Neutrino mass models

ELFT Winter School

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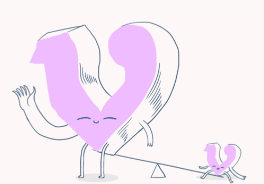
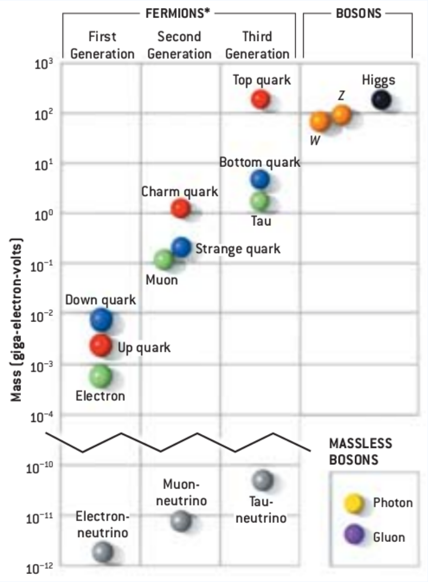
3.2.21



Type II seesaw mechanism



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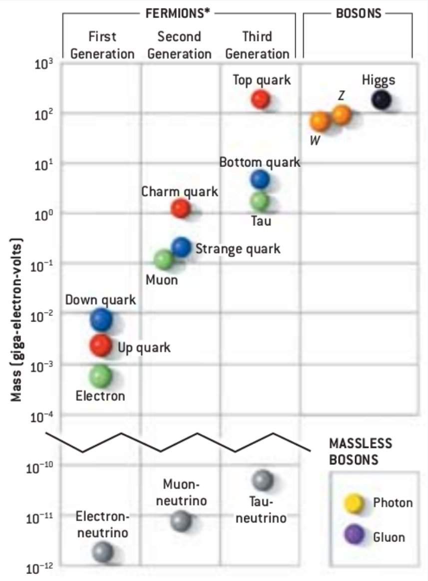

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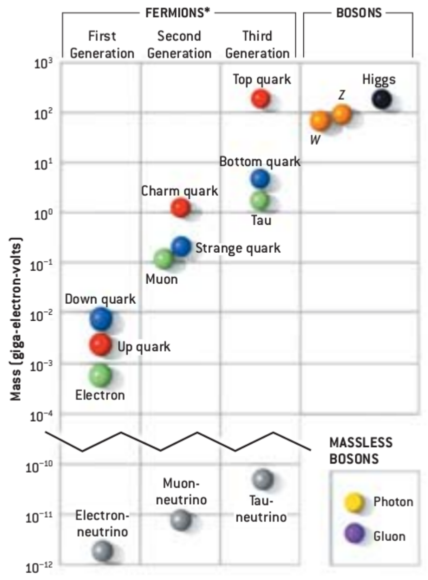

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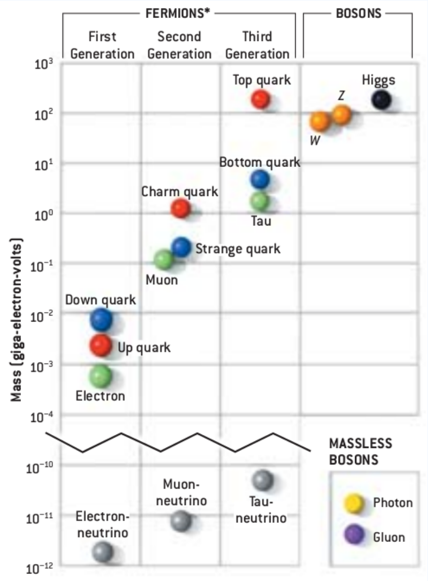

- Why neutrinos are so light? Maybe a different Higgs boson generates masses to neutrinos?
- Instead of sterile neutrinos, we may postulate the existence of a triplet scalar $\Delta = (\Delta_1, \Delta_2, \Delta_3)$.
- Neutrino mass is generated at tree-level.

[Magg, Wetterich, 1980; Schecter, Valle 1980; Cheng, Li, 1980; Mohapatra, Senjanović; 1980]

$$\mathcal{L}_{\text{scalar}} = (D_\mu H)^\dagger (D^\mu H) + \text{Tr}\left[(D_\mu \Delta)^\dagger (D^\mu \Delta)\right] + Y^\nu L_L^T C i\sigma_2 \Delta L_L - V(H, \Delta)$$

$$V(H,\Delta) = -m_H^2 H^\dagger H + \frac{\lambda}{4}(H^\dagger H)^2 + M_\Delta^2 \text{Tr}(\Delta^\dagger \Delta) + \left(\lambda_\phi\, H^T\, i\sigma_2\, \Delta^\dagger H + \text{h.c.}\right)$$

$$D_\mu \Delta = \partial_\mu \Delta + i g_2 \left[\boldsymbol{\tau} \cdot \mathbf{W}_\mu, \Delta\right] + i g_1 Y_\Delta B_\mu \Delta / 2$$

$$\Delta = \frac{1}{\sqrt{2}} \sigma_i \Delta_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \Delta_3 & \Delta_1 - i\Delta_2 \\ \Delta_1 + i\Delta_2 & -\Delta_3 \end{pmatrix} \equiv \begin{pmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{pmatrix}$$

Δ is represented here in a bidoublet form, so that the Lagrangian can be written in a compact way.

- Using Gell-Mann–Nishijima formula, we determine the electric charges of the new scalars.

$$Q\Delta = [\tau^3, \Delta] + \frac{Y}{2}\Delta = \begin{pmatrix} +\Delta_{11} & +2\Delta_{12} \\ 0 \cdot \Delta_{21} & +\Delta_{22} \end{pmatrix}$$

$$\Rightarrow \Delta = \frac{1}{\sqrt{2}} \begin{pmatrix} \Delta^{+} & \sqrt{2}\Delta^{++} \\ \sqrt{2}\Delta^{0} & -\Delta^{+} \end{pmatrix} \xmapsto{\text{VEV}} \begin{pmatrix} 0 & 0 \\ v' & 0 \end{pmatrix},$$

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  where v' is the vacuum expectation value (VEV) of Δ^0.

- The expression for v' can be derived by minimizing the effective Higgs potential:

$$\frac{\partial V(v, \Delta)}{\partial \Delta^{0*}} = 0 \Rightarrow \boxed{v' = \lambda_\phi \frac{v^2}{M_\Delta^2}}$$

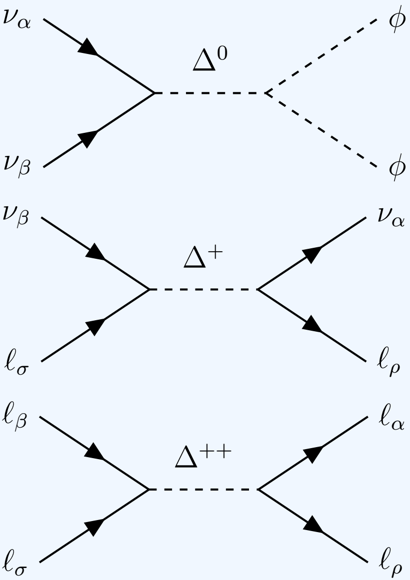
$$\mathcal{L} = Y_{\alpha\beta}\, L^T_{\alpha L}\, C\, i\sigma_2\, \Delta\, L_{\beta L} + \text{h.c.}$$

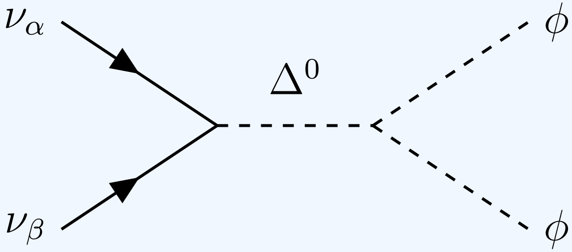
$$= Y_{\alpha\beta}\, \underbrace{\Big[\Delta^0\, \overline{(\nu_{\alpha L})^C}\, \nu_{\beta L}}_{\text{Majorana mass terms}}$$

$$\underbrace{-\frac{1}{\sqrt{2}}\Delta^+ \left(\overline{\ell^C_{\alpha R}}\, \nu_{\beta L} + \overline{\nu^C_{\alpha R}}\, \ell_{\beta L}\right)}_{\text{nonstandard interactions}}$$

$$\underbrace{-\Delta^{++}\, \overline{\ell^C_{\alpha R}}\, \ell_{\beta L}\Big]}_{\text{CLFV decays}} + \text{h.c.},$$

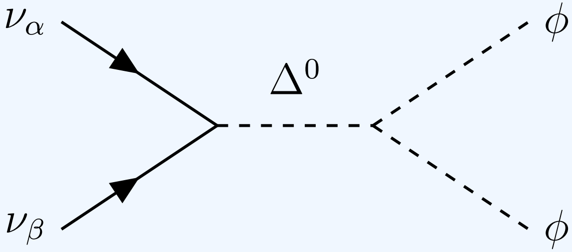
We obtain observable flavour physics!





- Use effective field theory to integrate out Δ^0, which must be heavy: $M_\Delta \gtrsim 750$ GeV [CMS collaboration, 2017]

$$\mathcal{L} = \frac{Y_{\alpha\beta}\,\lambda_\phi\,v^2}{M_\Delta^2}\left(\overline{(\nu_{L\alpha})^C}\,\nu_{\beta L}\right) \Rightarrow m_\nu = -\frac{Y\lambda_\phi v^2}{M_\Delta^2}$$



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- The negative sign can be removed via neutrino phase redefinition.

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  - Tiny trilinear coupling — can be motivated

$$m_\nu = {\scriptscriptstyle \lambda_\phi}\frac{Y v^2}{M_\Delta^2}$$

Naturalness criterion

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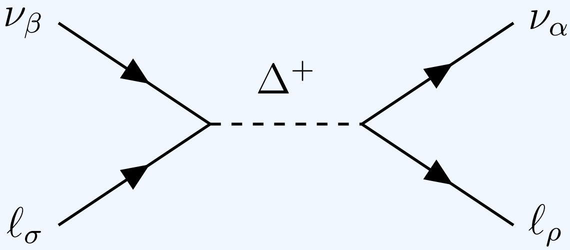
- A parameter α of a theory can be naturally small, if at the limit $\alpha \to 0$ the symmetry of the theory is enhanced. ['t Hooft, 1980]
- How we can enhance the symmetry? Consider how the lepton number L is broken in Type II seesaw.
- Consider the following terms from the full Lagrangian:

$$\mathcal{L} = -Y_{\alpha\beta}\Delta^0\,\overline{(\nu_{\alpha L})^C}\,\nu_{\beta L} - \lambda_\phi H^T i\sigma_2 \Delta^\dagger H + \text{h.c.}$$

The first term breaks L, but can be made L-conserving by assigning $L = -2$ to the triplet Δ. Then the second term breaks L, λ_ϕ can be naturally small, and therefore m_ν is naturally small.



- Use effective field theory to integrate out Δ^+, producing a dimension-6 operator

$$\mathcal{L}_{\mathrm{NSI}} = \frac{Y_{\sigma\beta}\, Y^\dagger_{\alpha\rho}}{M^2_\Delta} \left(\overline{\nu_{\alpha L}}\, \gamma_\mu\, \nu_{\beta L}\right) \left(\overline{\ell_{\rho L}}\, \gamma^\mu\, \ell_{\sigma L}\right)$$

Nonstandard interactions (NSI)

[Wolfenstein, 1978; Grossmann, 1995]

- The most general possible $V \pm A$-type NSI operators are

$$\mathcal{L}_{\text{NSI}}^{\text{CC}} = -2\sqrt{2}G_F \varepsilon_{\alpha\beta}^{ff',C}(\overline{\nu_{L\alpha}}\gamma^{\mu}\nu_{L\beta})(\bar{f}\gamma_{\mu}P_C f')$$
$$\mathcal{L}_{\text{NSI}}^{\text{NC}} = -2\sqrt{2}G_F \varepsilon_{\alpha\beta}^{f,C}(\overline{\nu_{L\alpha}}\gamma^{\mu}\nu_{L\beta})(\bar{f}\gamma_{\mu}P_C f),$$

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  where sum over chiralities ($C = L, R$), fermions (f, f') and flavours ($\alpha, \beta = e, \mu, \tau$) is implied.
- In general the complex parameters ε induce exotic decays and distort neutrino oscillation transition probabilites. For Type II seesaw, we have left-chiral couplings

$$\varepsilon_{\alpha\beta}^{\rho\sigma} = -\frac{M_\Delta^2}{2\sqrt{2}\, G_F\, v^4\, \lambda_\phi^2}\ (m_\nu)_{\sigma\beta}\ (m_\nu^\dagger)_{\alpha\rho}$$

  [Malinský, Ohlsson, Zhang, 2008]

- Neutrino flavour oscillations may be described with Schrödinger equation with Hamiltonian operator

$$H = \frac{1}{2E} U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger + \begin{pmatrix} V_{CC} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$+ \underbrace{\sum_{f=e,u,d} V_f \begin{pmatrix} \varepsilon_{ee}^{f} & \varepsilon_{e\mu}^{f} & \varepsilon_{e\tau}^{f} \\ \varepsilon_{e\mu}^{f*} & \varepsilon_{\mu\mu}^{f} & \varepsilon_{\mu\tau}^{f} \\ \varepsilon_{e\tau}^{f*} & \varepsilon_{\mu\tau}^{f*} & \varepsilon_{\tau\tau}^{f} \end{pmatrix}}_{\text{nonstandard interactions}}, \quad \varepsilon^f = \varepsilon^{f,L} + \varepsilon^{f,R}$$

where U is neutrino mixing matrix, $V_{CC} = \sqrt{2} G_F N_e(x)$ is the charged current background potential, and $N_e(x)$ electron density.

Correction to neutrino oscillations

- The total NSI contribution to neutrino matter potential is given by, assuming neutrality of matter,

$$\varepsilon^m_{ll'} \equiv \sum_f \varepsilon^f_{ll'} \frac{N_f}{N_e} = \varepsilon^e_{ll'} + 2\varepsilon^u_{ll'} + \varepsilon^d_{ll'} + \frac{N_n}{N_e}(\varepsilon^u_{ll'} + 2\varepsilon^d_{ll'})$$

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- From experiments, $|\varepsilon^{m}_{ll'}| \leq \mathcal{O}(0.1)$. For Type II seesaw, this gives a constraint

$$|\varepsilon^{m}_{ll'}| = |\varepsilon^{e}_{ll'}| = \left| -\frac{M_\Delta^2}{2\sqrt{2}\, G_F\, v^4\, \lambda_\phi^2}\, (m_\nu)_{el'}\, (m_\nu^\dagger)_{le} \right| \leq \mathcal{O}(0.1)$$

$\Rightarrow |\lambda_\phi| > 0.03$ eV, lower bound

[Huitu, Kärkkäinen, Maalampi, Vihonen, 2018]

Consider for example an NSI operator

$$\mathcal{L}' = -2\sqrt{2} G_F \varepsilon_{ee}^{e\mu} (\overline{\nu_e} \gamma^\nu P_L \nu_e)(\bar{e} \gamma_\nu \mu).$$

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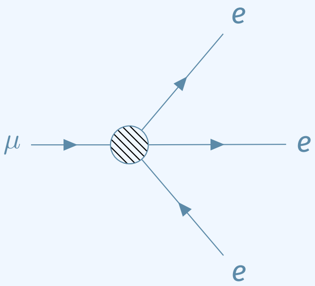
To retain gauge invariance, this operator must be part of a more general operator, namely

$$\mathcal{L} = -2\sqrt{2}G_F\varepsilon^{e\mu}_{ee}(\overline{L_e}\gamma^\nu P_L L_e)(\overline{L_e}\gamma_\nu L_\mu),$$

which indeed is present in Type II seesaw. The four-charged-lepton-operator then induces CLFV decays.



$$\mu^- \to 2e^- + e^+$$

CLFV decays constrain severely NSI

- For this decay, we have model-independently

$$\Gamma(\mu \to 3e) = \frac{m_\mu^5 G_F^2}{24\pi^3} |\varepsilon_{ee}^{e\mu}|^2$$
$$\mathrm{BR}(\mu \to 3e)_{\mathrm{exp}} \lesssim 10^{-12}$$
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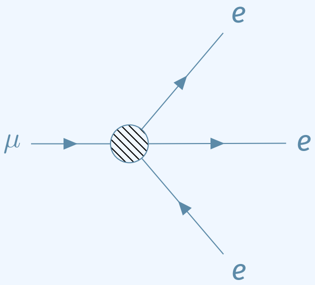

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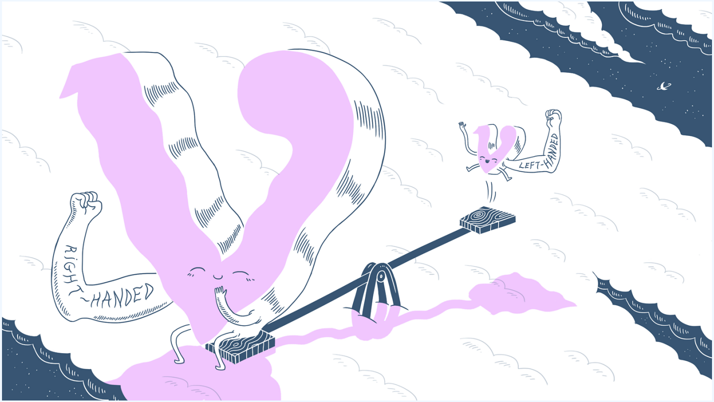
- $|\varepsilon_{ee}^{e\mu}| < 3.5 \times 10^{-7}$ [PDG 2020] is the most stringent constraint. Other CLFV NSI parameters have a few orders of magnitude more relaxed bounds.

Comparison between seesaws (Pic: Symmetry magazine)



Seesaw Type	Type I	Type II
Vacuum stability	No	Yes
Dark matter candidate	Yes	No
Enhanced Neutrino matter potential	No	Yes
Enhanced CLFV decays	No	Yes
PMNS unitarity	No	Yes

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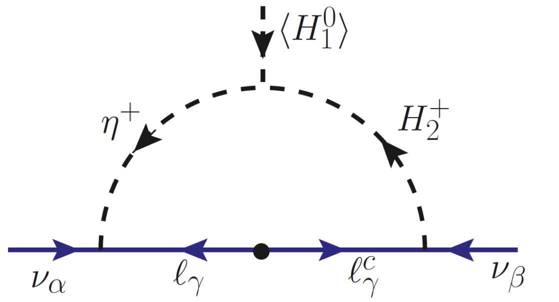

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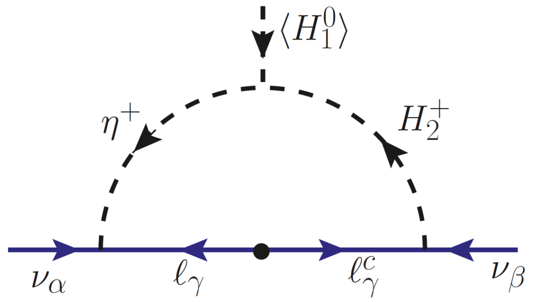

- In the SM, neutrinos are massless at all orders of perturbation theory.

Zee model [Zee, 1980]

$$P_{\odot}(\nu_e \to \nu_e)_{\text{Zee}} = \frac{1}{2}, \quad P_{\odot}(\nu_e \to \nu_e)_{\text{exp.}} \approx \frac{1}{3}$$

- Even though Zee model is ruled out [Frampton, Oh, Yoshikawa, 2001] by neutrino oscillation experiments, it is useful to look into the features of it, since
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- SM is extended only on scalar sector, which in Zee model consists of
  - two Higgs doublets $\phi_1, \phi_2 \sim (\mathbf{2}, \frac{1}{2})$ and
  - one charged scalar singlet $\eta^+ \sim (\mathbf{1}, 1)$.

Scalar sector of Zee model

- Vacuum expectation values are

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad \langle \eta^+ \rangle = 0.$$

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- We may however rotate the doublet of doublets to a new basis, where only one of the doublet develops a VEV:

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad v = \sqrt{v_1^2 + v_2^2}$$

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$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + H_1^0 + iG^0) \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^+ \\ \frac{1}{\sqrt{2}}(H_2^0 + iA) \end{pmatrix}$$

$\Rightarrow$ We gloss over many technicalities and intermediate steps to get to the point. More details in [Babu *et al.*, 2020].

Higgs potential contains a cubic term

$$V(H_1, H_2, \eta) = (\text{quadratic terms}) + (\text{quartic terms}) + (\mu \varepsilon_{ij} H_1^i H_2^j \eta^- + \text{h.c.})$$

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- Scalar content in the rotated basis:
  - Two Goldstone bosons (G^+, G^0).
  - Two neutral CP-even fields (H_1^0, H_2^0).
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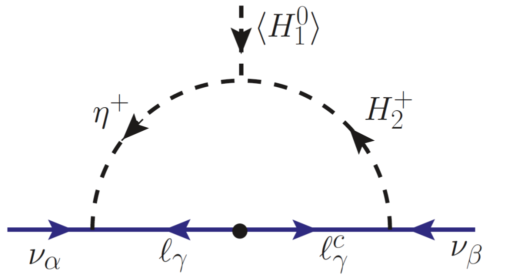
  - Two charged scalar fields (H_2^+, η^+).
- The scalar fields mix, and the mixing angle between charged scalars is given by

$$\sin 2\phi = \frac{-\sqrt{2} v \mu}{M_{H^+}^2 - M_{h^+}^2},$$

where H^+ and h^+ are the physical charged scalars, with $M_{H^+} > M_{h^+}^2$.



Here i, j are $SU(2)_L$ indices, α, β and flavour indices and $\phi' = i\sigma_2\phi^*$. The H_2 doublet is leptophilic and f is an antisymmetric matrix (Fermi statistics).

$$\mathcal{L}_{\text{Lepton Yukawa}} = -f_{\alpha\beta} L^i_\alpha L^j_\beta \varepsilon_{ij} \eta^+ - \tilde{Y}_{\alpha\beta} H'_{1i} L_{\alpha j} \ell^c_\beta \varepsilon_{ij} - Y_{\alpha\beta} H'_{2i} L_{\alpha j} \ell^c_\beta \varepsilon_{ij} + \text{h.c.}$$

$$V(H_1, H_2, \eta) = \mu \varepsilon_{ij} H^i_1 H^j_2 \eta^- + \text{h.c.}) + \cdots$$

- Charged lepton masses are given by diagonal mass matrix,

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After the dust settles...

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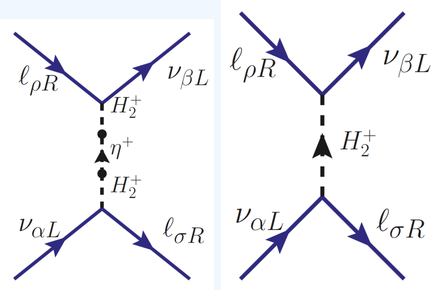
- Many free parameters:

$$f = \begin{pmatrix} 0 & f_{e\mu} & f_{e\tau} \\ -f_{e\mu} & 0 & f_{\mu\tau} \\ -f_{e\tau} & -f_{\mu\tau} & 0 \end{pmatrix}, Y = \begin{pmatrix} Y_{ee} & Y_{e\mu} & Y_{e\tau} \\ Y_{\mu e} & Y_{\mu\mu} & Y_{\mu\tau} \\ Y_{\tau e} & Y_{\tau\mu} & Y_{\tau\tau} \end{pmatrix}, \mu, M_{h^+}, M_{H^+}$$

Nonstandard interactions in Zee model



$$\mathcal{L}^{(h^+)}_{\text{NSI?}} = \sin^2\phi \frac{Y_{\alpha\rho} Y^*_{\beta\sigma}}{M^2_{h^+}} (\overline{\nu}_{\alpha L} \ell_{\rho R})(\overline{\ell}_{\sigma R} \nu_{\beta L})$$

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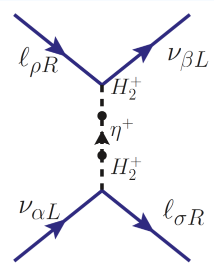

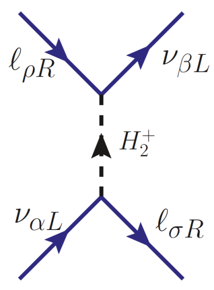

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Problem! This operator has scalar and pseudoscalar currents, but NSI has $(V-A)(V \pm A)$ current structure.

Five types of four-fermion operators: Scalar, Vector, Tensor, Axial vector and Pseudoscalar.

$$\mathcal{L}^S(\psi_1,\psi_2,\psi_3,\psi_4) = (\overline{\psi}_1\psi_2)(\overline{\psi}_3\psi_4)$$
$$\mathcal{L}^V(\psi_1,\psi_2,\psi_3,\psi_4) = (\overline{\psi}_1\gamma^\mu\psi_2)(\overline{\psi}_3\gamma_\mu\psi_4)$$
$$\mathcal{L}^T(\psi_1,\psi_2,\psi_3,\psi_4) = (\overline{\psi}_1\sigma^{\mu\nu}\psi_2)(\overline{\psi}_3\sigma_{\mu\nu}\psi_4)$$
$$\mathcal{L}^A(\psi_1,\psi_2,\psi_3,\psi_4) = (\overline{\psi}_1\gamma^\mu\gamma^5\psi_2)(\overline{\psi}_3\gamma_\mu\gamma^5\psi_4)$$
$$\mathcal{L}^P(\psi_1,\psi_2,\psi_3,\psi_4) = (\overline{\psi}_1\gamma^5\psi_2)(\overline{\psi}_3\gamma^5\psi_4)$$

$$C = \begin{pmatrix} -\frac{1}{4} & \frac{1}{4} & -\frac{1}{8} & \frac{1}{4} & -\frac{1}{4} \\ -1 & \frac{1}{2} & 0 & \frac{1}{2} & 1 \\ -3 & 0 & \frac{1}{2} & 0 & -3 \\ 1 & \frac{1}{2} & 0 & \frac{1}{2} & -1 \\ -\frac{1}{4} & \frac{1}{4} & -\frac{1}{8} & -\frac{1}{4} & -\frac{1}{4} \end{pmatrix}$$

$$\boxed{\mathcal{L}^X(\psi_1,\psi_2,\psi_3,\psi_4) = \sum_{Y=SVTAP} C_{XY}\mathcal{L}^Y(\psi_1,\psi_4,\psi_3,\psi_2)}$$

Fierz transform

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NSI parameters in Zee model

$$(\textit{Exercise}) \Rightarrow \mathcal{L}_{\text{NSI}}^{(h^+)} = -\frac{1}{2}\sin^2\phi \frac{Y_{\alpha\rho}Y^*_{\beta\sigma}}{M^2_{h^+}}(\overline{\nu}_\alpha\gamma^\mu P_L \nu_\beta)(\bar{\ell}_\sigma\gamma_\mu P_R \ell_\rho)$$

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Similarly,

$$\mathcal{L}_{\text{NSI}}^{(H^+)} = -\cos^2\phi \frac{Y_{\alpha\rho}Y^*_{\beta\sigma}}{2M^2_{H^+}}(\overline{\nu}_\alpha\gamma^\mu P_L\nu_\beta)(\overline{\ell}_\sigma\gamma_\mu P_R\ell_\rho)$$

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Finally, we get the total matter NSI in Zee model

$$\varepsilon^m_{\alpha\beta} \equiv \varepsilon^{ee,(h^+)}_{\alpha\beta} + \varepsilon^{ee,(H^+)}_{\alpha\beta} = \frac{Y_{\alpha e}Y^*_{\beta e}}{4\sqrt{2}G_F}\left(\frac{\sin^2\phi}{M^2_{h^+}} + \frac{\cos^2\phi}{M^2_{H^+}}\right)$$

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- Nonstandard interactions may distort neutrino oscillation transition probabilities and contribute to CLFV.

  Simplest loop-generated mass model is ruled out, but there are two- and three-loop models.

Thank you for you attention!

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