Is the Higgs elementary or composite?

Daniel Nogradi

ELFT Winter School 2021 Budapest

## Motivation

The Standard Model of particle physics is extraordinarily precise both experimentally and theoretically


Have already seen during this school of course ...

## Motivation

The visible Universe is correctly described by the Standard Model

Experimental and theoretical matching is in some cases $1: 10^{12}$


Yet the theory is very simple (in some sense...)

## Motivation

Last missing piece: Higgs boson

- July 4, 2012, LHC CERN, experimental confirmation
- 2013, Nobel prize to Peter Higgs, Francois Englert for theory


## Motivation

## But!

Gravity is not included $\rightarrow$
Standard Model expected to break down $\wedge_{\text {cutoff }}=10^{19} \mathrm{GeV}$
No problem for accelerators, etc.: sensitive to $\sim 10^{4} \mathrm{GeV}$

But as a result elementary Higgs boson mass is actually (in GeV )
$125=10000000000000000125-10000000000000000000$

-
$6 \cdot 10^{7}$.


## Motivation

Higgs mass (in GeV)
$125=10000000000000000125-10000000000000000000$

$$
m_{\text {Higgs }}^{2}=m_{\text {bare }}^{2}-\mathrm{const} \Lambda_{\text {cutoff }}^{2}
$$

Because of additive renormalization of mass
Top loop most significant


Even though theory is renormalizable, cutoff is physical

## Motivation

This is called Naturalness problem or fine tuning problem

Why is it present for elementary scalar?

No symmetry to protect $m_{\text {Higgs }}=0 \rightarrow$ additive mass renormalization

For example fermion masses are protected by chiral symmetry $\rightarrow$ multiplicative renormalization $\rightarrow$ no Naturalness problem

## Motivation

See for more: https://inspirehep.net/literature/144074

1979 Gerard 't Hooft: Naturalness, chiral symmetry, and spontaneous chiral symmetry breaking,

Citations per year


## Motivation

Why is this a problem?

Imagine tax laws are: if profits below $\$ 200 \rightarrow$ no tax, if profits above $\$ 200 \rightarrow 30 \%$ tax

- income: \$6000, costs: \$5500
profits: $\$ 500 \rightarrow 30 \%$ tax $\rightarrow$ natural
- income: \$600, costs: \$500
profits: $\$ 100 \rightarrow$ no tax $\rightarrow$ natural
- income: \$2000, costs: \$1900
profits: $\$ 100 \rightarrow$ no tax $\rightarrow$ natural
- income: \$100.000, costs: \$80.000 profits: $\$ 20.000 \rightarrow 30 \%$ tax $\rightarrow$ natural


## Motivation

Why is this a problem?

Imagine company filing with IRS (or NAV in Hungary):

- Revenue: \$ 10 thousand quadrillion +125
- Costs: \$ 10 thousand quadrillion
- Profit $=$ Revenue - Costs $=\$ 125 \rightarrow$ no tax

IRS (or NAV) might not say definitely something is illegal, but would find it suspicious ...

## Motivation

Similarly, such a fine tuning of the Higgs mass: suspicious

Haven't seen anything like this elsewhere in Nature

Fine Tuning Problem - Hierarchy Problem - Naturalness Problem

Only present if elementary Higgs, not if composite!

Example: QCD, composite hadrons, no problem

## Motivation

Suggestion: Higgs is a composite particle

Basic building blocks can build up other composite particles

Similarly to QCD, lots of hadrons

Prediction for LHC: lots of new particles

Key: testable

## Motivation

Not the only possible solution to Fine Tuning Problem

Example: supersymmetry

Challenge to all proposals: reproduce the extraordinarily precise results we already know from Standard Model $\rightarrow$ at low energies extensions of the Standard Model should not differ much from Standard Model, only at higher energies $\sim O(10 \mathrm{TeV})$

Also: should provide testable predictions for LHC

## Motivation

Summary: replace elementary Higgs by composite particle of new, so far undetected, gauge theory coupled to new, so far undetected, fermions

Sometimes called technigluons, technifermions (but not always)

Large set of ideas

Often use QCD terminology and QCD analogy

- Motivation, Naturalness (done already)
- QCD review
- Elementary quarks, gluons
- Composite hadrons
- No fine tuning problem
- Symmtries and their breaking (chiral symmetry): explicit, spontaneous, anomalous
- Spontaneous breaking, Goldstone theorem, effective theory, chiral perturbation theory
- Composite Higgs models
- New elementary building blocks, new gauge fields and new fermions
- Predictions of new particles
- Two large classes of models: Little Higgs, Strong Dynamics
- Problems


## QCD review

Elementary quarks and gluons

$$
\mathcal{L}=-\frac{1}{2 g^{2}} \operatorname{Tr} F_{\mu \nu} F_{\mu \nu}+\sum_{i=1}^{N_{f}} \bar{\psi}_{i}\left(D+m_{i}\right) \psi_{i}
$$

$g$ : single dimensionless coupling, $D$ Dirac operator, $m_{i}$ masses for each flavor $i=1, \ldots, N_{f}$

In Nature: $N_{f}=6,(u, d, s, c, b, t)$

Symmetries: vector + axial

## QCD review - symmetries

$$
\delta \psi_{i}=i \omega_{i j} \psi_{j} \quad \delta \bar{\psi}_{i}=-i \bar{\psi}_{j} \omega_{j i}
$$

Vector symmetry, $\omega_{i j}$ Hermitian, $U=e^{i \omega}$ unitary $\rightarrow S U\left(N_{f}\right)$

Symmetry even if $m_{i} \neq 0$

$$
\delta \psi_{i}=i \omega_{i j} \gamma_{5} \psi_{j} \quad \delta \bar{\psi}_{i}=i \bar{\psi}_{j} \gamma_{5} \omega_{j i}
$$

Axial symmetry, $\omega_{i j}$ Hermitian, $U=e^{i \omega}$ unitary $\rightarrow S U\left(N_{f}\right)$
Only symmetry if $m_{i}=0$, reminder: $\left\{\gamma_{5}, \gamma_{\mu}\right\}=0,\left\{\gamma_{5}, D\right\}=0$

QCD review - symmetries

$$
S U\left(N_{f}\right) \times S U\left(N_{f}\right)
$$

Symmetry if $m_{i}=0$ : the Lagrangian $\mathcal{L}$ is invariant

Spontaneous symmetry breaking: $\mathcal{L}$ (or Hamiltonian) invariant with some $G$ but vacuum $|0\rangle$ is not

Subgroup $H \subset G$ leaves $|0\rangle$ invariant: $G / H$ coset

## QCD review - spontaneous symmetry breaking

Goldstone theorem: there are as many massless particles as the dimension of $G / H$ : one for each breaking direction


Breaking direction is flat because $\mathcal{L}$ invariant $\rightarrow$ no quadratic term $\rightarrow$ massless mode

QCD review - spontaneous chiral symmetry breaking

In massless QCD: $G=S U\left(N_{f}\right) \times S U\left(N_{f}\right)$ broken spontaneously, $H=S U\left(N_{f}\right)$ corresponding to axial

There are $N_{f}^{2}-1$ massless particles $\rightarrow$ pions
Two light flavors, $(u, d)$, there are 3 pions

Order parameter $\Sigma=\langle 0| \bar{\psi}_{i} \psi_{i}|0\rangle \neq 0$

If symmetry would not be spontaneously broken: $\Sigma=0$

QCD review - explicit symmetry breaking

In QCD $m_{i} \neq 0$ but for $(u, d, s)$ they are small, with $(u, d)$ even smaller
$\mathcal{L}$ is not invariant with $m_{i} \neq 0$ but change $\delta \mathcal{L}$ small

Massless Goldstones become massive but in $m_{i} \rightarrow 0$ limit $M_{\text {Goldstone }}=$ 0

$$
M_{\text {Goldstone }} \sim m^{\alpha}(1+\ldots)
$$

With $\alpha>0$

## QCD review - symmetry breaking

Consequence for the spectrum of particles

Observable particles: gauge singlet because of confinement: hadrons

$\Lambda=\wedge_{Q C D}$ dynamically generated scale

## QCD review - symmetry breaking

QCD with $N_{f}=2$ flavors $(u, d): 3$ Goldstones: $3 \pi$

QCD with $N_{f}=3$ flavors $(u, d, s): 8$ Goldstones: $3 \pi, 3 K, \eta, \eta^{\prime}$

More about $\eta, \eta^{\prime}$ Iater

In QCD: $m_{\pi}=135 \mathrm{MeV}, m_{K}=497 \mathrm{MeV}, m_{\varrho}=775 \mathrm{MeV}$
$m_{u, d}=O(M e V)$ really small, $m_{s}$ order of magnitude larger

Goldstone picture applies to $3 \pi$, less so to $3 K$

## QCD review - symmetry breaking

Summary: with $N_{f}=2$ QCD there are 3 light particles $M_{\pi} \sim m^{\alpha}$ and all the rest are heavy $M \sim \wedge$


## QCD review - Iow energy effective theory

$3 \pi$ very light $\rightarrow$ Iow energy effective theory only for them

Same situation in general with $G / H$ spontaneous symmetry breaking and Goldstone bosons

What can this low energy EFT be?

Start from the original symmetry $S U\left(N_{f}\right) \times S U\left(N_{f}\right)$, look for a variable transforming appropriately and representing the pions (Goldstones)

$$
U \in S U\left(N_{f}\right) \quad\left(g_{1}, g_{2}\right) \in S U\left(N_{f}\right) \times S U\left(N_{f}\right) \quad U \rightarrow g_{1} U g_{2}^{-1}
$$

In field theory $U(x)$

What is $\mathcal{L}_{\text {eff }}(U)$ ?

QCD review - Iow energy effective theory

What is $\mathcal{L}_{e f f}(U)$ ? (assume $m_{i}=m$ )

- Dimension 4
- Lorentz invariant scalar
- Contain 2 derivatives for kinetic term
- $S U\left(N_{f}\right) \times S U\left(N_{f}\right)$ invariant if $m=0$
- $S U\left(N_{f}\right) \times S U\left(N_{f}\right)$ breaking to $S U\left(N_{f}\right)$ if $m \neq 0$

QCD review - Iow energy effective theory

$$
\mathcal{L}_{e f f}=\frac{F^{2}}{2} \operatorname{Tr} \partial_{\mu} U(x) \partial_{\mu} U^{\dagger}(x)-m \Sigma \operatorname{Re} \operatorname{Tr} U(x)
$$

$U$ is dimensionless, $\operatorname{dim}(F)=1, \operatorname{dim}(\Sigma)=3$ (remember $\left.\Sigma=\bar{\psi}_{i} \psi_{i}\right)$

Note: $F$ is decay constant of $\pi$

Last term $\operatorname{Tr} U$ breaks $S U\left(N_{f}\right) \times S U\left(N_{f}\right)$ to $S U\left(N_{f}\right)$ :

$$
\operatorname{Tr} U \rightarrow \operatorname{Tr}\left(g_{1} U g_{2}^{-1}\right)=\operatorname{Tr}\left(g_{2}^{-1} g_{1} U\right)
$$

equals $\operatorname{Tr} U$ only if $g_{1}=g_{2} \rightarrow S U\left(N_{f}\right) \subset S U\left(N_{f}\right) \times S U\left(N_{f}\right)$ unbroken
$\mathcal{L}_{\text {eff }}$ fixed by symmetry considerations

QCD review - Iow energy effective theory

Low energy effective theory: should be good description if we only ask about properties of pions
$U(x)=e^{i T_{a} \phi_{a}(x) / F}$ where $\phi_{a}$ scalar fields, $T_{a}$ generators of $S U\left(N_{f}\right)$, $a=1 \ldots N_{f}^{2}-1$

In calculations expand $U=1+i T_{a} \frac{\phi_{a}}{F}-T_{a} T_{b} \frac{\phi_{a} \phi_{b}}{2 F^{2}}+\ldots$

Assume normalized basis $\operatorname{Tr} T_{a} T_{b}=\delta_{a b}$

## Home work

Expand $\mathcal{L}_{e f f}$ to quadratic order in $\phi$

Obtain free theory of scalar fields with mass term
$\rightarrow$ at low energy pions are free

Obtain $\alpha$ in $M_{\text {Goldstone }}=M_{\pi} \sim m^{\alpha}$

Low energy EFT can give properties of $\pi$

This was simple example, many more can be given

Note: $\mathcal{L}_{\text {eff }}$ only leading order EFT, there are loop corrections

## QCD review - anomalous symmetry breaking

Note: there is a $U(1)$ component for both vector and axial, flavor singlet

$$
\begin{array}{cc}
\delta \psi_{i}=i \psi_{i} & \delta \bar{\psi}_{i}=-i \bar{\psi}_{i} \\
\delta \psi_{i}=i \gamma_{5} \psi_{i} & \delta \bar{\psi}_{i}=i \bar{\psi}_{i} \gamma_{5}
\end{array}
$$

The axial $U(1)$ is anomalous

Anomalous symmetry breaking: $\mathcal{L}$ invariant but path integration measure not invariant

Similar to explicit breaking
No Goldstone theorem $\rightarrow \eta^{\prime}$ not light but heavy, $M_{\eta^{\prime}}=958 \mathrm{MeV}$

Summary so far: QCD has very special spectrum, light particles separated from heavy particles

All of this because of Goldstone theorem, spontaneous symmetry breaking

No elementary particles in spectrum

No fine tuning, everything is natural, no quadratic divergences

## Motivation

Class of theories considered:
new non-abelian gauge theory sector + new (massless) fermions

Spontaneous chiral symmetry breaking $\rightarrow$ electroweak symmetry breaking

Goldstone bosons $\rightarrow$ eaten by $W$ and $Z$

Spin 0 scalar composite particle (like $\sigma$-meson): Higgs

Non-perturbative dynamics (like QCD)

## Motivation

This is an old idea! (Weinberg, Susskind, ..., Iate 70's)

Many early problems

- scaled up QCD doesn't work $\left(\wedge_{Q C D}=\wedge \sim O(100) G e V\right)$
- S-parameter large?
- Higgs heavy (or Higgsless)
- many new massless particles?
- large FCNC vs. quark masses


## Motivation

Problems may be due to QCD intuition and/or perturbation theory

We have lattice tools now to address them

Let's use lattice QCD techniques to do first principle calculations

Close to conformal window $\rightarrow$ very different properties from QCD

## Motivation

Typical quantity: $\langle\mathcal{O}(x) \mathcal{O}(0)\rangle$

QFT vacuum expectation value: quantum mechanics for infinitely many degrees of freedom

$$
\langle\mathcal{O}(x) \mathcal{O}(0)\rangle=\frac{\int D A_{\mu} D \psi D \bar{\psi} \mathcal{O}(x) \mathcal{O}(0) e^{-S}}{\int D A_{\mu} D \psi D \bar{\psi} e^{-S}}
$$

Perturbation theory or other analytical approach doesn't work

Numerical evaluation (as in QCD)

## Motivation

Space-time lattice $\rightarrow$ finite number of variables $\rightarrow$ path integral finite dimensional

Reasonable lattice: $32^{4} \rightarrow 33554432$ dimensional integral (for SU(3))

Even numerical integration is hopeless

Only approach: stochastic evaluation $\rightarrow$ Monte-Carlo methods

## Motivation

Key numerical difficulty: fermions (Grassmann variables)

Numerical cost: orders of magnitude more than without fermions

Main ingredient: inversion of Dirac operator

Dirac equation $D \psi=\eta$, where $D=\gamma_{\mu}\left(\partial_{\mu}+A_{\mu}\right)+m$
Need to invert $10^{7} \times 10^{7} \ldots \ldots .10^{10} \times 10^{10}$ matrices

## Motivation

High performance computing


Supercomputers - very expensive \$\$\$

## Motivation

High performance computing


Much better price/performance

## Motivation

Computers give vacuum expectation values with statistical errors

Can extract physical quantities $\rightarrow$ compare with experiment

Key steps:

- (technical) $m \rightarrow 0$ limit need massless Goldstones, eaten by $W$ and $Z$
- $N^{4}$ lattice, $N \rightarrow \infty$, infinite volume extrapolation
- lattice spacing $a \rightarrow 0$, continuum limit

All 3 are tricky in their own way

Continuum

```
Lattice - continuum limit
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Lattice - continuum limit


Lattice - continuum limit


Lattice - continuum limit


Continuum

## Motivation

Many theories to choose from

Hope to convince you that $S U(3)$ with

$$
N_{f}=2 \text { and } R=\text { sextet is a minimal model }
$$ and is promising phenomenologically

Outline and summary

- Sextet model - expectations, conformal window
- Particle spectrum from lattice
- Light scalar - Higgs
- Running coupling
- Conclusion (caveats, difficulties, questions)

Why $S U(3)$ sextet $N_{f}=2$ ?

Dietrich, Sannino, Tuominen: hep-ph/0405209, hep-ph/0611341

- Asymptotically free
- Perturbatively: just below conformal window (Schwinger-Dyson)
- Slowly changing coupling? (FCNC vs. quark masses)
- Perturbatively: small $S$-parameter
- Complex representation: exactly 3 Goldstones $\rightarrow$ eaten by $W$ and $Z$

Why $S U(3)$ sextet $N_{f}=2$ ?

Very similar to $N_{f}=2$ QCD
$R=$ fundamental replaced by $R=$ sextet $=2-$ index - symm

But very different properties
$f_{\pi}=250 \mathrm{GeV}$
Much closer to the conformal window than QCD

## Conformal window

$S U(N)$ gauge theory with $N_{f}$ fermions in $R$

$$
\begin{aligned}
\beta(g) & =\mu \frac{d g}{d \mu}=\beta_{1} \frac{g^{3}}{16 \pi^{2}}+\beta_{2} \frac{g^{5}}{\left(16 \pi^{2}\right)^{2}} \\
\beta_{1} & =-\frac{11}{3} N+\frac{4}{3} N_{f} T(R) \\
\beta_{2} & =-\frac{34}{3} N^{2}+\left(\frac{5}{3} N+C_{2}(R)\right) 4 T(R) N_{f}
\end{aligned}
$$

Asymptotic freedom: $\beta_{1}<0$, perturbation theory reliable

$$
N_{f}<\frac{11 N}{4 T(R)}
$$

Asymptotic freedom


Conformal window, $N_{f}$-dependence

Non-trivial fixed point $\beta\left(g_{*}\right)=0$ :

Exists if $\beta_{1}<0$ and $\beta_{2}>0$
Banks-Zaks
$g_{*}=4 \pi \sqrt{-\frac{\beta_{1}}{\beta_{2}}}$
$N_{f}^{l o w}=\frac{34 N^{2}}{4 T(R)\left(5 N+3 C_{2}(R)\right)}<N_{f}<\frac{11 N}{4 T(R)}=N_{f}^{u p}$
This $N_{f}$ range is the conformal window in 2-Ioop approximation

Fixed point $g_{*}$ an IR fixed point.

## Infrared fixed point



## Conformal window, $N_{f}$-dependence

How trustworthy is this?
$N_{f}^{l o w}=\frac{34 N^{2}}{4 T(R)\left(5 N+3 C_{2}(R)\right)}<N_{f}<\frac{11 N}{4 T(R)}=N_{f}^{u p}$
Upper end of the conformal window: loss of asymptotic freedom $\rightarrow$ perturbation theory is trustworthy, even 1-loop is enough
$g_{*}=4 \pi \sqrt{-\frac{\beta_{1}}{\beta_{2}}}$ is small because $\beta_{1}$ is small
Lower end of the conformal window: 2-loop is suspect
$g_{*}=4 \pi \sqrt{-\frac{\beta_{1}}{\beta_{2}}}$ is large because $\beta_{2}$ is small

## Conformal window, $N_{f}$-dependence

Where we know what we are doing: close to upper end of the conformal window
E.g. $N=3, R=$ fund, $N_{f}^{u p}=16.5$
E.g. $N=3, R=$ sextet, $N_{f}^{u p}=3.3$

For example $N_{f}=16$ fundamental or $N_{f}=3$ sextet: 2-loop result is probably okay, a non-trivial weakly interacting 4D CFT

## Conformal window, $N_{f}$-dependence

Even though 2-loop result is unreliable for $N_{f}^{l o w}$ the lesson is that there exists an $N_{f}^{l o w}$ but we can't compute it in perturbation theory

Is real $N_{f}^{l o w}$ smaller or larger than 2-Ioop $N_{f}^{l o w}$ ?
Probably Iarger.

As $N_{f}$ decreases from upper end of conformal window $g_{*}$ grows $\rightarrow$ if not too large still CFT $\rightarrow$ as it gets large chiral symmetry breaks $\rightarrow$ scale is generated $\rightarrow$ conformal symmetry lost $\rightarrow$ no IR fixed point $\rightarrow$ we are outside the conformal window.

## Conformal window, $N_{f}$-dependence summary


$N_{f}$ increases from left to right

## Examples

Perturbative 2-loop $N_{f}^{\text {low }}$
$S U(2)$

- $R: j=1 / 2, \quad 5.551 \ldots<N_{f}<11$
- $R: j=1, \quad 1.0625<N_{f}<2.75$
- $R: j=3 / 2, \quad 0.32<N_{f}<1.1$


## Examples

Perturbative 2-Ioop $N_{f}^{l o w}$
$S U(3)$

- $R=$ fund, $\quad 8.05 \ldots<N_{f}<16.5$
- $R=$ sextet, $1.224<N_{f}<3.3$
- $R=a d j, \quad 1.0625<N_{f}<2.75$


## $N_{f}$ just below lower end of conformal window



## Non-perturbative (lattice) studies

We only study the model in isolation as $S U(3)$ gauge theory with $N_{f}=2$ fermions in sextet

Forget about rest of Standard Model

Questions for this talk

- Does chiral symmetry breaking happen?
- Particles in the spectrum? Light Higgs?
- Running coupling (is it walking?)


## Lattice setup

## Particle spectrum

- Finite lattice spacing $a$
- Finite volume $L$
- Finite fermion mass $m>0$
- Chiral limit $m \rightarrow 0$ in large volumes
- Decrease lattice spacing (2 values at the moment)
- Express things in chiral limit in dimensionless combinations
- $f_{\pi}=250 G e V$ scale setting


## Lattice setup

Particle spectrum

- Staggered fermions (fast!)
- Need rooting trick for $N_{f}=2$ from QCD: as long as $m$ finite, not too small, it's okay
- Stout-improvement
- Symanzik tree level improved gauge action
- $\beta=3.20$ and 3.25


## Lattice

## Particle spectrum (staggered fermions)

## Using QCD terminology consider

$$
m_{\pi} \quad f_{\pi} \quad m_{a_{0}} \quad m_{\rho} \quad m_{a_{1}} \quad m_{N} \quad m_{\eta^{\prime}} \quad m_{f_{0}}=m_{0++}=m_{\text {Higgs }}
$$

## Lattice - finite volume effects



Already at $\beta=3.20$ and $m=0.003,32^{3}$ is not enough, $m_{\pi} L>6-7$ needed

## Lattice - pseudo-scalar meson




Unable to resolve chiral logs

## Lattice - pseudo-scalar meson



Much stronger m-dependence than in QCD

## Lattice - pseudo-scalar meson



Note the different slopes, in QCD parallel

## Lattice - vector mesons $\varrho$ and $a_{1}$




Within reach of LHC Run 2

Small splitting: $S \sim V V-A A$, small?

## Lattice - scalar mesons $f_{0}$ and $a_{0}$



Remember $f_{0}$ is the Higgs!
Difficult channel, disconnected fermion graphs $\beta=3.25$ preliminary, topology?

## Lattice - baryons

Baryon states very diferent from QCD
$3 \otimes 3 \otimes 3=1 \oplus 2 \times 8 \oplus 10$
$6 \otimes 6 \otimes 6=1 \oplus 2 \times 8 \oplus 10 \oplus \overline{10} \oplus 3 \times 27 \oplus 28 \oplus 2 \times 35$

But!
singlet in QCD: $\quad \varepsilon_{a b c} \psi_{a} \psi_{b} \psi_{c}, \quad \epsilon_{a b c}$ anti-symmetric
singlet in sextet:
$a, b, \ldots=1,2,3$
$\varepsilon_{a b c} \varepsilon_{d e f} \psi_{a d} \psi_{b e} \psi_{c f}=T_{A B C} \psi_{A} \psi_{B} \psi_{C}$
$A, B, C=1,2,3,4,5,6$
$T_{A B C}$ symmetric

## Lattice - baryons

As a result, very different wave functions
"color": symmetric, spin-flavor: anti-symmetric

Non-relativistic notation (suppress "color" index):

$$
\begin{gathered}
|\psi\rangle=|\uparrow u, \uparrow d, \downarrow u\rangle+|\downarrow u, \uparrow u, \uparrow d\rangle+|\uparrow d, \downarrow u, \uparrow u\rangle- \\
|\downarrow u, \uparrow d, \uparrow u\rangle-|\uparrow d, \uparrow u, \downarrow u\rangle-|\uparrow u, \downarrow u, \uparrow d\rangle
\end{gathered}
$$

## Lattice - baryons




Dark matter?

## Lattice - $\eta^{\prime}$

## Extract from gluonic operator

$$
q(x)=\frac{1}{32 \pi^{2}} \varepsilon_{\mu \nu \rho \sigma} F_{\mu \nu}(x) F_{\rho \sigma}(x)
$$

$$
-\langle q(x) q(y)\rangle \sim \frac{K_{1}\left(m_{\eta^{\prime}} r\right)}{r} \quad r=|x-y|
$$

Measure on gradient flow $\rightarrow$ less noisy

## Lattice - $\eta^{\prime}$

$\beta=3.20$, preliminary



## Lattice - $\eta^{\prime}$

$\beta=3.25$, preliminary


## Spectrum summary 1

$$
\left.\begin{array}{rlrl}
m_{f_{0}} / f_{\pi} & \sim 1-2 & m_{f_{0}} & \sim 250-500 \mathrm{GeV} \\
m_{a_{0}} / f_{\pi} & \sim 6-8 & m_{a_{0}} & \sim 1.5-2 \mathrm{TeV} \\
m_{\varrho} / f_{\pi} & \sim 7-8 & m_{\varrho} & \sim 1.8-2 \mathrm{TeV} \\
m_{a_{1}} / f_{\pi} & \sim 10-11 & m_{\varrho} & \sim 2.5-2.7 \mathrm{TeV} \\
m_{N} / f_{\pi} & \sim 11-14 & & m_{N} \sim 2.7-3.5 \mathrm{TeV} \\
m_{\eta^{\prime}} / f_{\pi} & \sim 13-18 & & m_{\eta^{\prime}}
\end{array}\right)
$$

Higgs at $250-500 \mathrm{GeV}$ ? ?

What we measure is not "the" Higgs

Coupling to SM: top loop

$$
m_{\text {Higgs }}^{2}=m_{\text {sextet } f_{0}}^{2}-\text { const } m_{\text {top }}^{2}
$$

Foadi, et al.

Other particles expected to be effected less

## Spectrum summary 2

Model seems consistent with $\chi P T$

Model gives rise to a light scalar

New particles with definite properties in 2-3 TeV region

Potential dark matter candidate as well

## Important caveats

- Slow topology change
- Unestimated systematics
- Only $20-30 \%$ change in lattice spacing
- Coupled scalar-pion dynamics ignored in $\chi P T$
- etc.

More lattice results - running coupling

Running scale: $\mu$

Need: $1 / L<\mu<1 / a$

Separating 3 scales difficult, instead

$$
1 / L=\mu<1 / a
$$

Running scale is finite volume

## Running coupling

Running scale: $\mu=1 / L$, using gradient flow

$$
g^{2}(L) \sim\left\langle t^{2} E(t)\right\rangle \quad c=\frac{\sqrt{8 t}}{L}=\mathrm{const}
$$

Discrete $\beta$-function: $\frac{g^{2}(s L)-g^{2}(L)}{\log \left(s^{2}\right)} \quad s=3 / 2,2$

## Running coupling, extrapolated to continuum



## Running coupling summary (for sextet)

No sign of fixed point in the $0<g^{2}<6.5$ range

3-loop fixed point in $\overline{\mathrm{MS}}: g^{2}=6.28$

4-loop fixed point in $\overline{M S}: g^{2}=5.73$

Schwinger-Dyson: no fixed point

## Summary and questions

- Sextet model is a minimal composite Higgs model
- Particle spectrum shows chiral symmetry breaking
- Light scalar emerges
- Running coupling consistent with it


## Summary and questions

- Lower end of conformal window $\rightarrow$ light scalar?
- Slow running $\rightarrow$ light scalar?
- Why light? Dilatation symmetry?
- $m_{\rho} / f_{\pi} \sim 8$ for $S U(3)$ largely $N_{f}$ and $R$ independent?


## Work in progress and future outlook

Haven't talked about lots of things

- Chiral condensate from Dirac eigenvalues (GMOR)
- Mass anomalous dimension
- Thermodynamics
- etc.

Thank you for your attention!

