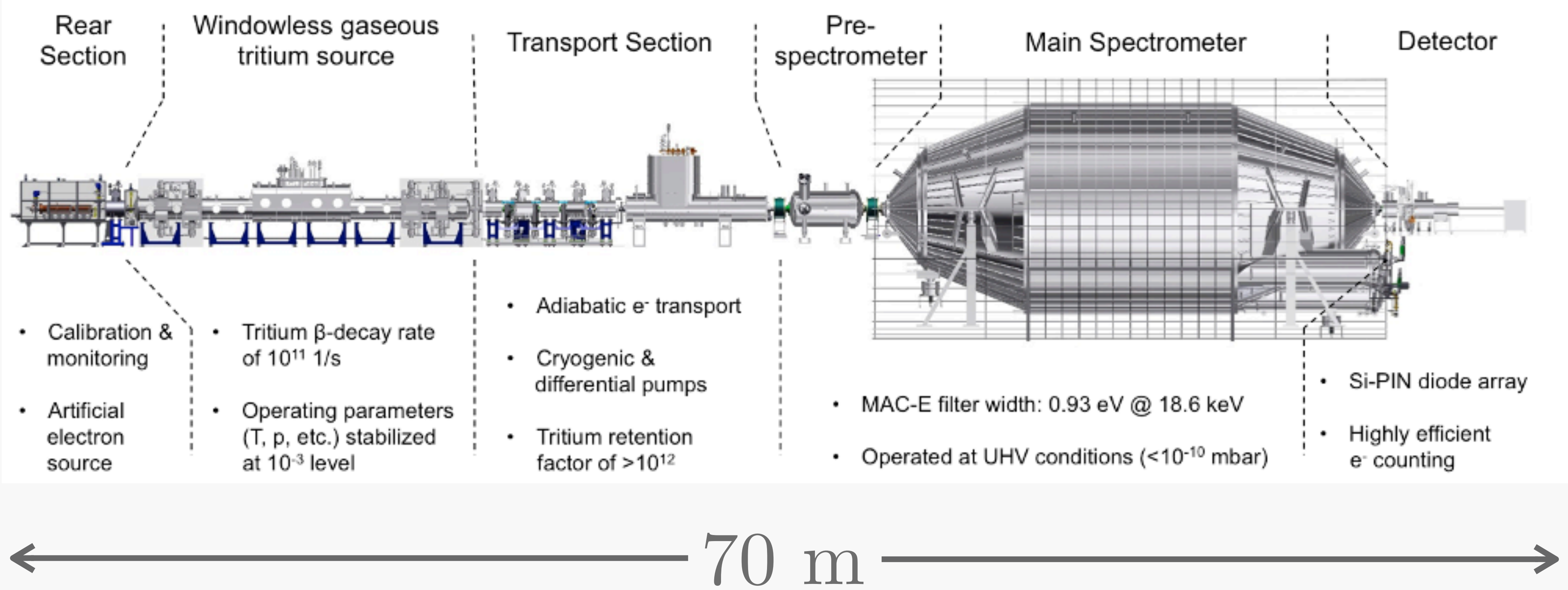
KATRIN aims to explore the mass region up to 0.2 eV



Karlsruhe
Tritium
Neutrino
experiment

Neutrino Masses in the SM

zoom.us video

- Dirac neutrino masses

All fermions get masses through the Yukawa interaction

$$\overline{\psi_L} y_\psi \phi \psi_R \xrightarrow[\text{EWSB}]{\text{after}} \boxed{y_\psi \frac{v_{\text{EW}}}{\sqrt{2}}} \overline{\psi_L} \psi_R$$

$$m_\psi \equiv y_\psi \frac{v_{\text{EW}}}{\sqrt{2}}$$

Remember that... → lecture by Timo Kärkkäinen

Neutrino Masses in the SM

zoom.us video

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$$m_\psi \equiv y_\psi \frac{v_{\text{EW}}}{\sqrt{2}}$$

For neutrinos

$$\overline{\nu_L} y_\nu \phi \nu_R \Rightarrow m_D = y_\nu \frac{v_{\text{EW}}}{\sqrt{2}}$$

Remember that... → lecture by Timo Kärkkäinen

Neutrino Masses in the SM

zoom.us video

- Dirac neutrino masses

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Not present in the SM

Remember that... → lecture by Timo Kärkkäinen

Neutrino Masses in the SM

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- Majorana neutrino masses

Since neutrinos are the only neutral fermions

$$\hat{m}\overline{\nu_L^c}\nu_L$$

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Neutrino Masses in the SM

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$d = 5 \Rightarrow$ Is SM low energy remnant of higher energy theory?

Remember that... $\rightarrow$ lecture by Timo Kärkkäinen

Neutrino Masses in the SM

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And therefore, neutrinos are strictly massless in the SM.

The SM must be extended to account for neutrino oscillations.

Remember that... → lecture by Timo Kärkkäinen

Neutrino Masses in the SM

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The Weinberg operator effectively generated by new particles.

At tree level the 3 possible realizations of the Weinberg operator are

- Type-I Seesaw: heavy fermionic singlets N_R

Neutrino Masses in the SM

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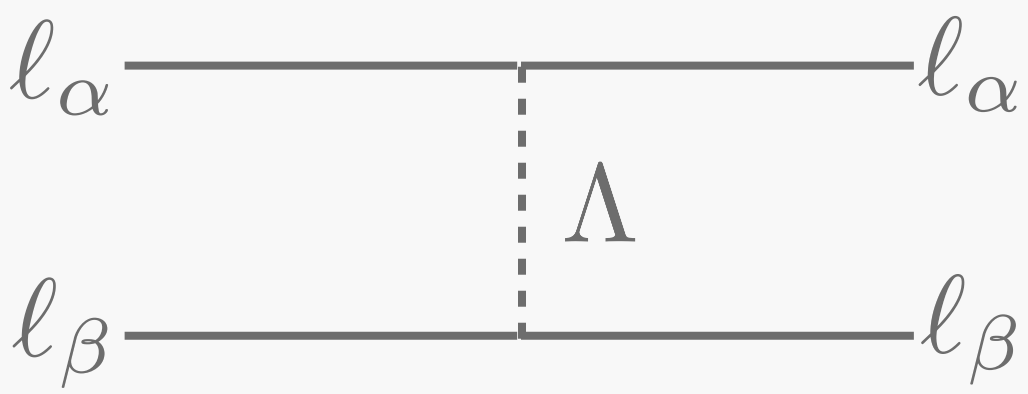

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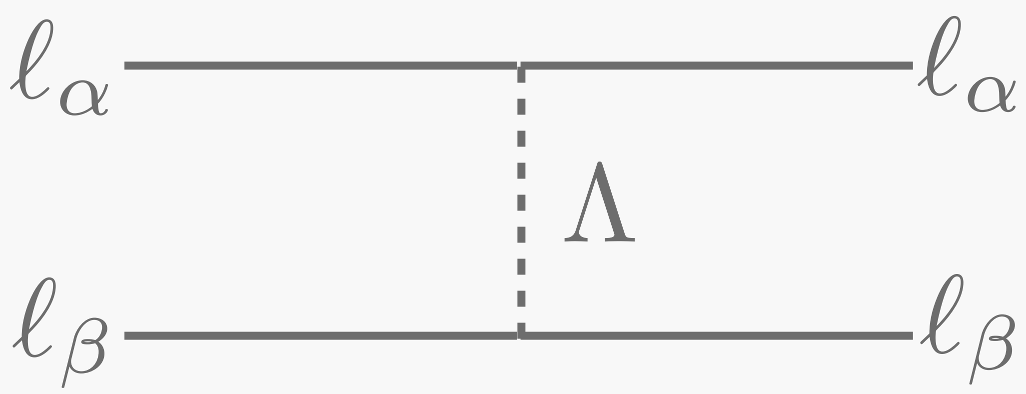
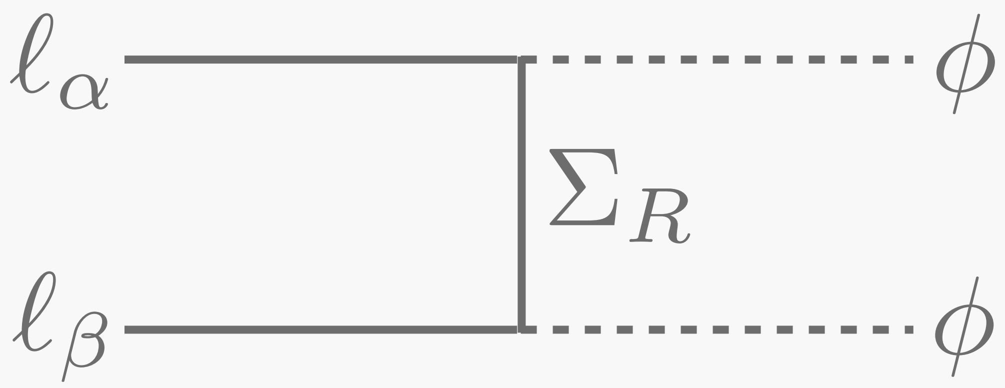


- Type-III Seesaw: heavy fermionic $SU(2)_L$ triplets Σ_R



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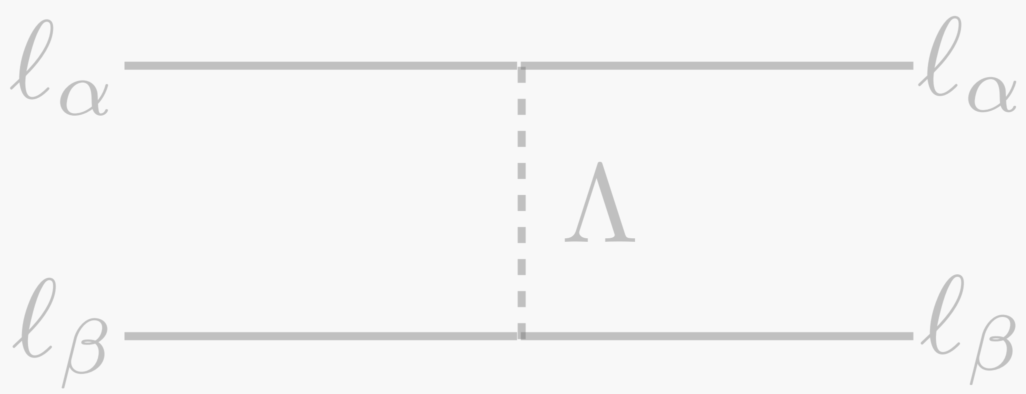

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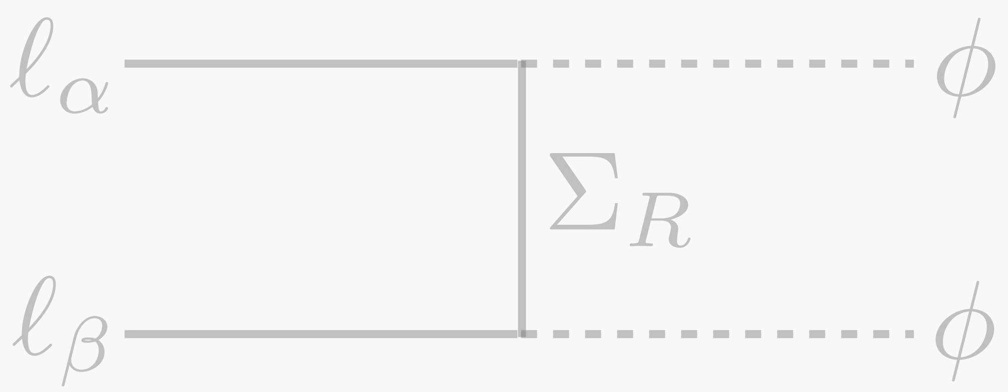

SM + type-I Seesaw

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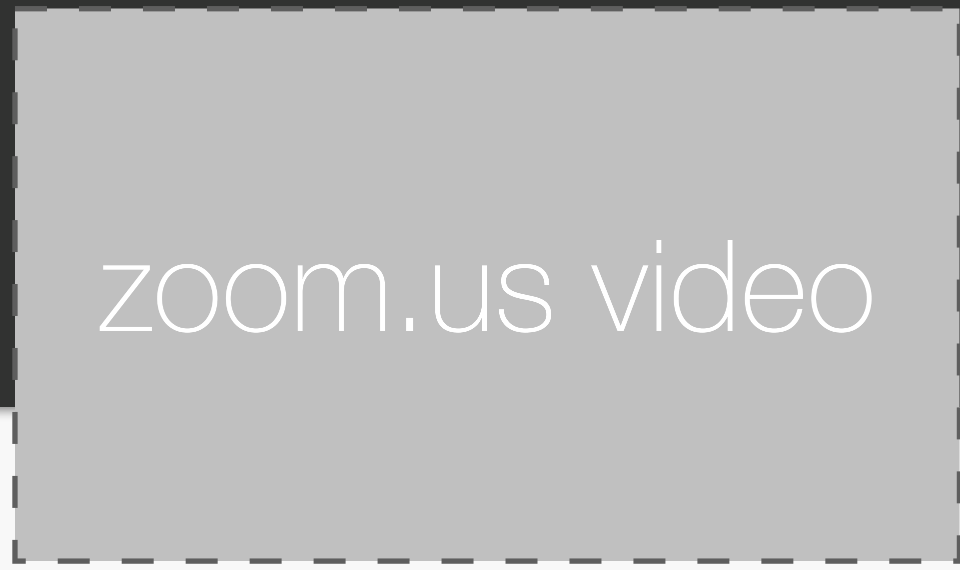
The SM is enlarged by an arbitrary number of N_R

$$\mathcal{L} = \mathcal{L}_{\rm SM} + i\overline{N_R}\partial\!\!\!/ N_R - \left(\frac{1}{2}\overline{N_{Ri}}(M_N)_{ij}N^c_{Rj} + (y_N)_{i\alpha}\overline{N_{Ri}}\phi^\dagger \ell_{L\alpha}\right) + \text{h.c.}$$

since N_R are singlets and $Y = 0$, $D_\mu = \partial_\mu$ in the kinetic term.

SM + type-I Seesaw



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Yukawa interaction

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SM + TYPE-I SEESAW

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Allowed Majorana mass for N_R

SM + TYPE-I SEESAW

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M_N not related to the EWSB. New Physics scale where L is broken.

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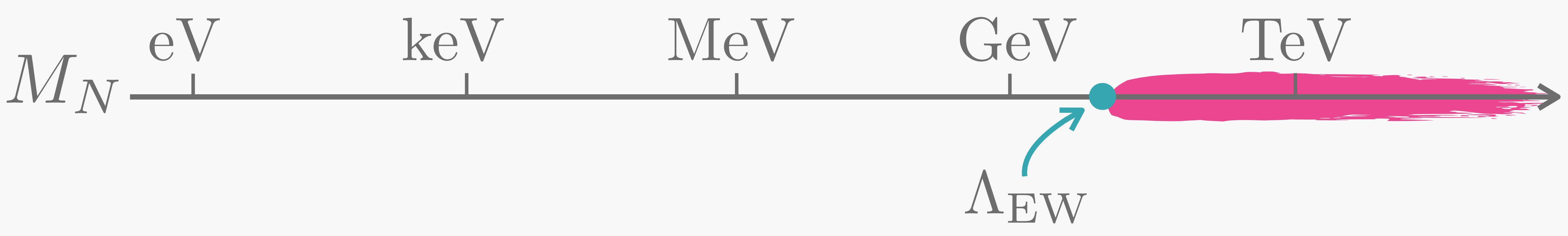

If $M_N \gg \Lambda_{\rm EW} \Rightarrow$ the new fields can be integrated out.

SM + type-I Seesaw

zoom.us video

The resulting effective field theory, built from a set of effective operators, can be used to study the low energy phenomenology.

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda}\mathcal{L}^{d=5} + \frac{1}{\Lambda^2}\mathcal{L}^{d=6} + \cdots \simeq \mathcal{L}_{\text{SM}} + \delta\mathcal{L}^{d=5} + \delta\mathcal{L}^{d=6} \qquad \Lambda \sim M_N$$

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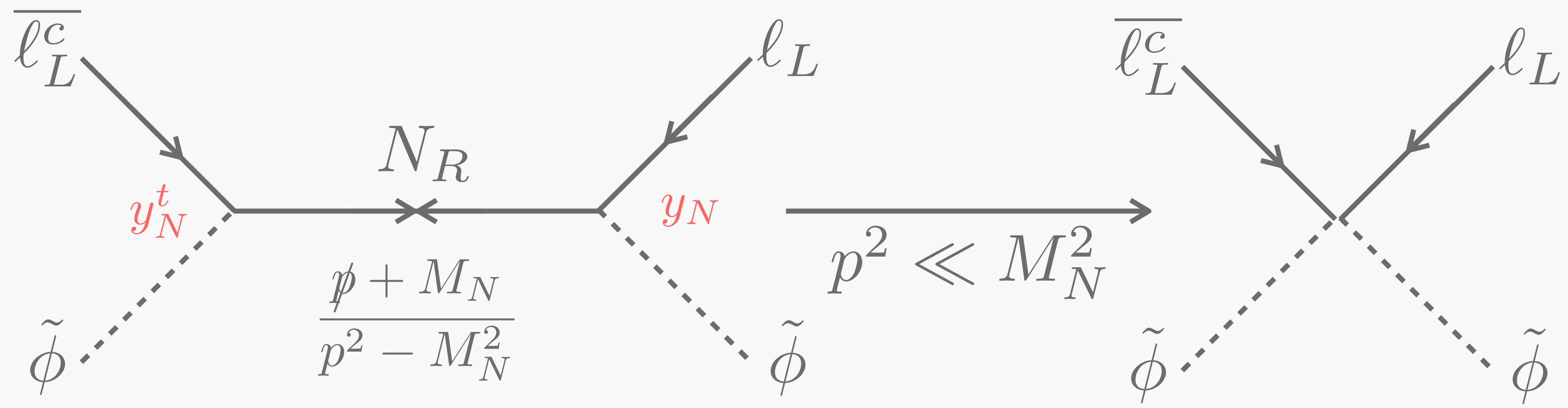

SM + TYPE-I SEESAW

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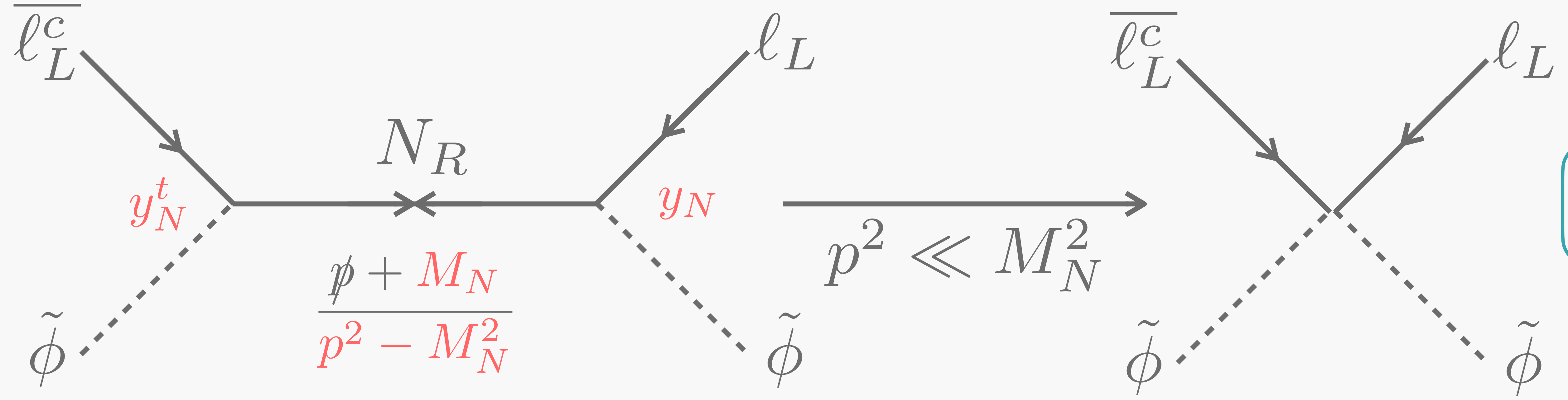

$$\boxed{-y^t_N M_N^{-1} y_N}\left(\overline{\ell^c_L}\,\tilde{\phi}^*\right)\left(\tilde{\phi}^\dagger \ell_L\right)$$

$1/\Lambda$

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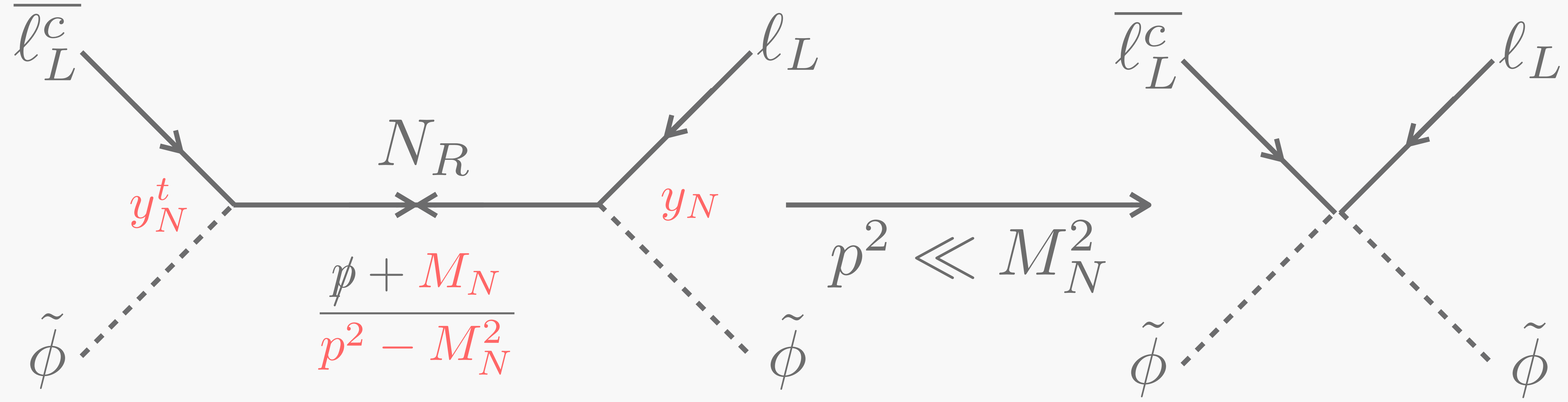

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  There will be a $d = 6$ operator from the $\frac{\not{p}}{M_N^2}$ term

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it induces corrections to the left-handed neutrino kinetic term $\Rightarrow$

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SM + type-I Seesaw

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We define the Hermitian matrix

$$\eta \equiv \frac{v_{\text{EW}}^2}{4} c^{d=6} = \frac{m_D^t M_N^{-2} m_D}{2}$$

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  With the following transformation

$$\nu_{\alpha L} \to \nu_{\alpha L} \equiv \left(\delta_{\alpha\beta} + 2\eta_{\alpha_\beta}\right)^{-1/2} \nu_{L}{}'_{\beta}$$

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As a result, the leptonic CC and NC Lagrangians take the form

$$\mathcal{L}_{\rm CC} \simeq \frac{g}{\sqrt{2}} \overline{\ell_\alpha} \gamma^\mu P_L (\delta_{\alpha\beta} - \eta_{\alpha\beta}) \left(U_{\rm PMNS}\right)_{\beta i} \nu_i W^-_\mu + \text{h.c.}$$
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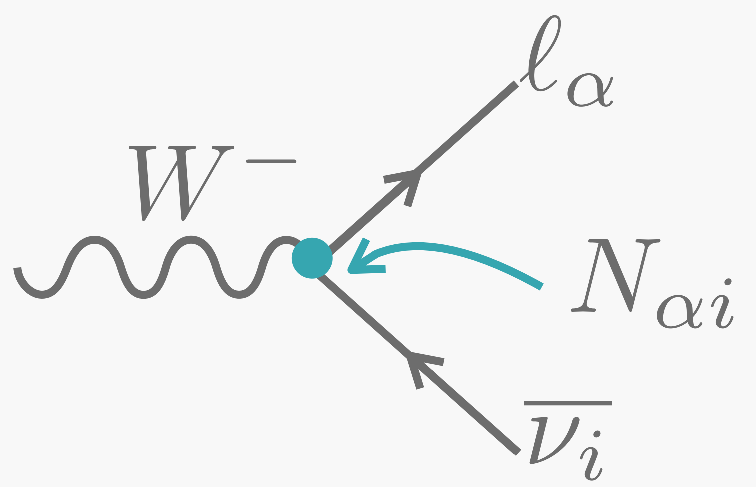

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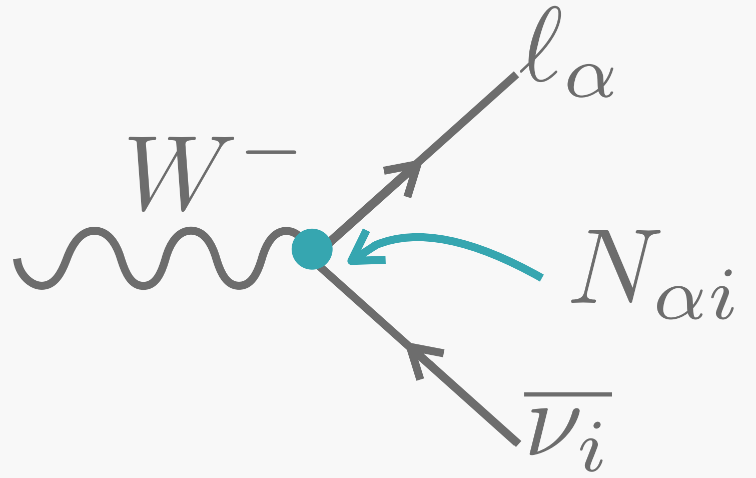
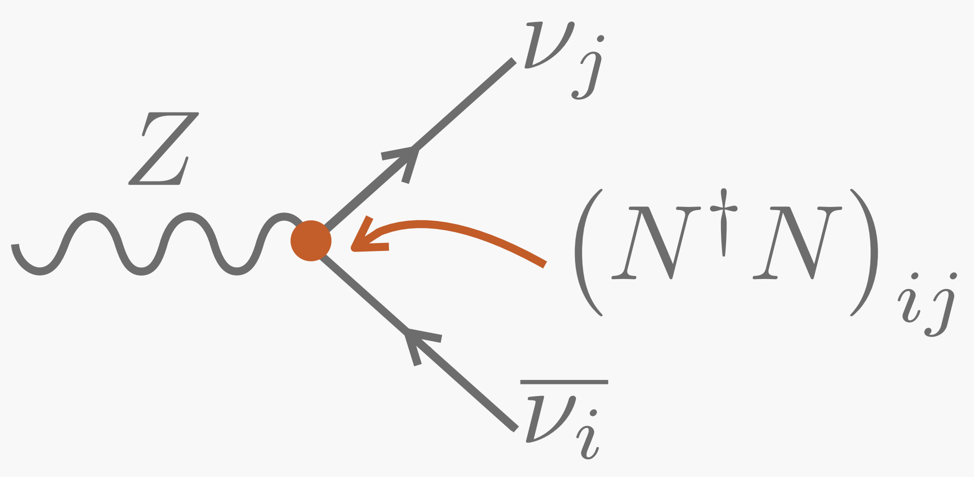


$$\mathcal{L}_{\rm NC} \simeq \frac{g}{2c_{\rm W}} \left\{ \overline{\nu_i} \left(U^\dagger_{\rm PMNS}\right)_{i\alpha} (\delta_{\alpha\beta} - \eta_{\alpha\beta}) \gamma^\mu P_L (\delta_{\beta\gamma} - \eta_{\beta\gamma}) \left(U_{\rm PMNS}\right)_{\gamma j} \nu_j \right.$$

$$\left. - \overline{\ell_\alpha} \gamma^\mu P_L \left(1 - 2 s_{\rm W}^2\right) \ell_\alpha \right\} Z_\mu$$

$$\equiv \frac{g}{2c_{\rm W}} \left\{ \overline{\nu_i} \gamma^\mu P_L \left(N^\dagger N\right)_{ij} \nu_j - \overline{\ell_\alpha} \gamma^\mu P_L \left(1 - 2 s_{\rm W}^2\right) \ell_\alpha \right\} Z_\mu$$



SM + type-I Seesaw

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- $d = 6$: operator $\rightarrow$ non-unintary neutrino mixing

  We find deviations from unitarity in the leptonic mixing matrix

$$N = (I - \eta)\, U_{\text{PMNS}}$$

(Hermitian matrix) (Unintary matrix) $\Rightarrow$ general parametrization of the non-unitarity

SM + type-I Seesaw

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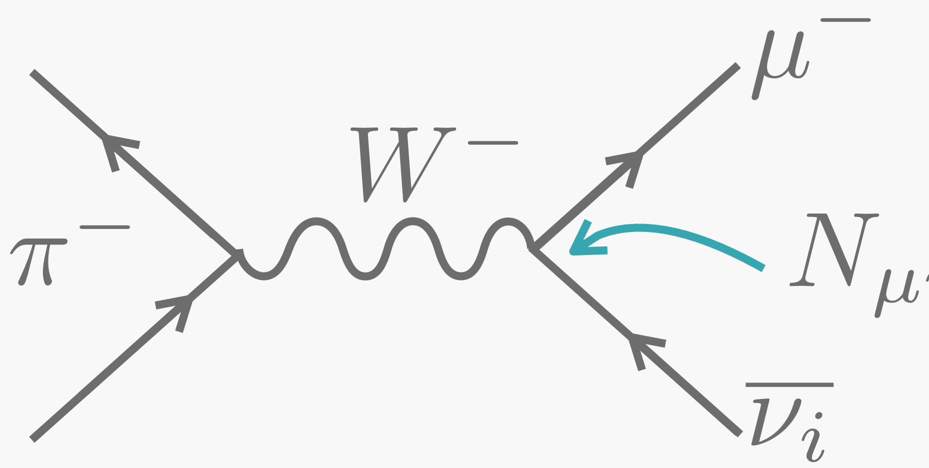
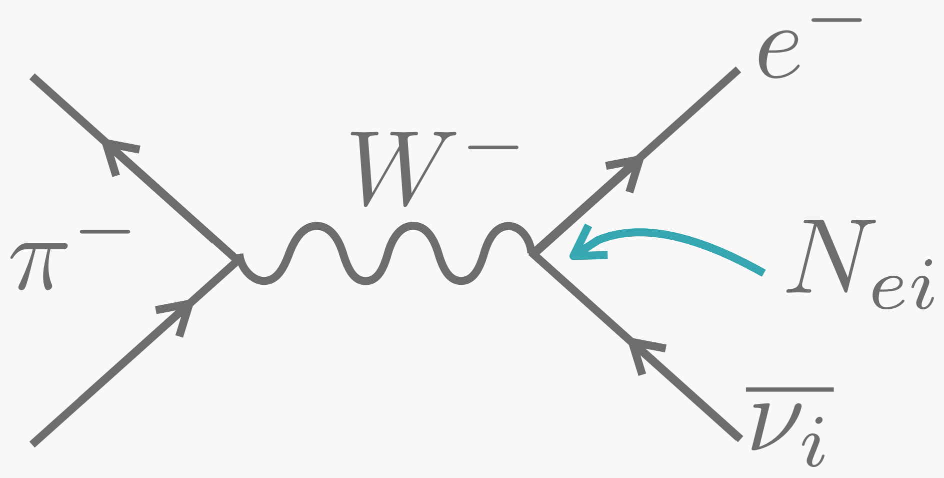
Eg: $\pi^- \to \mu^- \overline{\nu}$ vs $\pi^- \to e^- \overline{\nu}$



$$N_{\mu i} = (\delta_{\mu\alpha} - \eta_{\mu\alpha})\, (U_{\text{PMNS}})_{\alpha i}$$



$$\Gamma_{\pi\mu} \propto \sum_{\alpha,i} |N_{\mu i}|^2 = 1 - 2\eta_{\mu\mu} \quad \text{while} \quad \Gamma_{\pi e} \propto 1 - 2\eta_{ee} \quad \Rightarrow$$

CC non-universal!

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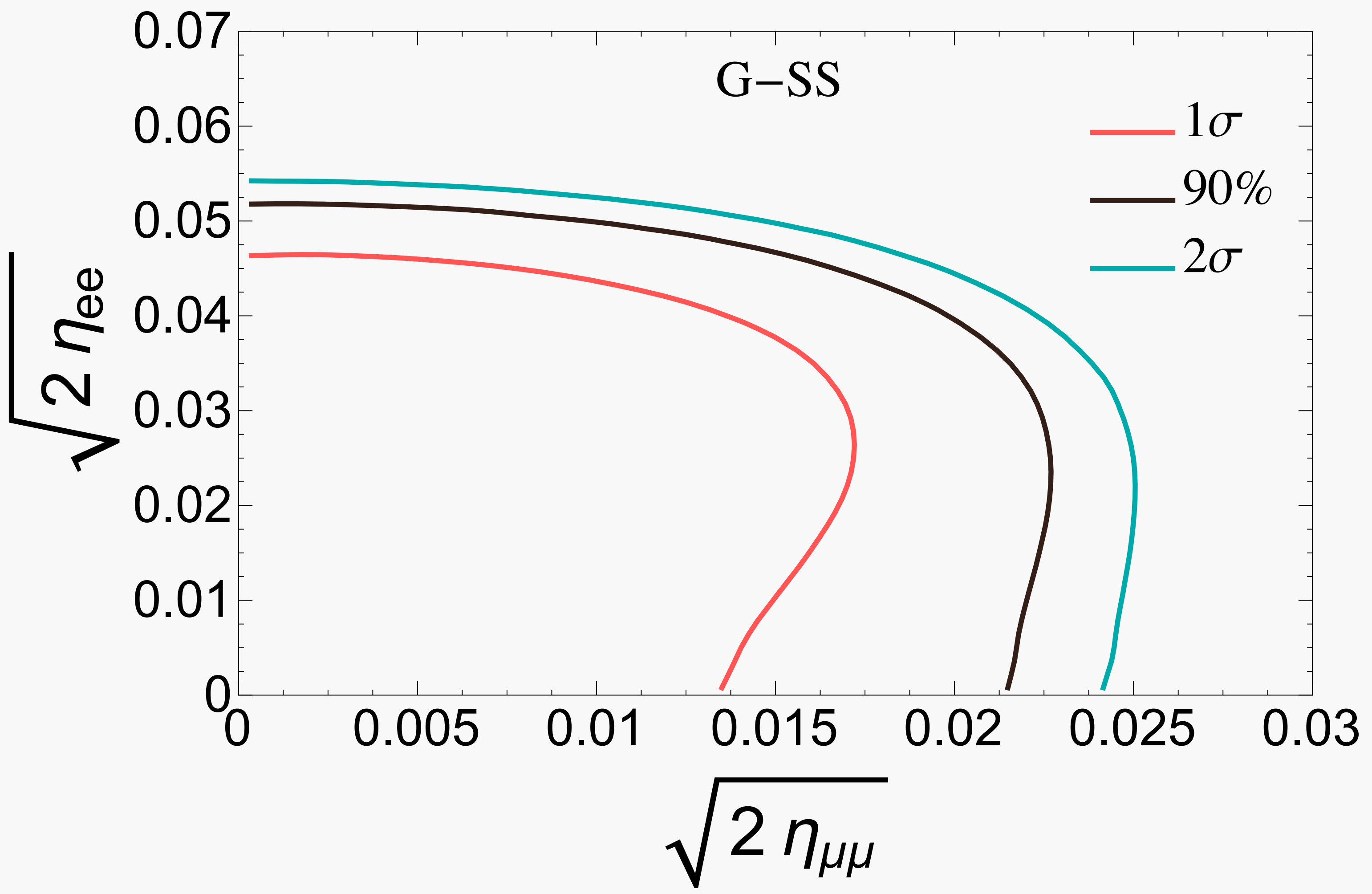
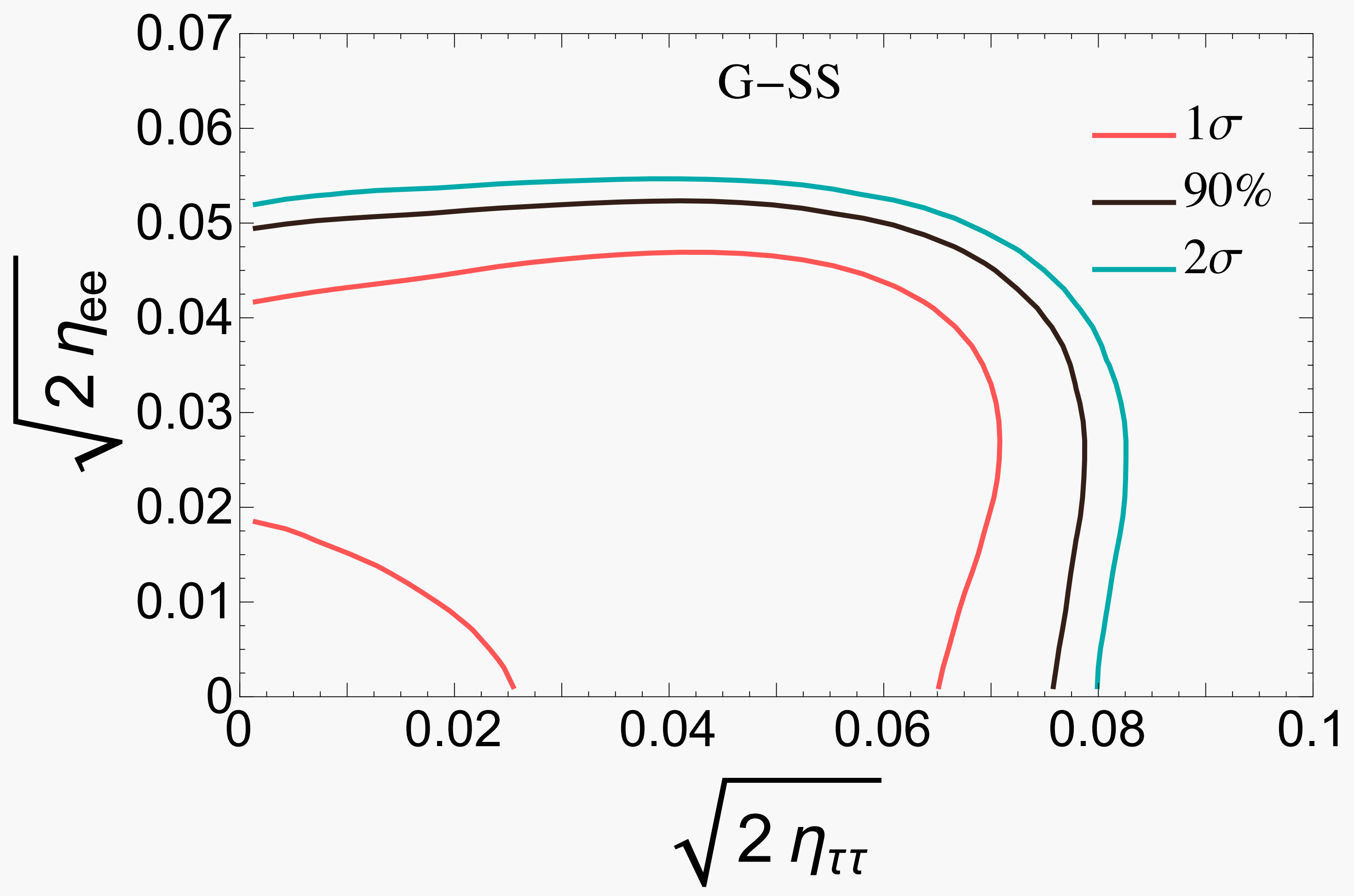
MCMC analysis with 28 observables scanning over the free parameters





Diagonal

$$\sqrt{2\eta_{ee}} = 0.031^{+0.010}_{-0.020}$$

$$\sqrt{2\eta_{\mu\mu}} < 0.011$$

$$\sqrt{2\eta_{\tau\tau}} = 0.044^{+0.019}_{-0.027}$$

Off-diagonal

LFC	LFV
$\sqrt{2\lvert\eta_{e\mu}\rvert} < 0.018$	$\sqrt{2\lvert\eta_{e\mu}\rvert} < 4.1 \cdot 10^{-3}$
$\sqrt{2\lvert\eta_{e\tau}\rvert} < 0.045$	$\sqrt{2\lvert\eta_{e\tau}\rvert} < 0.107$
$\sqrt{2\lvert\eta_{\mu\tau}\rvert} < 0.024$	$\sqrt{2\lvert\eta_{\mu\tau}\rvert} < 0.115$
Schwarz inequality	

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Matter-antimatter Asymmetry

zoom.us video

Matter-antimatter asymmetry in the Universe well measured using cosmic microwave background (CMB) radiation. From Planck data

$$Y_B^{\mathrm{CMB}} \simeq (8.67 \pm 0.09) \cdot 10^{-10}$$

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zoom.us video

Matter-antimatter asymmetry in the Universe well measured using cosmic microwave background (CMB) radiation. From Planck data

$$Y_B^{\text{CMB}} \simeq (8.67 \pm 0.09) \cdot 10^{-10}$$

Baryon asymmetry (baryogenesis) can be dynamically generated if the Sakharov conditions are satisfied

1) $\not{B}$ interactions

   $B + L$ is anomalous and transitions that violate B and L can happen via sphalerons

2) $\not{C}$ and $\not{CP}$

   $J_q \simeq 2.9 \cdot 10^{-5}$

3) Departure from thermal equilibrium

   In equilibrium, the production and destruction of a baryon asymmetry

Matter-antimatter Asymmetry

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When baryogenesis occurs at energies higher than the EW scale, besides the necessary Sakharov conditions, $B - L$ symmetry must be violated too, so that sphalerons do not wash out the baryon asymmetry.

Matter-antimatter Asymmetry

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Idea: create L in the early Universe that will be converted into B via sphaleron transitions. It is knowns as baryogenesis through leptogenesis.

Matter-antimatter Asymmetry

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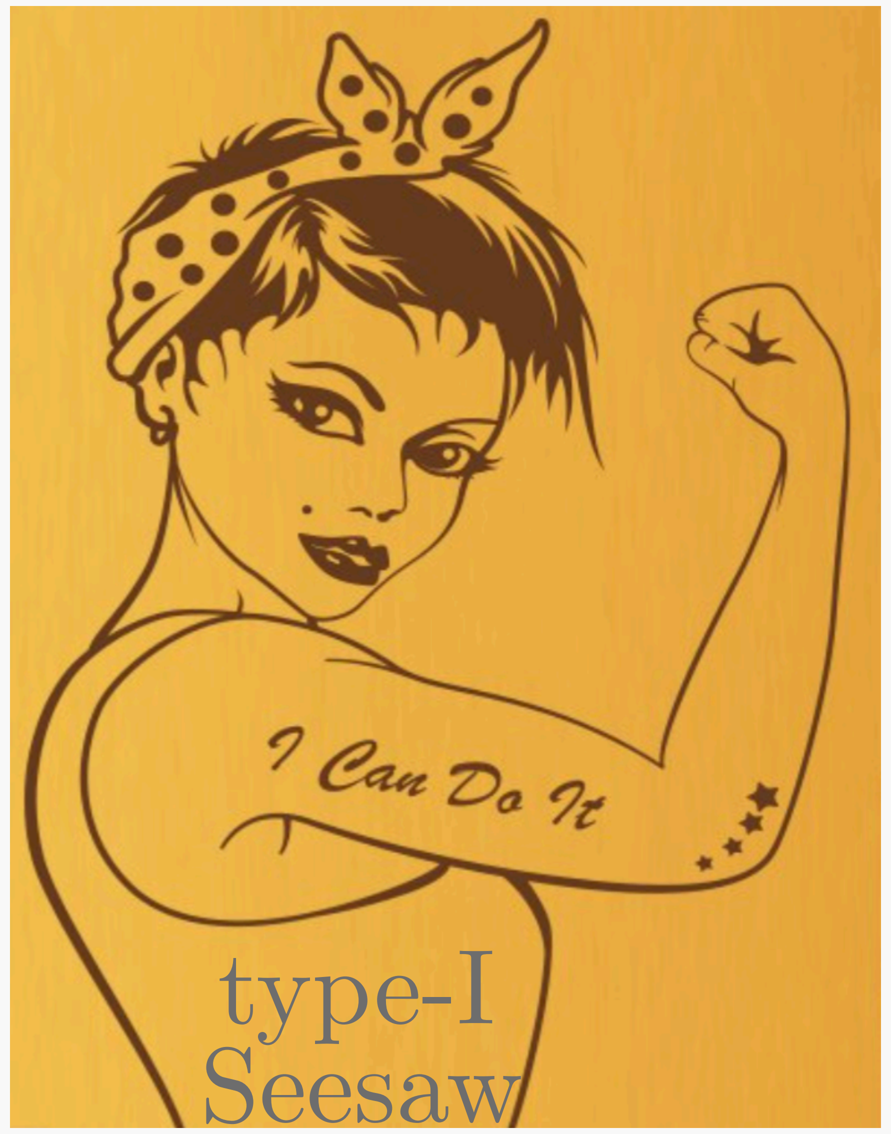

The type-I Seesaw could do that!

1) $\not{L}$ by the Majorana mass

2) $J_\nu \simeq 0.03 \sin\delta$ + additional phases if $y_N \in \mathbb{C}$

3) N decays out of eq. in the expanding Universe once $T < M_N$

Matter-antimatter Asymmetry

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At very high temperature (T)

$$N_i \leftrightarrow \phi^0 \nu_L \qquad \text{and} \qquad N_i \leftrightarrow \phi^+ \ell^-$$

Matter-antimatter Asymmetry

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If N_i are Majorana particles, then both decays are allowed

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Matter-antimatter Asymmetry

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Imagine that $\not{CP}$ and $\Gamma_{N\to l\phi} > \Gamma_{N\to \bar{l}\phi} \Rightarrow -L_0$ is created

L	$-L_0$	3	6	$\ldots$	$-L_0/2$
B	0	3	6	$\ldots$	$+L_0/2$

$$\Delta L = \Delta B = 3\Delta n_{\rm CS}$$

Matter-antimatter Asymmetry

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However, this process is not instantaneous and washout effects will partly erase the asymmetry. The remaining L asymmetry can then be converted by sphaleron processes into a B asymmetry

$$\eta_B \propto k c_s \epsilon_1$$

Matter-antimatter Asymmetry

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c_s: sphaleron constant, quantifies how much L asymmetry is converted to B

ϵ_1: CP asymmetry given by

Matter-antimatter Asymmetry

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ϵ_1: CP asymmetry given by

$$\epsilon_1 = \frac{\Gamma_{N_1 \to \ell\phi} - \Gamma_{N_1 \to \bar{\ell}\phi}}{\Gamma_{N_1 \to \ell\phi} + \Gamma_{N_1 \to \bar{\ell}\phi}} \simeq \frac{3}{16\pi v_{\text{EW}}^2} \sum_{j \neq 1} \frac{\text{Im}\left[\left(m_D m_D^\dagger\right)^2_{1j}\right]}{\left(m_D m_D^\dagger\right)_{11}} \frac{M_{N_1}}{M_{N_j}}$$

for $T > 10^{12}$ GeV and assuming $M_{N_1} \ll M_{N_2} \ll M_{N_3}$.

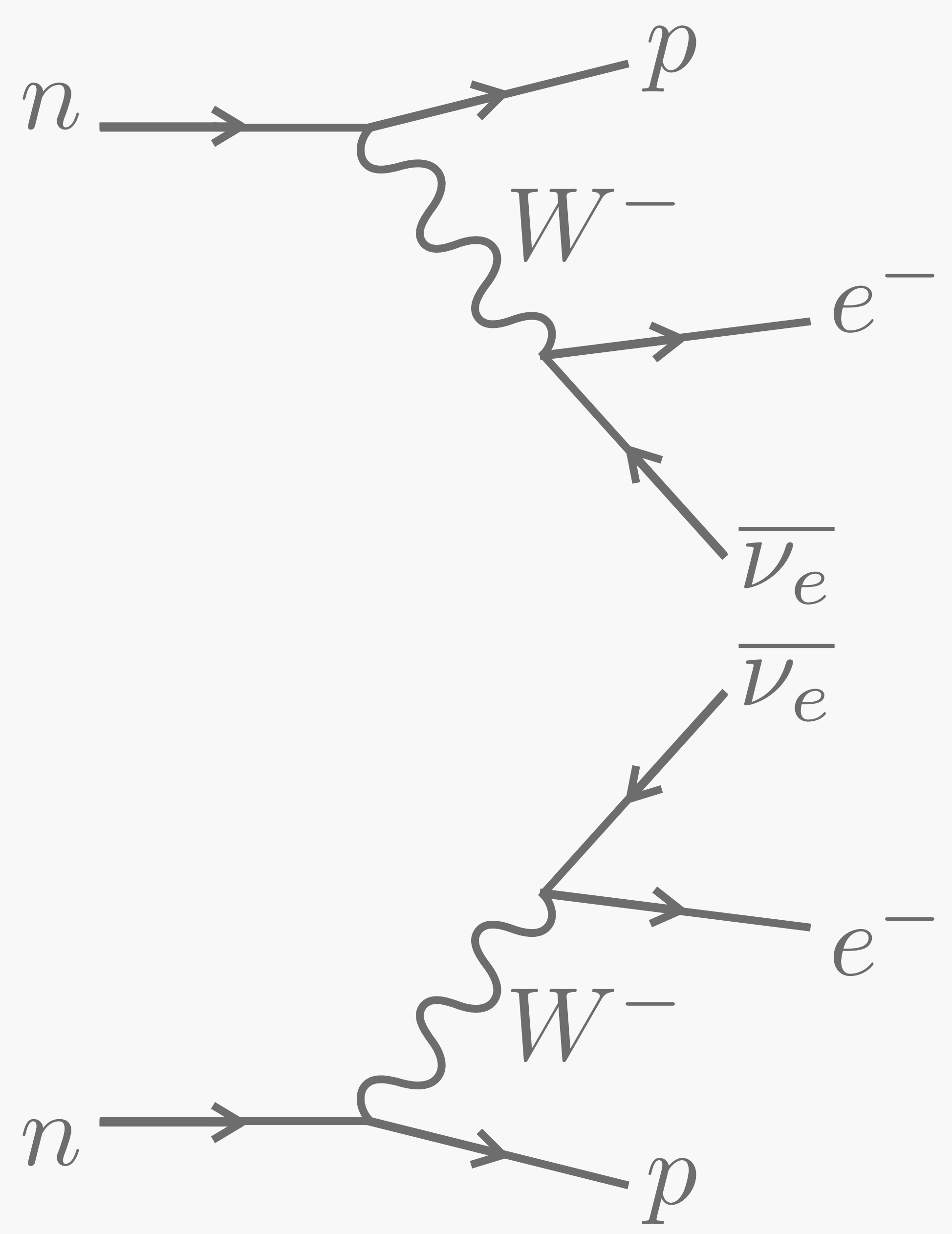
Testing the Majorana nature of ν

zoom.us video

In order to test the Majorana character of ν, we have to look for $\not{L}$ processes.

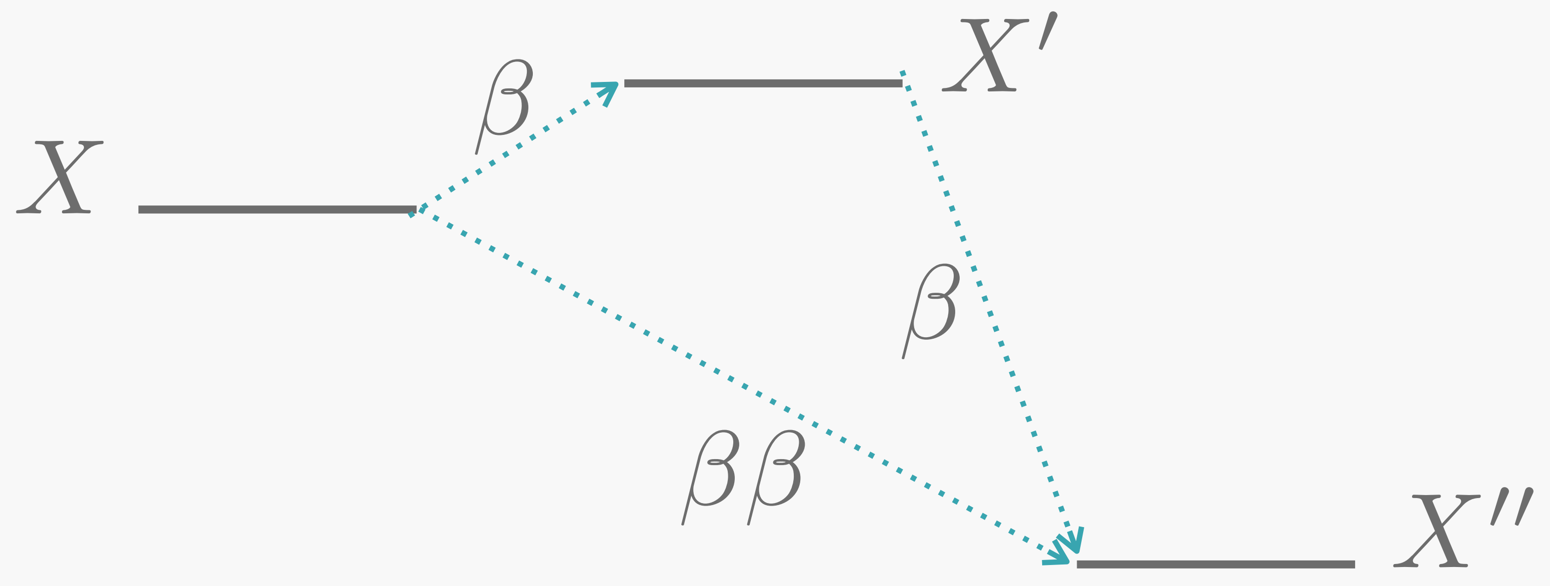
The most promising window is the neutrinoless double β-decay $(0\nu\beta\beta)$.

- Double β-decay $(2\nu\beta\beta)$



$$N(A, Z) \to N(A, Z+2) + 2e^- + 2\overline{\nu_e}$$

possible for some rare isotopes



But if ν are Majorana particles ...

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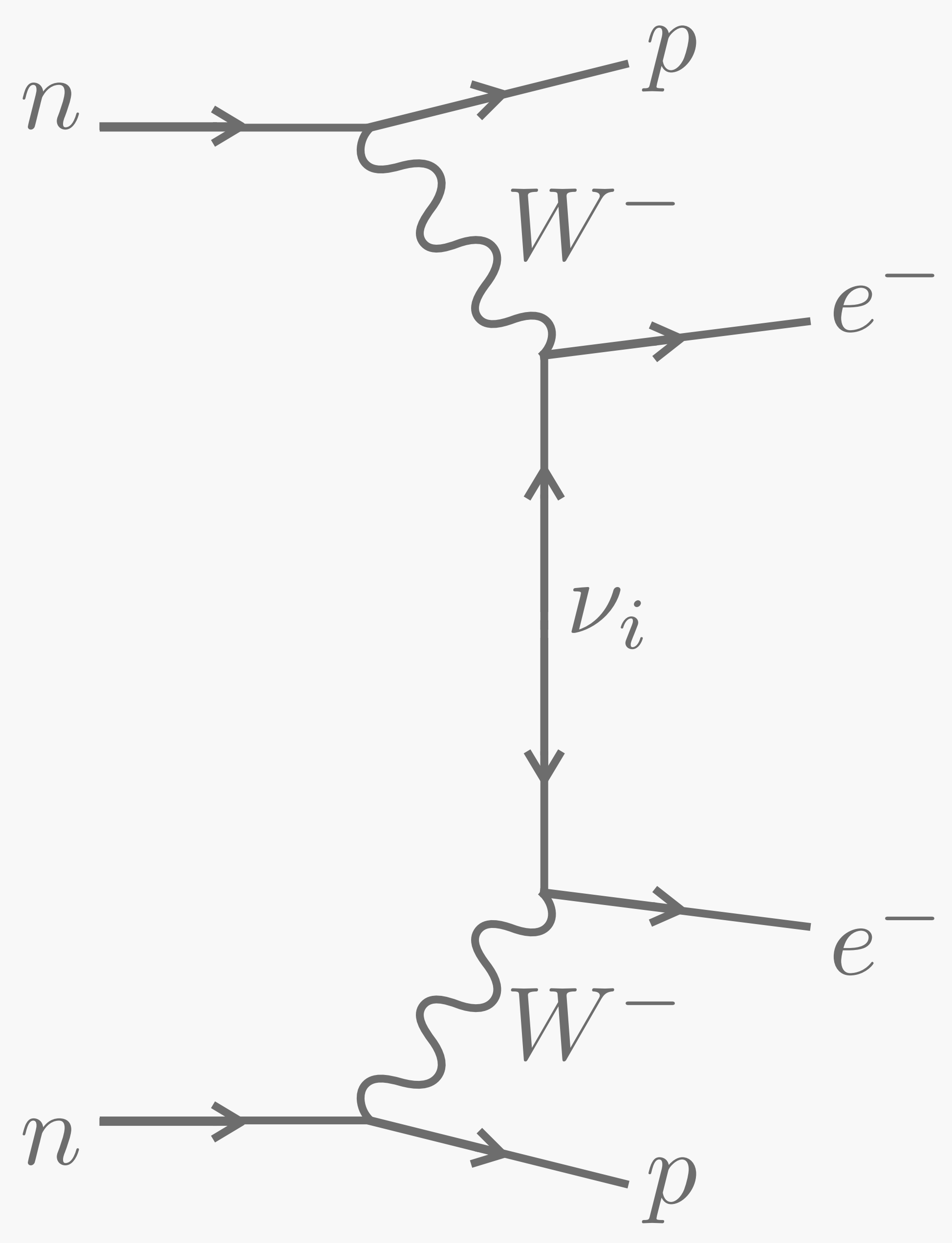


$$N(A,Z) \to N(A,Z+2) + 2e^-$$

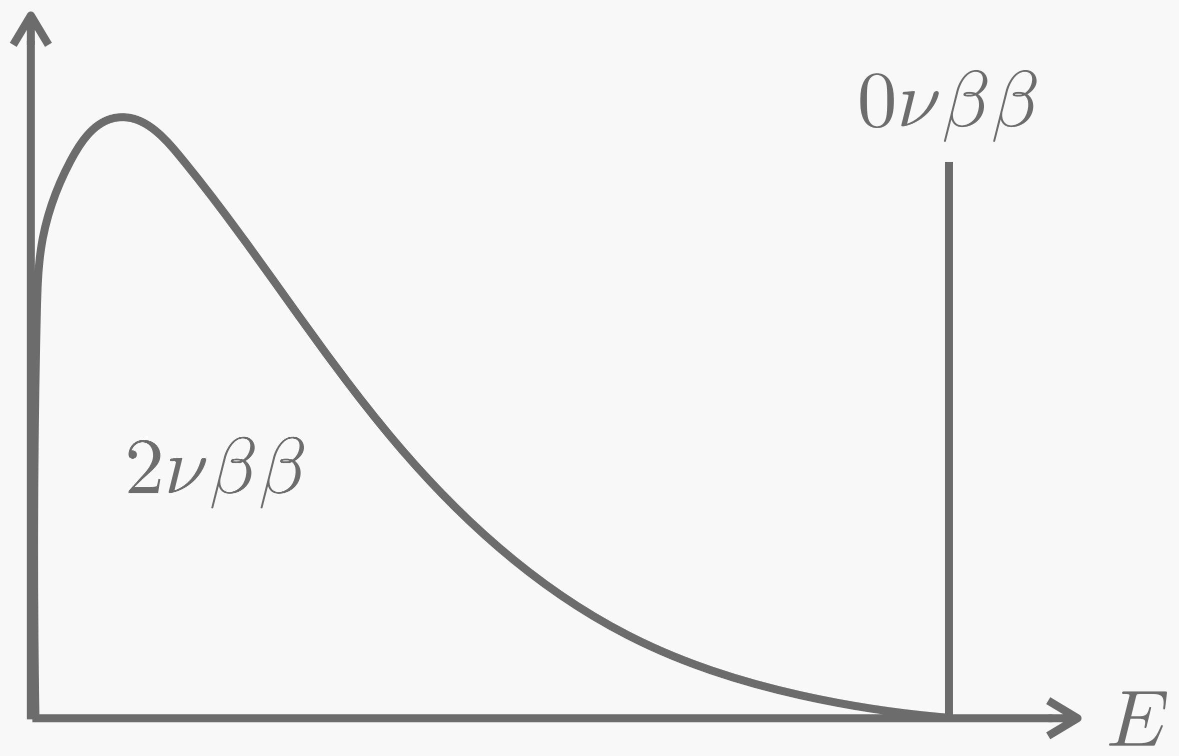
$$\propto \sum_i (U_{\mathrm{PMNS}})^2_{ei}\, m_i \equiv m_{\beta\beta}$$

effective Majorana mass

Testing the Majorana nature of ν

zoom.us video

The two processes produce different spectra



By measuring the half-life the effective Majorana mass can be constrained

$$T_{0\nu\beta\beta} \simeq \left(\frac{G_{0\nu}}{m_e} |m_{\beta\beta}|^2 \mathcal{M}_{\text{nuc}} \right)^{-1}$$

Testing the Majorana nature of ν

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We see that

$$|m_{\beta\beta}| = \left|(U_{\text{PMNS}})^2_{e1}\, m_1 + (U_{\text{PMNS}})^2_{e2}\, m_2 + (U_{\text{PMNS}})^2_{e3}\, m_3\right|$$

$$= \left|c^2_{12}c^2_{13}m_1 + s^2_{12}c^2_{13}e^{i\alpha_2}m_2 + s^2_{13}e^{i(\alpha_3-2\delta)}m_3\right|$$

Majorana phases ($e^{i\alpha_2}$, $e^{i(\alpha_3}$)

Testing the Majorana nature of ν

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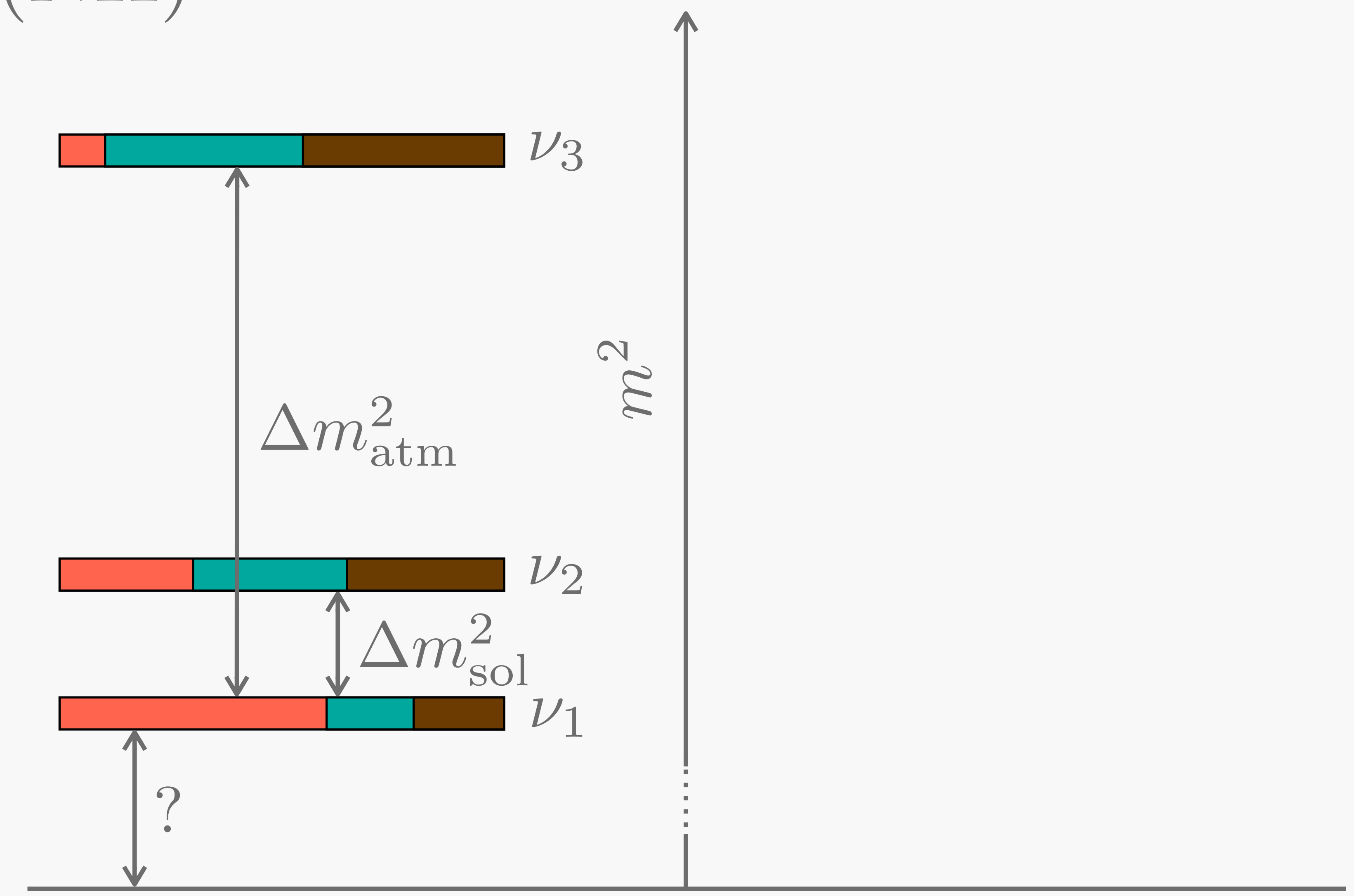
- If Normal Hierarchy (NH)

3 small terms
$\Rightarrow \ |m_{\beta\beta}| \ll$



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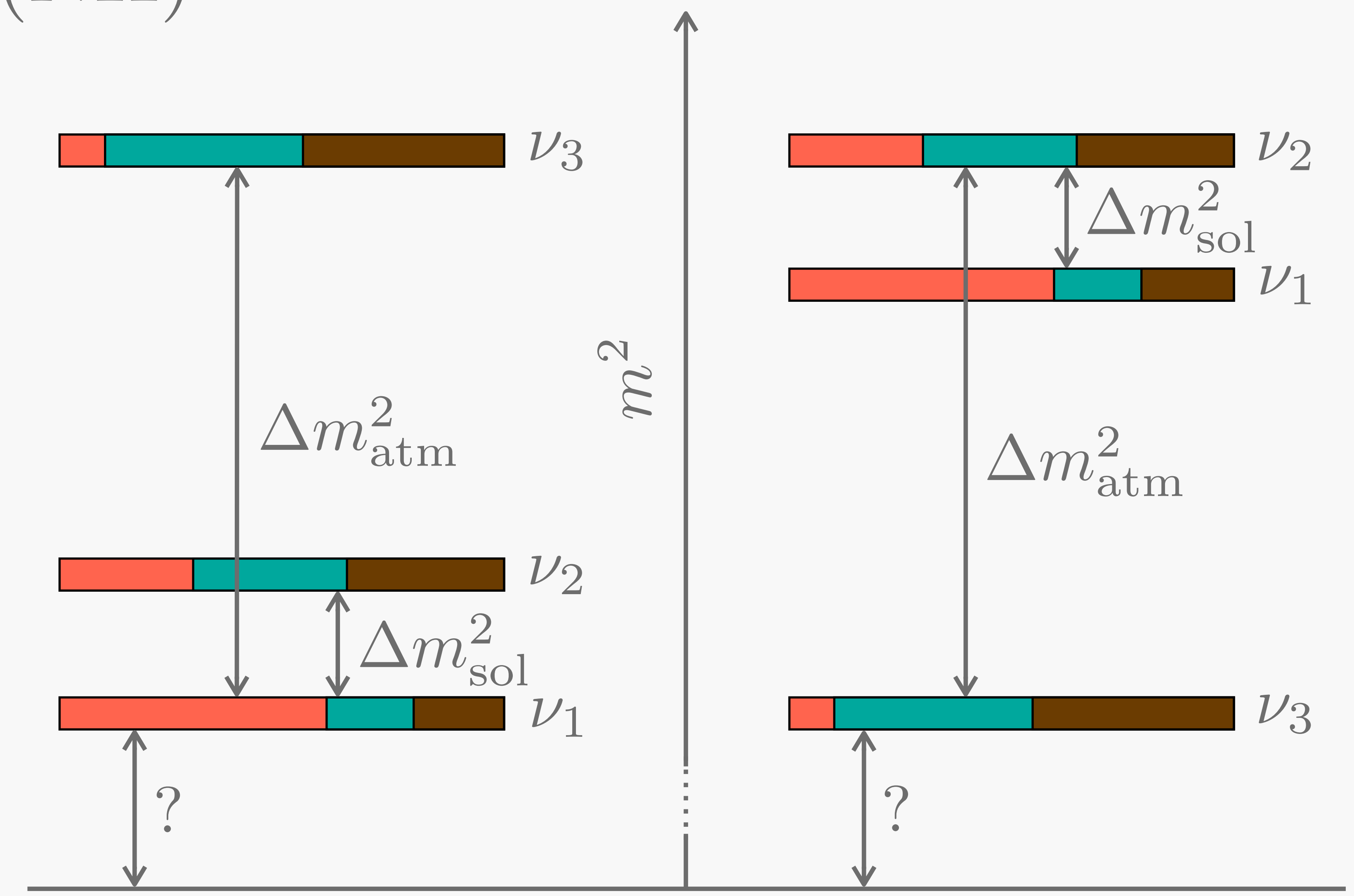
- If Normal Hierarchy (NH)
- If Inverted Hierarchy (IH)

3 small terms
$\Rightarrow \quad |m_{\beta\beta}| \ll$

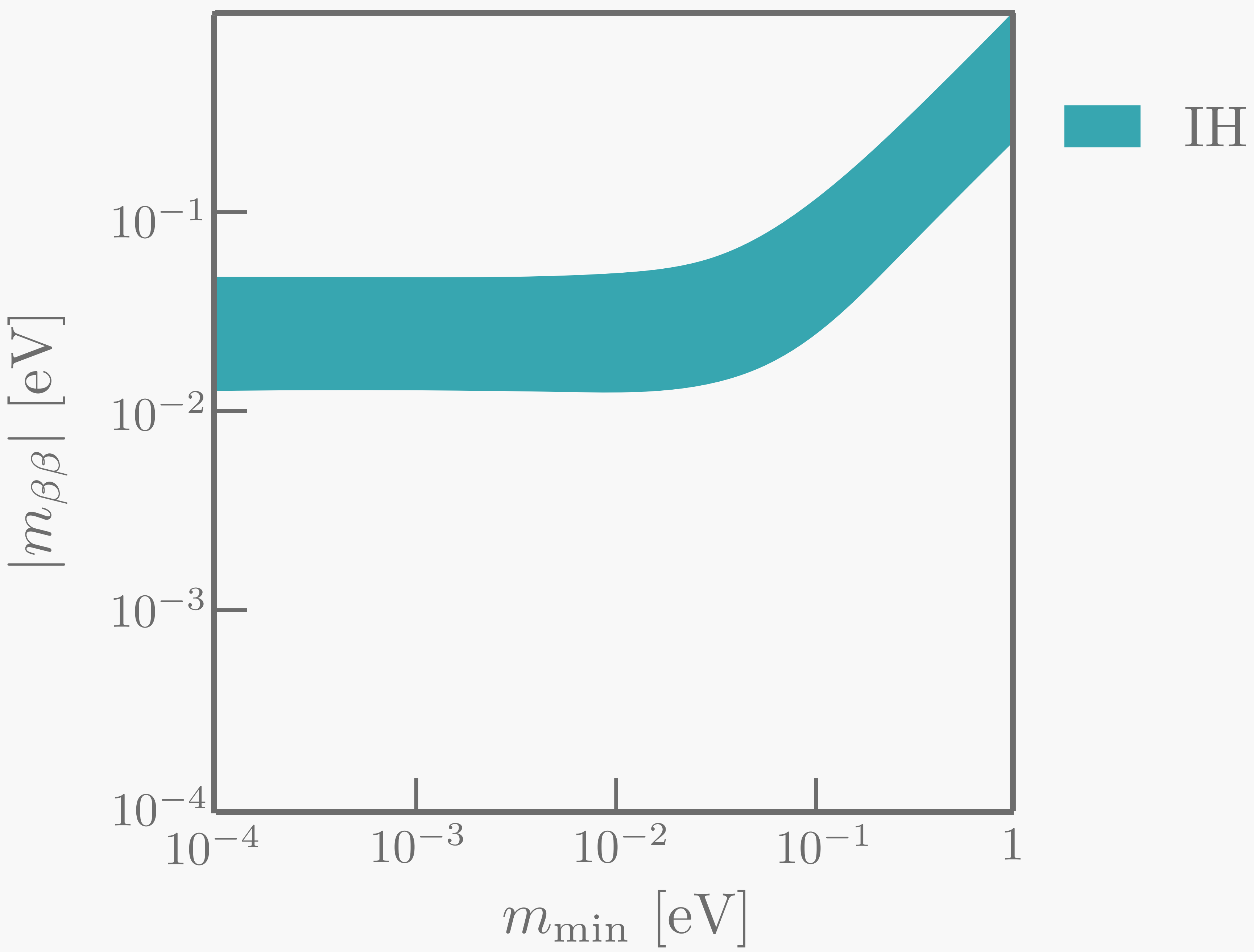


2 big terms

Testing the Majorana nature of ν

zoom.us video

By scanning over the free parameters the following allowed regions for NH and IH are obtained



Testing the Majorana nature of ν

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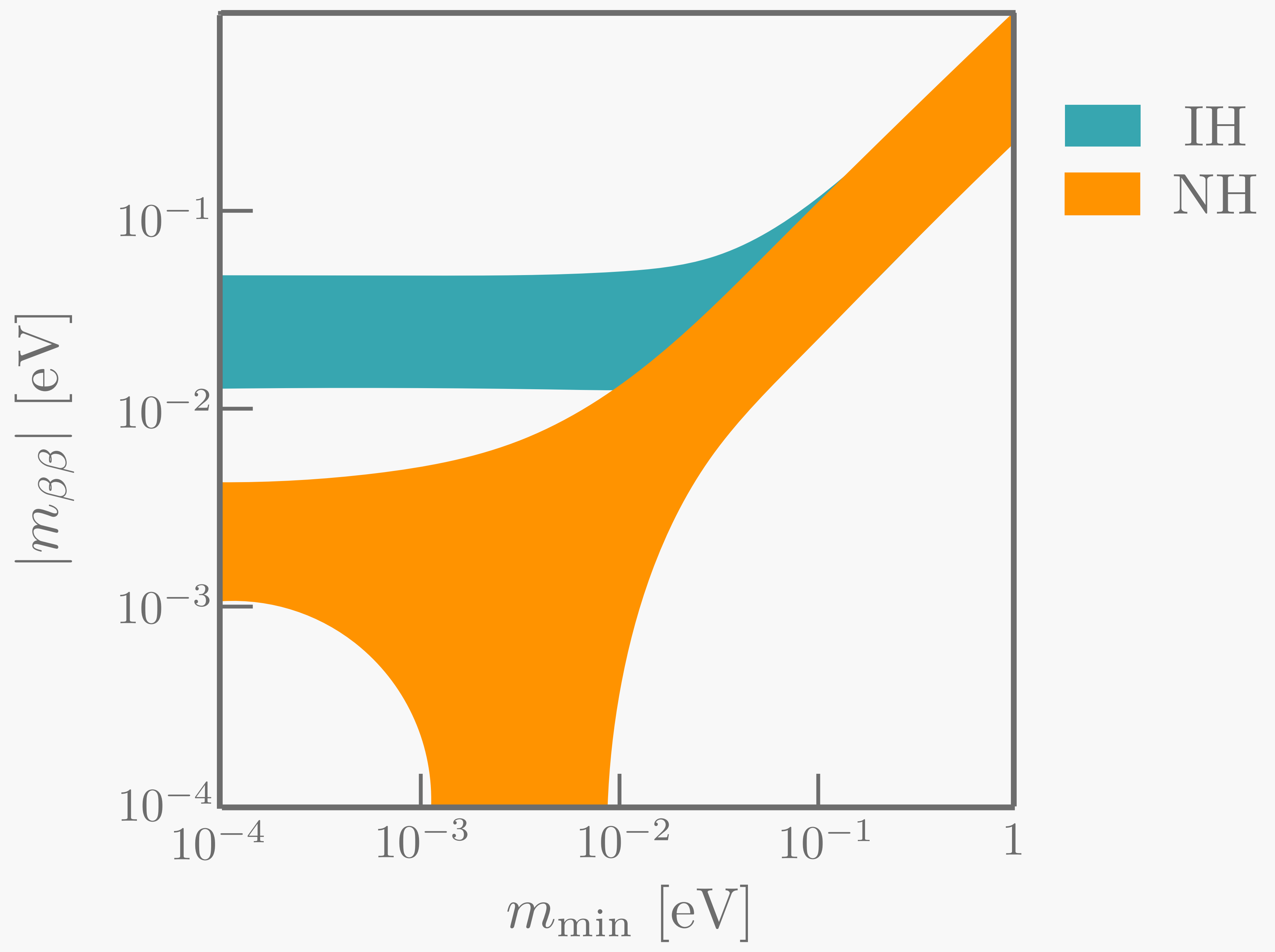

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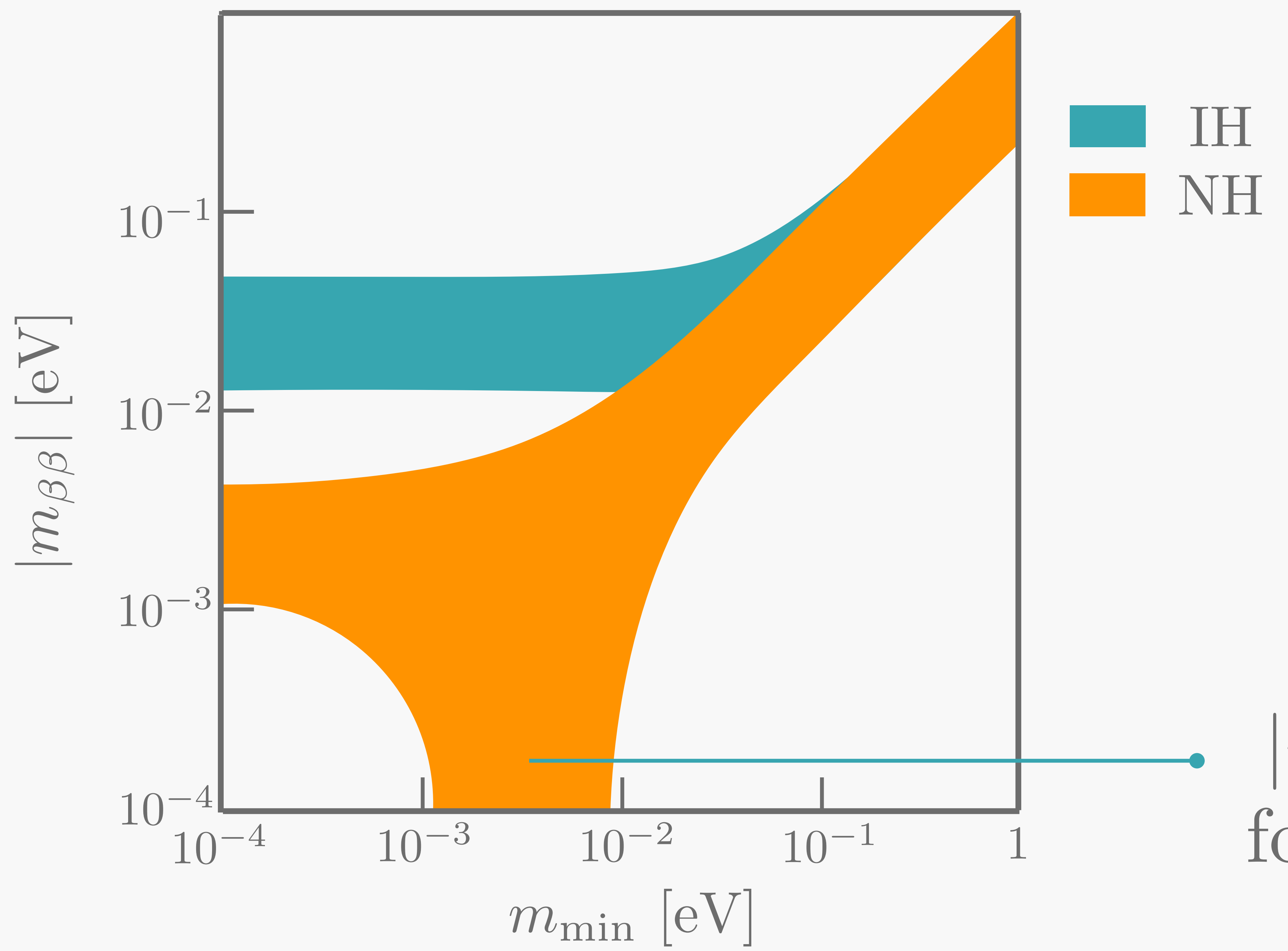

Bad news for NH:
$|m_{\beta\beta}|$ could be 0 even
for Majorana neutrinos

Testing the Majorana nature of ν

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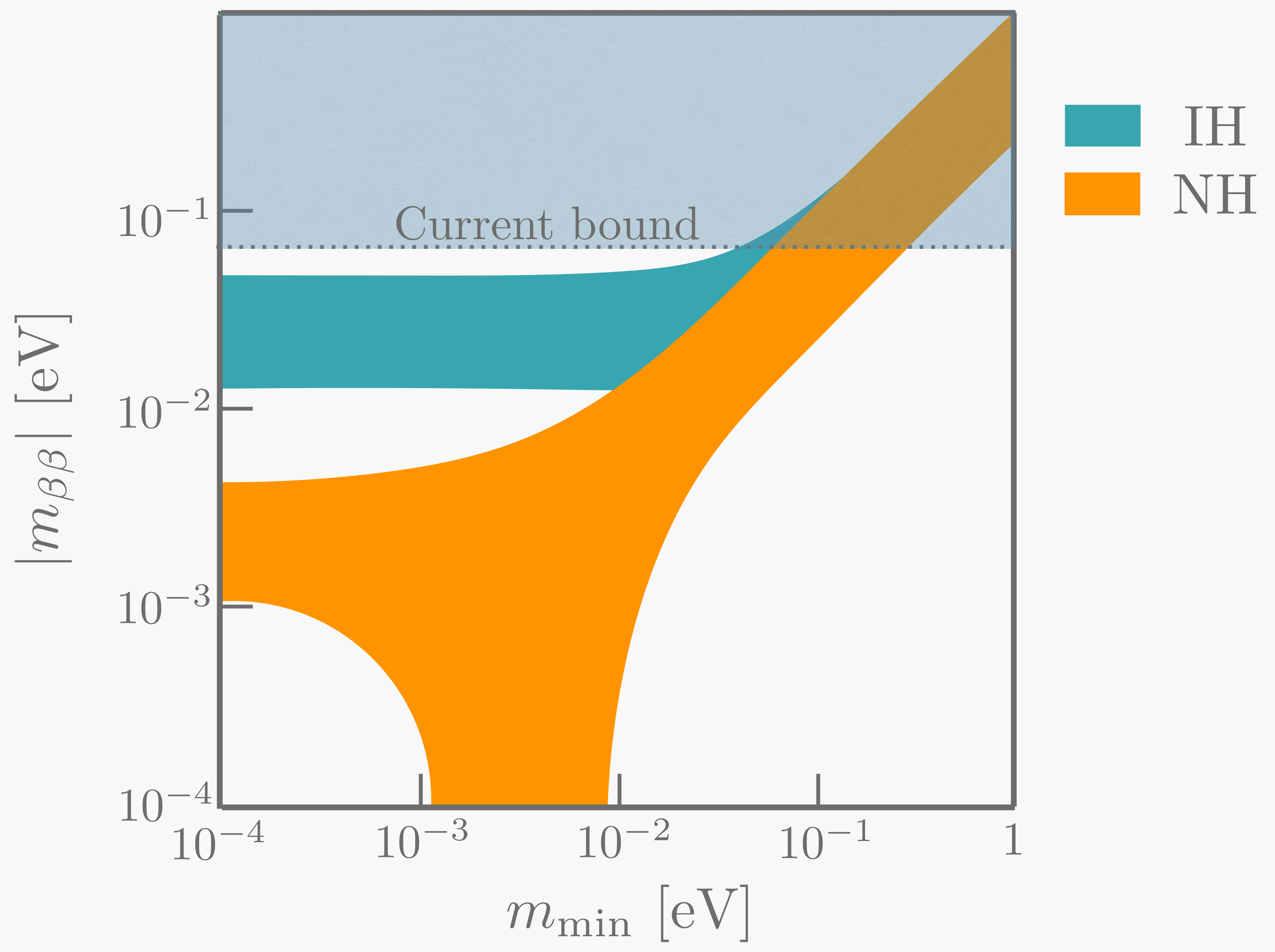

Present constrains

- GERDA (^{76}Ge)

  $T_{0\nu\beta\beta} > 0.8 \cdot 10^{26}$ y (90% CL)

- KamLAND-Zen (^{136}Xe) (present best constrain)

  $T_{0\nu\beta\beta} > 1.07 \cdot 10^{26}$ y $\Rightarrow |m_{\beta\beta}| < (61, 165)$ meV (90% CL)

Testing the Majorana nature of ν

zoom.us video

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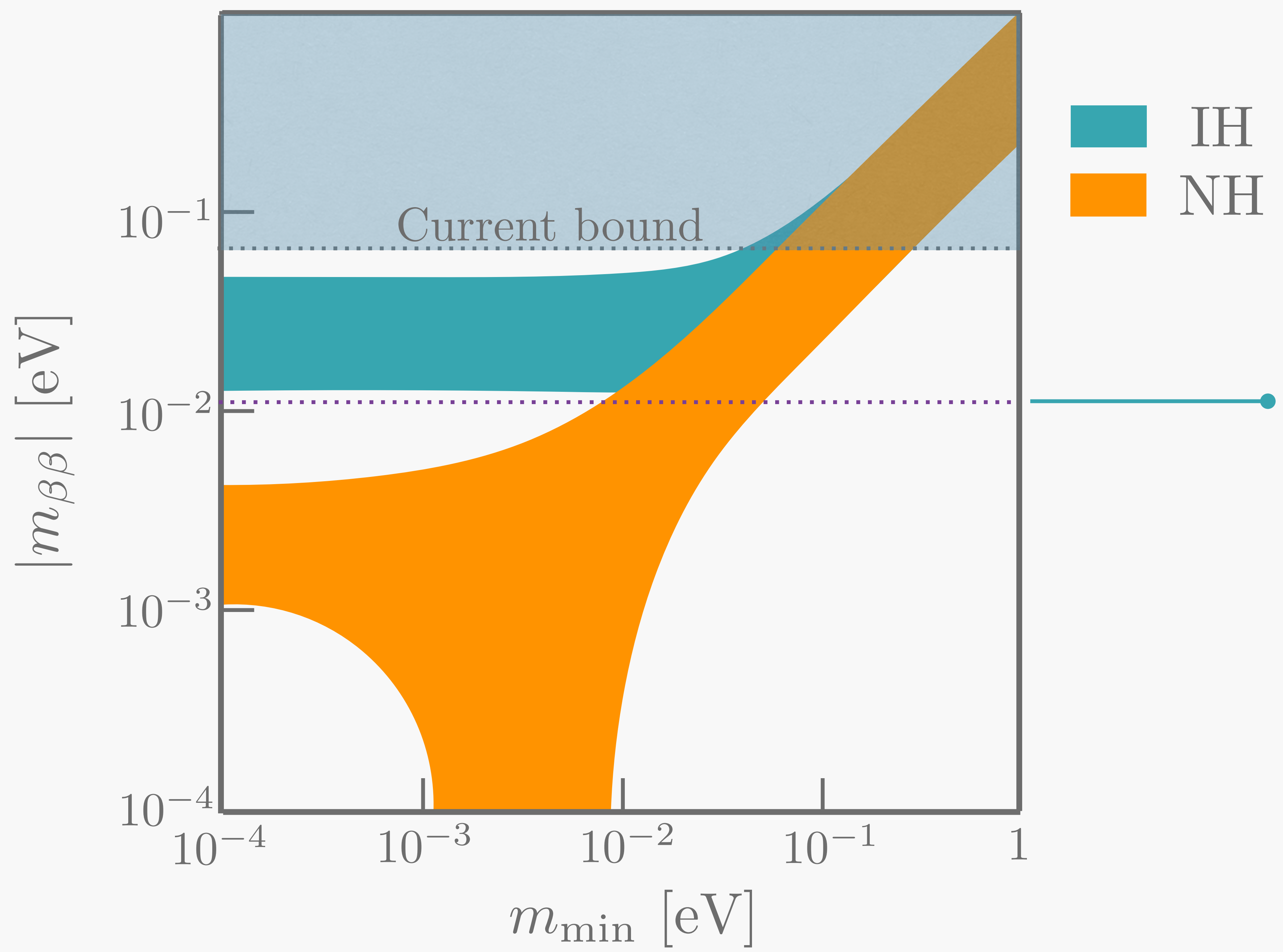

sensitivity goal next generation experiments

Future perspective

KamLAND2-Zen

SNO+

intention to cover the IH region

$|m_{\beta\beta}| < (5, 20)$ meV

Testing the Majorana nature of ν

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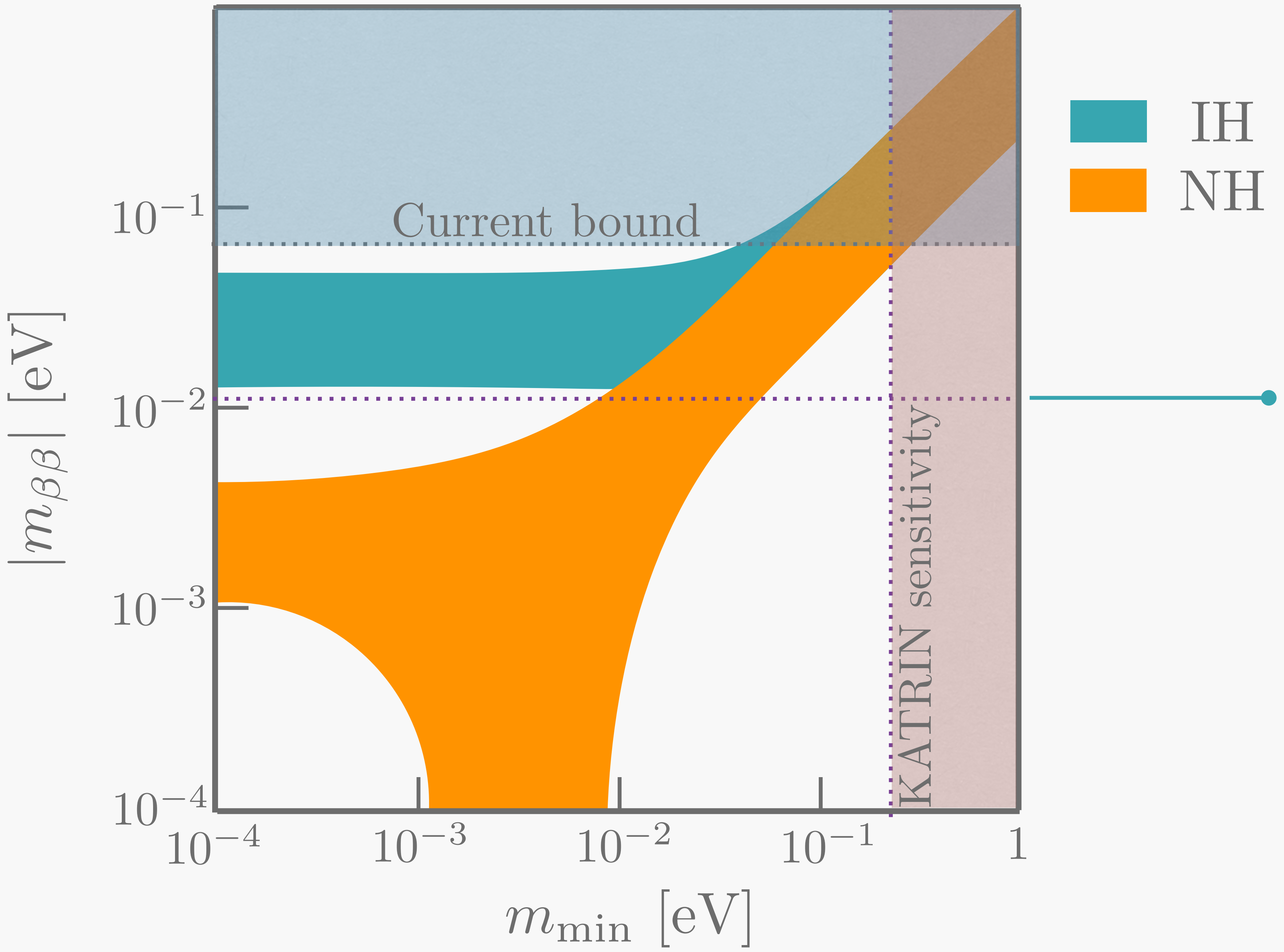

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Thanks