Introduction to SARAH

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What is SARAH?

"SARAH is a Mathematica package for building and analyzing SUSY and non-SUSY models. It calculates all vertices, mass matrices, tadpoles [sic] equations, one-loop corrections for tadpoles and self-energies, and two-loop RGEs for a given model. SARAH writes model files for FeynArts, CalcHep/CompHep, which can also be used for dark matter studies using MicrOmegas, the UFO format which is supported by MadGraph 5 and for WHIZARD and OMEGA."

- SARAH webpage (2.2.21)

SARAH is a Mathematica extension

- Developed by Florian Staub and first version released in 2008. Since 2019, the software is maintained by Mark Goodsell and Werner Porod.
- Version: 4.14.4, last update 17.12.20.
- Webpage: `https://sarah.hepforge.org/`
- Discussion forum: `http://wernersarah.de/sarah_forum/`
  - Developers are active and answer quite soon!
  - `http` instead of `https`, so do not re-use your bank/university password...
- Online GitLab reference: `https://gitlab.in2p3.fr/goodsell/sarah/wikis/home/`
- Manuals by <Florian Staub> and <Avelino Vicente>.

How do I implement my model?

- Symmetries and Lagrangian $\Rightarrow$ Model file
- Particle content $\Rightarrow$ Particle file
- Model parameters $\Rightarrow$ Parameter file
- SARAH comes with more than 50 complete models already implemented.
- **Nota bene!** Most of them are supersymmetric, so more than likely your favourite model is not included in the way you wish.
- **Nota bene!** Effective operators are not supported.
- Do not re-invent the wheel! There is probably a model resembling your favourite model in the catalogue. It is easier to tinker with working model than to construct everything from the start.

```
Off[General::spell] (* Switch off warnings *)

Model`Name = "Hungarion";
Model`NameLaTeX ="Model to tax $X_{17}$";
Model`Authors = "Viktor Orbán";
Model`Date = "2010-05-29";

(* 2020-11-11 : model implemented, based on
arXiv:1910.10459 *)
```

Let's get started! What symmetries you have?

- SARAH was created for supersymmetric models, but it works just fine for non-supersymmetric models, too. I will not cover SUSY models.
- Global symmetries
  - Supported: U(1) and $\mathbb{Z}_n$.
- Gauge symmetries
  - U(1), SU(n), SO(n) Sp(2n), E_n
- SARAH automatically checks gauge anomalies, generates the kinetic and self-interaction terms of gauge bosons:

$$\mathcal{L} = \sum_{\text{group } G} \sum_{a=1}^{\dim G} -\frac{1}{4}(F_G)^a_{\mu\nu}(F_G)^{a\mu\nu}$$

$$(F_G)^a_{\mu\nu} = \partial_\mu (V_G)^a_\nu - \partial_\nu (V_G)^a_\mu + g_G f_G^{abc} (V_G)_{\mu b} (V_G)_{\nu c}$$

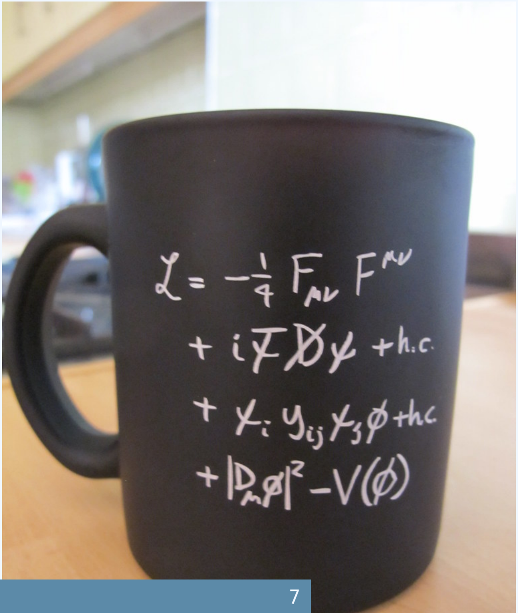
Gauge coupling variable name should start with "g".

```
(* Standard model  *)
Gauge[[1]]={B,   U[1], hypercharge, g1,False};
Gauge[[2]]={WB, SU[2], left,        g2, True};
Gauge[[3]]={G,  SU[3], color,       g3,False};

FermionFields[[1]] = {q, 3, {uL, dL},  1/6, 2,  3};
FermionFields[[2]] = {d, 3, conj[dR],  1/3, 1, -3};
...
```

1. Name
2. Number of generations
3. $SU(2)_L$ components. All fermions must be defined as LH.
4. Gauge group charges in order of appearance. Adjoint: $\mathbf{3}^* \equiv -3$.
5. Global charges (not in SM).

You should know the Lagrangian



⇐ Gauge sector

⇐ Kinetic sector

⇐ Yukawa sector

⇐ Scalar sector

Let's write the Lagrangian in gauge eigenstates

```
(* Standard model *)
NameOfStates={GaugeES, EWSB};

DEFINITION[GaugeES][LagrangianInput]= {
    {LagHC,  {AddHC->True}},
    {LagNoHC,{AddHC->False}}
};

LagNoHC = - mu2 conj[H].H
          - 1/2 \[Lambda] conj[H].H.conj[H].H;
LagHC = -(Yd conj[H].d.q + Ye conj[H].e.l + Yu u.q.H);
```

- Split the Lagrangian to parts with and without Hermitian conjugate. Parameters: $\mu^2, \lambda, Y_d, Y_e, Y_u$.
- SARAH expands the $SU(2)_L$ fields and take cares of generations, kinetic terms and gauge sector.

Gauge boson rotations

```
(* Standard model *)
Gauge[[1]]={B,   U[1], hypercharge, g1,False};
Gauge[[2]]={WB, SU[2], left,        g2, True};
...

DEFINITION[EWSB][GaugeSector] =
{
    {{VB,VWB[3]},{VP,VZ},ZZ},
    {{VWB[1],VWB[2]},{VWp,conj[VWp]},ZW}
};
```

Vector bosons are denoted by joining "V" and the group name.

$$\begin{pmatrix} \gamma \\ Z \end{pmatrix} = Z^Z \begin{pmatrix} B \\ W_3 \end{pmatrix}, \quad \begin{pmatrix} W^+ \\ (W^+)^* \end{pmatrix} = Z^W \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}$$

Scalar field decomposition

```
(* Standard model *)
ScalarFields[[1]] =  {H, 1, {Hp, H0}, 1/2, 2, 1};
...

DEFINITION[EWSB][VEVs]={{H0,
{v, 1/Sqrt[2]},
{Ah, \[ImaginaryI]/Sqrt[2]},
{hh, 1/Sqrt[2]}}}};
```

$$H = \begin{pmatrix} H^+ \\ H_0 \end{pmatrix}, \quad H_0 = \frac{1}{\sqrt{2}}(v + h_h + iA_h)$$

For each neutral scalar, define the VEV, CP-odd and CP-even parts with normalization, after EWSB.

Fermion mass eigenstates

We move from gauge eigenstates to mass eigenstates by diagonalizing the mass matrix and redefining the fields.

$$\overline{\psi_L} M \psi_R = \overline{\psi_L} V^T \underbrace{V^* M U^\dagger}_{M_{\text{diag}}} \underbrace{U \psi_R}_{\psi'_R} = \overline{\psi'_L} M_{\text{diag}} \psi'_R$$

```
(* Standard model *)
DEFINITION[EWSB][MatterSector]= {
{{{dL}, {conj[dR]}}, {{DL,Vd}, {DR,Ud}}},
{{{uL}, {conj[uR]}}, {{UL,Vu}, {UR,Uu}}},
{{{eL}, {conj[eR]}}, {{EL,Ve}, {ER,Ue}}}};
```

$$V_u^* M_u U_u^\dagger = m_u^{\text{diag}} \qquad V_d^* M_d U_d^\dagger = m_d^{\text{diag}} \qquad V_e^* M_\ell U_e^\dagger = m_\ell^{\text{diag}}$$

$$u_L = V_u^* U_L \qquad d_L = V_d^* D_L \qquad e_L = V_e^* E_L$$

$$u_R = U_u U_R \qquad d_R = U_d D_R \qquad e_R = U_e E_R$$

All the fermions previously defined are two-component Weyl spinors. We need to combine them to four-component Dirac spinors for SARAH. It is customary to set prefix "F".

```
(* Standard model *)
DEFINITION[EWSB][DiracSpinors]={
    Fd ->{  DL, conj[DR]},
    Fe ->{  EL, conj[ER]},
    Fu ->{  UL, conj[UR]},
    Fv ->{  vL, 0}};
```

$$d = \begin{pmatrix} D_L \\ D_R \end{pmatrix}, \quad e = \begin{pmatrix} E_L \\ E_R \end{pmatrix}, \quad u = \begin{pmatrix} u_L \\ u_R \end{pmatrix}, \quad \nu = \begin{pmatrix} \nu_L \\ 0 \end{pmatrix}$$

Most of the job is done: Parameter file

Before running the model file in SARAH, we should write all the defined parameters to `parameters.m` with short human-readable descriptions. It is advisable to edit a complete parameter file for a similar model.

```
(* Standard model, parameter file *)
ParameterDefinitions = {
{Yu,{Description -> "Up-Yukawa-Coupling",
     DependenceNum -> Sqrt[2]/v* {{Mass[Fu,1],0,0},
                                  {0, Mass[Fu,2],0},
                                  {0,0,Mass[Fu,3]}}}},
    ...
```

$$Y_u = \frac{\sqrt{2}}{v}\begin{pmatrix} M_{u1} & 0 & 0 \\ 0 & M_{u2} & 0 \\ 0 & 0 & M_{u3} \end{pmatrix}$$

For gauge eigenstates, really LaTeX-name is the only which should be written (for output PDF), since calculations are done in EWSB.

```
     (* Standard model particle file *)
     ParticleDefinitions[GaugeES] = {
     {H0,{ PDG -> {0},
           Width -> 0,
           Mass -> Automatic,
           FeynArtsNr -> 1,
           LaTeX -> "H^0",
           OutputName -> "H0" }},
...
     ParticleDefinitions[EWSB] = {
     {hh,{Description -> "Higgs",
          PDG -> {25}},
```

Features not in SM

Global symmetries

First, one writes all global symmetries and then all gauge symmetries.

```
(* 331-model *)
Global[[1]]={U[1], Lnumber};
Global[[2]]={Z[2], Parity};

Gauge[[1]]={B,  U[1],  xcharge, g1,False, 0, 1};
Gauge[[2]]={WB, SU[3], left,    g2,True,  0, 1};
Gauge[[3]]={G,  SU[3], color,   g3,False, 0, 1};
FermionFields[[1]] = {q1, 1, {dpL, dL, uL},
                     0, 3, 3, -2/3, 1};
```

- Boolean: SARAH will expand $SU(3)_L$ triplets.
- Gauge bosons have $L = 0$ and are even under $\mathbb{Z}_2$ (odd $= -1$).
- Other fields: gauge charges and global charges in order.

Scalar and Goldstone mixing

In a model with several scalar fields, SARAH needs to now how they mix. Let H_0 be the lower component of SM Higgs doublet and χ_0 a singlet scalar.

```
     (* Super-weak model *)
     DEFINITION[EWSB][VEVs] = {
          {H0,{vH, 1/Sqrt[2]}, {sigmaH,I/Sqrt[2]},
{h,1/Sqrt[2]}},
          {X0,{vX, 1/Sqrt[2]}, {sigmaX,I/Sqrt[2]},
{s,1/Sqrt[2]}}};

     DEFINITION[EWSB][MatterSector] = {
          {{h,s}, {hh, ZH}},
          {{sigmaH, sigmaX}, {Ah, ZA}},...
```

$$hh = \begin{pmatrix} hh_1 \\ hh_2 \end{pmatrix} = Z_H \begin{pmatrix} h \\ s \end{pmatrix}, \quad Ah = \begin{pmatrix} Ah_1 \\ Ah_2 \end{pmatrix} = Z_A \begin{pmatrix} \sigma_H \\ \sigma_X \end{pmatrix}$$

Gauge kinetic mixing

Consider super-weak model with $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$:

```
(* Super-weak model *)
Gauge[[1]] = {B,   U[1], hypercharge, gY,  False};
Gauge[[2]] = {WB, SU[2], left,        g2,  True};
Gauge[[3]] = {G,  SU[3], color,       g3,  False};
Gauge[[4]] = {Bp,  U[1], superweak,   gX, False};
```

Two gauged U(1)'s lead to kinetic mixing $\Rightarrow$ Zoltán Trócsányi's talk.

$$D_\mu \phi = \left(\partial_\mu + i(y_\phi, z_\phi) \begin{pmatrix} g_Y & 0 \\ 0 & g_X \end{pmatrix} \begin{pmatrix} B_\mu \\ B'_\mu \end{pmatrix} \right) \phi \equiv (\partial_\mu - i Q_\phi^T G_{\text{diag}} F_\mu) \phi$$

SARAH automatically rotates away the kinetic mixing term, $\mathcal{L} = -\frac{\varepsilon}{2} F_{\mu\nu} Z^{\mu\nu}$, and produces new gauge couplings, named automatically: $(g_Y, g_X) \mapsto (g_{YY}, g_{YX}, g_{XY}, g_{XX})$.

$\Rightarrow$ need to write them in parameter file.

$$D_\mu \phi = \left(\partial_\mu + i(y_\phi, z_\phi) \begin{pmatrix} g_Y & 0 \\ 0 & g_X \end{pmatrix} \begin{pmatrix} B_\mu \\ B'_\mu \end{pmatrix}\right) \phi \equiv (\partial_\mu - i Q_\phi^T G_{\text{diag}} F_\mu) \phi$$

$$\mapsto \left(\partial_\mu + i(y_\phi, z_\phi) \begin{pmatrix} g_{YY} & g_{YX} \\ g_{XY} & g_{XX} \end{pmatrix} \begin{pmatrix} A_\mu \\ A'_\mu \end{pmatrix}\right) \phi \equiv (\partial_\mu - i Q_\phi^T G F'_\mu) \phi$$

Beta functions are calculated to four $U(1)_X \otimes U(1)_Y$ gauge couplings.

BSM parameter file

If we wish to write down the transformed couplings with respect to the original parameters, we must add a dependence.

$$\mathcal{L}_{\text{k-mix}} = -\frac{1}{2}\sin\theta_\varepsilon B_{\mu\nu}Z^{\mu\nu}$$

$$\Rightarrow g_{YY} = g_Y, \quad g_{YX} = g_Y \tan\theta_\varepsilon, \quad g_{XY} = 0, \quad g_{XX} = g_X \sec\theta_\varepsilon$$

```
(* Super-weak model, parameter file *)
{gXX,{Description -> "U(1) X-X gauge coupling",
      Real -> True,
      OutputName -> gXX,
      LaTeX -> "g_{XX}",
      Dependence -> gX Sec[ThetaE]}},
...
```

All the new particles should be written in particle file with short description and LaTeX-name. Nothing else is really needed, since we are not going to generate input to other analysis tools in this demo.

```
(* Super-weak particle file *)
ParticleDefinitions[GaugeES] = {
  {X0,{Description -> "Neutral Extra Higgs",
       PDG -> {0},
       Width -> {0},
       Mass -> Automatic,
       FeynArtsNr -> 12,
       LaTeX -> "\\chi_0",
       OutputName -> X0}}
...
```

Let's demonstate SARAH with SM and U(1)-extended SM.